Stable Design of a Class of Nonlinear Discrete-Time MIMO Fuzzy Control Systems

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Abstract: This paper presents a new stability analysis approach dedicated to a class of nonlinear discrete-time multi input-multi output (MIMO) Takagi-Sugeno fuzzy control systems (FCSs). The theorem presented in this paper offers sufficient conditions for the global stability of the FCSs. The applicability of the theoretical results is illustrated by the stable design of Takagi-Sugeno fuzzy controllers for the level control of spherical three tank systems as nonlinear MIMO processes. Digital simulation results are included.

Keywords: eigenvalues; MIMO fuzzy control systems; stability analysis; Takagi-Sugeno fuzzy controllers; three tank systems

1 Introduction

The stable design of fuzzy control systems (FCSs) is important because it contributes to the fulfilment of very good performance. Many popular stability analysis solutions concerning Takagi-Sugeno (T-S) FCSs are offered in this context, and their usual formulation is done in the linear matrix inequality (LMI) framework. The main features of these solutions are:

- The linearization can result in uncertainties and inaccuracies of fuzzy models.
- The quadratic Lyapunov functions may lead usually to conservative stability conditions.

- Although the LMIs are computationally solvable, they require numerical algorithms implemented by software tools.

Some approaches to the stability analysis of multi input-multi output (MIMO) T-S FCSs have been reported recently in the literature. Based on a novel fuzzy Lyapunov-Krasovskii functional, a stability analysis and stabilization for a class of discrete-time Takagi-Sugeno fuzzy systems is developed in [1]. A useful property of the staircase membership functions and a set of linear-matrix-inequality (LMI), the stability conditions for fuzzy control systems are offered in [2]-[4]. Sufficient conditions for the exponential stability of type-1 and type-2 T-S FCSs are given in [5]-[7] in fuzzy positive systems formulations. Fuzzy control design based on adaptive control schemes are proposed in [8]-[11].

The new contribution of this paper with respect to the state of the art is a stability analysis theorem dedicated to nonlinear MIMO processes controlled by T-S fuzzy controllers (FCs). Our original proof of the stability analysis theorem is based on the eigenvalues of the matrices of quadratic forms. Since these matrices are actually vector functions of vector arguments, their eigenvalues are functions of state variables. Similar approaches but with different stability formulations and proofs are reported in [12]-[15].

The specific features of the stability analysis theorem proposed in this paper concern the avoidance of both process linearization and the LMIs in the derivation and proof of the stability conditions because there is no need to calculate common positive definite matrices. Those are the reasons why the suggested approach proves to be advantageous with respect to LMI-based stability analysis solutions. Furthermore, the stability analysis method is formulated here so as to be well suited for T-S FC designs dedicated to a wide class of nonlinear processes [16]-[26].

This paper is organized as follows. Section 2 defines the structure of T-S FCSs which control a class of nonlinear MIMO processes. Section 3 gives the stability theorem for discrete-time MIMO FCSs. A case study presented in Section 4 offers the stable design of T-S FCSs dedicated to the level control of spherical three tank systems and digital simulation results. The conclusions are discussed in Section 5.

2 Fuzzy Control System Structure

The MIMO FCS structure is presented in Figure 1. Let $X \subset R^n$ ($n \in N$, $n > 0$) be the universe of discourse. The nonlinear MIMO process is characterized by the discrete-time input affine state-space model

$$\begin{aligned} &\mathbf{x}(t+1) = \mathbf{f}(\mathbf{x}(t)) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}(t),\ t \in N,\ \mathbf{x}(0) = \mathbf{x}_0 \in X, \\ &\mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t)). \end{aligned} \tag{1}$$

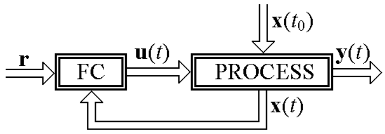

Figure 1
MIMO FCS structure

Figure 1 illustrates: $\mathbf{r}$ – the reference input vector which is constant for stabilizing control systems, $\mathbf{y}$ – the controlled output vector, $\mathbf{x} \in X$ – the state vector, $\mathbf{x}(t) = [x_1(t) \quad x_2(t) \quad \dots \quad x_n(t)]^T \in X$; the superscript T stands for matrix transposition, t is the time variable (with the initial time moment $t_0 = 0$), $\mathbf{x}_0$ is the initial condition vector, $\mathbf{f} : R^n \to R,\ \ \mathbf{B} : R^n \to R^{n \times m}$ – the continuous vector-valued functions which describe the dynamics of the process,

$$\mathbf{f}(\mathbf{x}(t)) = \begin{bmatrix} f_1(\mathbf{x}(t)) \\ f_2(\mathbf{x}(t)) \\ \dots \\ f_n(\mathbf{x}(t)) \end{bmatrix},\ f_i : R^n \to R,\ i = 1...n,$$
$$\mathbf{B}(\mathbf{x}(t)) = \begin{bmatrix} \mathbf{b}_1^T(\mathbf{x}(t)) \\ \mathbf{b}_2^T(\mathbf{x}(t)) \\ \dots \\ \mathbf{b}_n^T(\mathbf{x}(t)) \end{bmatrix},\ \mathbf{b}_i^T(\mathbf{x}(t)) = [b_{i1}(\mathbf{x}(t)) \quad b_{i2}(\mathbf{x}(t)) \quad \dots \quad b_{im}(\mathbf{x}(t))], \tag{2}$$

and $\mathbf{u}(t) = [u_1(t) \quad u_2(t) \quad \dots \quad u_m(t)]^T$ – the control signal vector applied to the process. The actuators and measuring instrumentation are included in the nonlinear process.

The i^{th} fuzzy control rule in the rule base of the T-S FC, referred to as $\mathrm{R}^i, i = 1...n_{RB}, n_{RB} \geq 2$ (n_{RB} – the number of rules), is expressed as

$$\begin{aligned} &\mathrm{R}^i : \text{IF } x_1(t) \text{ IS } \tilde{X}_{1i} \text{ AND } x_2(t) \text{ IS } \tilde{X}_{2i} \text{ AND} \dots \text{AND } x_n(t) \text{ IS } \tilde{X}_{ni} \\ &\qquad \text{THEN } \mathbf{u} = \mathbf{u}^i(\mathbf{x}(t)),\ i = 1...n_{RB}, \end{aligned} \tag{3}$$

where $\tilde{X}_{ki}$ are fuzzy sets with the universes X_{ki}, $k = 1...n$, corresponding to the linguistic terms (LTs) afferent to the state variables x_i, $u^i(\mathbf{x})$ is the control signal produced by the rule R^i with the firing strength $\alpha^i = \alpha^i(\mathbf{x})$

$$\begin{aligned} &\alpha^i(\mathbf{x}) = \text{AND}(\mu_{\tilde{X}_{1i}}(x_1), \mu_{\tilde{X}_{2i}}(x_2), \dots, \mu_{\tilde{X}_{ni}}(x_n)), \forall \mathbf{x} \in X\ \ \exists i = 1...n_{RB}, \\ &0 < \alpha^i(\mathbf{x}) \leq 1, \end{aligned} \tag{4}$$

where the function AND is a t-norm, and $\mu_{\tilde{X}_{ki}}$ are the membership functions of the fuzzy sets of LTs $\tilde{X}_{ki}$. An active region of the rule R^i is defined as the set $X_i^A = \{\mathbf{x} \in X \mid \alpha^i(\mathbf{x}) \neq 0\}$.

The control signal vector $\mathbf{u}$ is a function of α^i and $\mathbf{u}^i$ which depends on the inference engine and on the defuzzification method. The weighted sum defuzzification method produces the control signal vector $\mathbf{u}(\mathbf{x}(t))$, which will also be referred to as $\mathbf{u}(t)$ in the sequel for the sake of simplicity:

$$\mathbf{u}(\mathbf{x}(t)) = \frac{\sum_{i=1}^{n_{RB}} \alpha^i(\mathbf{x}(t))\mathbf{u}^i(\mathbf{x}(t))}{\sum_{i=1}^{n_{RB}} \alpha^i(\mathbf{x}(t))}. \tag{5}$$

3 Stability Analysis Theorem

Let the process be characterized by the state-space model defined in (1), and let V be a radially unbounded function $V: X \to R,\ V(\mathbf{x}) > 0, \forall \mathbf{x} \in X, \mathbf{x} \neq \mathbf{0}$. The first difference of the function $V(\mathbf{x}(t))$ along the trajectory of (1), denoted by $\Delta V(\mathbf{x}(t))$, is

$$\Delta V(\mathbf{x}(t)) = V(\mathbf{x}(t+1)) - V(\mathbf{x}(t)). \tag{6}$$

Using the notation $V_i(\mathbf{x}(t))$ for the Lyapunov function candidate $V(\mathbf{x}(t))$, which is considered along the trajectory of the system (1) for $\mathbf{u}(t) = \mathbf{u}^i(\mathbf{x}(t))$, the first difference of $V_i(\mathbf{x}(t))$ is $\Delta V_i(\mathbf{x}(t))$:

$$\Delta V_i(\mathbf{x}(t)) = V_i(\mathbf{x}(t+1)) - V_i(\mathbf{x}(t)), \forall\, \mathbf{x} \in X_i^A. \tag{7}$$

The following original stability analysis theorem is derived on the basis of Lyapunov's theorem for discrete-time systems using the formulation given in [27]:

Theorem 1. Let the FCS be described by the discrete-time input affine MIMO system modelled in (1), the T-S FC characterized by equations (3)–(7), and $\mathbf{x} = \mathbf{0}$ be an equilibrium point of (1). If there exists

$$V: X \to R,\ V(\mathbf{x}(t)) = \mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t), \text{ continuous in } \mathbf{x}, \tag{8}$$

where $\mathbf{P} \in R^{n \times n}$ is a positive definite matrix such that

$$\Delta V_i(\mathbf{x}(t)) \leq 0,\ \forall\, \mathbf{x} \in X_i^A,\ i = 1...n_{RB}, \tag{9}$$

then $\mathbf{x} = \mathbf{0}$ is stable.

Proof. The hypotheses of the theorem result in

$$\Delta V_i(\mathbf{x}(t)) = V_i(\mathbf{x}(t+1)) - V_i(\mathbf{x}(t)) < 0,\ \forall\, \mathbf{x} \in X_i^A,\ i = 1...n_{RB}. \tag{10}$$

The term $\mathbf{x}(t+1)$ is next substituted from (1) into (10):

$$\begin{aligned}
&\Delta V_i(\mathbf{x}(t)) = V_i(\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)) - V_i(\mathbf{x}(t)) \\
&= [\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)]^T\ \mathbf{P}\,[\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)] \\
&- \mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t) = [\mathbf{f}^T(\mathbf{x}(t)) + (\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))]\,\mathbf{P}\,[\mathbf{f}(\mathbf{x}(t)) \\
&+ \mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)] - \mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t) = [\mathbf{f}^T(\mathbf{x}(t))\mathbf{P} \\
&+ (\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}][\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)] - \mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t) \\
&= \mathbf{f}^T(\mathbf{x}(t))\mathbf{P}\mathbf{f}(\mathbf{x}(t)) + \mathbf{f}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t) \\
&+ (\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{f}(\mathbf{x}(t)) + (\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t) \\
&- \mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t) < 0,\ i = 1...n_{RB}.
\end{aligned} \tag{11}$$

The multiplication of (11) by $\alpha^i(\mathbf{x}(t))$ and the calculation of the sum result in

$$\begin{aligned}
&[\mathbf{f}^T(\mathbf{x}(t))\mathbf{P}\mathbf{f}(\mathbf{x}(t))]\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t)) + [\mathbf{f}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))]\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))\mathbf{u}^i(t) \\
&+ \sum_{i=1}^{n_{RB}}[(\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{f}(\mathbf{x}(t))\alpha^i(\mathbf{x}(t))] \\
&+ \sum_{i=1}^{n_{RB}}[(\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)\alpha^i(\mathbf{x}(t))] \\
&- [\mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t)]\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t)) < 0.
\end{aligned} \tag{12}$$

Equation (12) is divided by $\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t)) > 0$ and equation (5) is applied to transform the resulted sums as follows:

$$\begin{aligned}
&\mathbf{f}^T(\mathbf{x}(t))\mathbf{P}\mathbf{f}(\mathbf{x}(t)) + \mathbf{f}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}(t) + \mathbf{u}^T(t)\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{f}(\mathbf{x}(t)) \\
&+ \frac{\sum_{i=1}^{n_{RB}}[(\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)\alpha^i(\mathbf{x}(t))]}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))} - \mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t) < 0.
\end{aligned} \tag{13}$$

The expression of $\Delta V(\mathbf{x}(t))$ results from (1) and (6):

$$\begin{aligned}
&\Delta V(\mathbf{x}(t)) = V(\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}(t)) - V(\mathbf{x}(t)) \\
&= [\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}(t)]^T\ \mathbf{P}\,[\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}(t)] - \mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t) \\
&= [\mathbf{f}^T(\mathbf{x}(t))\mathbf{P} + \mathbf{u}^T(t)\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}][\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}(t)] - \mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t) \\
&= \mathbf{f}^T(\mathbf{x}(t))\mathbf{P}\mathbf{f}(\mathbf{x}(t)) + \mathbf{f}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}(t) + \mathbf{u}^T(t)\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{f}(\mathbf{x}(t)) \\
&+ \mathbf{u}^T(t)\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}(t) - \mathbf{x}^T(t)\,\mathbf{P}\,\mathbf{x}(t) < 0.
\end{aligned} \tag{14}$$

In the following we prove that

$$\mathbf{u}^T(t)\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}(t) \le \frac{\sum_{i=1}^{n_{RB}}[(\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)\alpha^i(\mathbf{x}(t))]}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))}. \quad (15)$$

The terms $\mathbf{u}^T(t)\mathbf{B}^T(\mathbf{x}(t))\,\mathbf{P}\,\mathbf{B}(\mathbf{x}(t))\mathbf{u}(t)$ and $(\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\,\mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)$ are quadratic forms because the matrix

$$\mathbf{M}(\mathbf{x}(t)) = \mathbf{B}^T(\mathbf{x}(t))\,\mathbf{P}\,\mathbf{B}(\mathbf{x}(t)) \in R^{m\times m} \quad (16)$$

is symmetric. The matrix $\mathbf{M}(\mathbf{x}(t))$ has the following spectral decomposition (Jordan decomposition):

$$\mathbf{M}(\mathbf{x}(t)) = \mathbf{\Gamma}(\mathbf{x}(t))\,\mathbf{\Lambda}(\mathbf{x}(t))\,\mathbf{\Gamma}^T(\mathbf{x}(t)) = \sum_{i=1}^{m}[\lambda_i(\mathbf{x}(t))\boldsymbol{\gamma}_i(\mathbf{x}(t))\boldsymbol{\gamma}_i^T(\mathbf{x}(t))], \quad (17)$$

where

$$\mathbf{\Lambda}(\mathbf{x}(t)) = \mathrm{diag}(\lambda_1(\mathbf{x}(t)),\lambda_2(\mathbf{x}(t)),...,\lambda_m(\mathbf{x}(t))), \quad (18)$$

$\lambda_j(\mathbf{x}(t)),\ j=1...m,$ are the eigenvalues of $\mathbf{M}(\mathbf{x}(t))$. The orthogonal matrix

$$\mathbf{\Gamma}(\mathbf{x}(t)) = [\boldsymbol{\gamma}_1(\mathbf{x}(t))\quad \boldsymbol{\gamma}_2(\mathbf{x}(t))\quad ...\quad \boldsymbol{\gamma}_m(\mathbf{x}(t))] \quad (19)$$

consists of the eigenvectors $\boldsymbol{\gamma}_i(\mathbf{x}(t))$ of $\mathbf{M}(\mathbf{x}(t))$.

Considering the linear transformation

$$\mathbf{u} \mapsto \Gamma^T\mathbf{u} = \mathbf{w} = [w_1\quad w_2\quad ...\quad w_m]^T,\ w_j = \boldsymbol{\gamma}_j^T\mathbf{u},\ j=1...m, \quad (20)$$

equations (16), (17) and (20) lead to

$$\begin{aligned}&\mathbf{u}^T(t)\,\mathbf{B}^T(\mathbf{x}(t))\,\mathbf{P}\,\mathbf{B}(\mathbf{x}(t))\,\mathbf{u}(t) = \mathbf{u}^T(t)\,\mathbf{M}(\mathbf{x}(t))\,\mathbf{u}(t)\\ &= \mathbf{u}^T(t)\,\mathbf{\Gamma}(\mathbf{x}(t))\,\mathbf{\Lambda}(\mathbf{x}(t))\,\mathbf{\Gamma}^T(\mathbf{x}(t))\,\mathbf{u}(t) = (\mathbf{w}(t))^T\,\mathbf{\Lambda}(\mathbf{x}(t))\,\mathbf{w}(t)\\ &= \sum_{j=1}^{m}[\lambda_j(\mathbf{x}(t))w_j^2(\mathbf{x}(t))].\end{aligned} \quad (21)$$

The expression of w_j from (20) is substituted into (21) leading to the following result:

$$\mathbf{u}^T(t)\,\mathbf{B}^T(\mathbf{x}(t))\,\mathbf{P}\,\mathbf{B}(\mathbf{x}(t))\,\mathbf{u}(t) = \sum_{j=1}^{m}\{\lambda_j(\mathbf{x}(t))[\boldsymbol{\gamma}_j^T(\mathbf{x}(t))\mathbf{u}(t)]^2\}\cdot \quad (22)$$

The following relationship results from (22) by replacing $\mathbf{u}(t)$ with $\mathbf{u}^i(t)$:

$$(\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\,\mathbf{P}\,\mathbf{B}(\mathbf{x}(t))\,\mathbf{u}^i(t) = \sum_{j=1}^{m}\{\lambda_j(\mathbf{x}(t))[\boldsymbol{\gamma}_j^T(\mathbf{x}(t))\mathbf{u}^i(t)]^2\}\cdot \quad (23)$$

The expression of $\mathbf{u}(t)$ is substituted from (5) into the right-hand side of (22), and the sums are manipulated as follows:

$$\mathbf{u}^T(t)\,\mathbf{B}^T(\mathbf{x}(t))\,\mathbf{P}\,\mathbf{B}(\mathbf{x}(t))\,\mathbf{u}(t) = \sum_{j=1}^{m}\{\lambda_j(\mathbf{x}(t))[\boldsymbol{\gamma}_j^T(\mathbf{x}(t))\left(\frac{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))\mathbf{u}^i(t)}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))}\right)]^2\} \quad (24)$$

$$= \sum_{j=1}^{m}\{\frac{\lambda_j(\mathbf{x}(t))}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))}[\frac{\left(\sum_{i=1}^{n_{RB}}[\alpha^i(\mathbf{x}(t))\boldsymbol{\gamma}_j^T(\mathbf{x}(t))\mathbf{u}^i(t)]\right)^2}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))}]\}\,.$$

The application of Cauchy-Buniakovski-Schwarz's inequality to the second fraction in the right-hand side of (24) leads to

$$\mathbf{u}^T(t)\,\mathbf{B}^T(\mathbf{x}(t))\,\mathbf{P}\,\mathbf{B}(\mathbf{x}(t))\,\mathbf{u}(t) \le \sum_{j=1}^{m}\left\{\frac{\lambda_j(\mathbf{x}(t))}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))}\sum_{i=1}^{n_{RB}}\{\alpha^i(\mathbf{x}(t))[\boldsymbol{\gamma}_j^T(\mathbf{x}(t))\mathbf{u}^i(t)]^2\}\right\}, \quad (25)$$

and next to

$$\mathbf{u}^T(t)\,\mathbf{B}^T(\mathbf{x}(t))\,\mathbf{P}\,\mathbf{B}(\mathbf{x}(t))\,\mathbf{u}(t) \le \frac{\sum_{j=1}^{m}\sum_{i=1}^{n_{RB}}\{\lambda_j(\mathbf{x}(t))[\boldsymbol{\gamma}_j^T(\mathbf{x}(t))\mathbf{u}^i(t)]^2\alpha^i(\mathbf{x}(t))\}}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))}. \quad (26)$$

The multiplication of (23) by $\alpha^i(\mathbf{x}(t))$, the calculation of the sum and the division by $\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t)) > 0$ result in the expression of the right-hand side of (15):

$$\frac{\sum_{i=1}^{n_{RB}}[(\mathbf{u}^i(t))^T\mathbf{B}^T(\mathbf{x}(t))\mathbf{P}\mathbf{B}(\mathbf{x}(t))\mathbf{u}^i(t)\alpha^i(\mathbf{x}(t))]}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))} = \frac{\sum_{i=1}^{n_{RB}}\sum_{j=1}^{m}\{\lambda_j(\mathbf{x}(t))[\boldsymbol{\gamma}_j^T(\mathbf{x}(t))\mathbf{u}^i(t)]^2\alpha^i(\mathbf{x}(t))\}}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))}$$

$$= \frac{\sum_{j=1}^{m}\sum_{i=1}^{n_{RB}}\{\lambda_j(\mathbf{x}(t))[\boldsymbol{\gamma}_j^T(\mathbf{x}(t))\mathbf{u}^i(t)]^2\alpha^i(\mathbf{x}(t))\}}{\sum_{i=1}^{n_{RB}}\alpha^i(\mathbf{x}(t))}. \quad (27)$$

Therefore equations (26) and (27) demonstrate the inequality (15). The inequality (15) is applied to (13) and (14), which result finally in

$$\Delta V(\mathbf{x}(t)) \leq 0 . \quad (28)$$

Therefore, the equilibrium point at the origin will be stable. The proof is now complete. Concluding, Theorem 1 offers sufficient stability conditions concerning the class of fuzzy control systems defined in Section 2.

4 Case Study

The case study applies Theorem 1 to the design of T-S FCSs dedicated to the level control of spherical three tank systems. The process structure presented in Figure 2 illustrates the three spherical tanks, T1, T2 and T3, with the same radius R, in series connection by two connecting pipes of inner area S. All three tanks are equipped with piezo-resistive pressure sensors (viz. the level sensors LS1, LS2 and LS3) to measure the liquid levels. The FC actuates (by means of the pumps P1 and P2) the flow rates q_{p1} and q_{p2} in order to control independently the levels in the tanks T1 (h_1) and T2 (h_2), and the following constraints imposed to the levels result from the process structure:

$$0 < h_i < 2R, \ i = 1...3 . \quad (29)$$

A typical control objective pointed out in [13] is to keep the liquid levels h_1 and h_2 at the imposed levels while the liquid level in the tank T3 (h_3) is uncontrollable. The level sensors give the measured levels h_{m1}, h_{m2} and h_{m3} used by the FC. The connecting pipes and tanks are equipped with manually adjustable valves and outlets to simulate clogs and leaks.

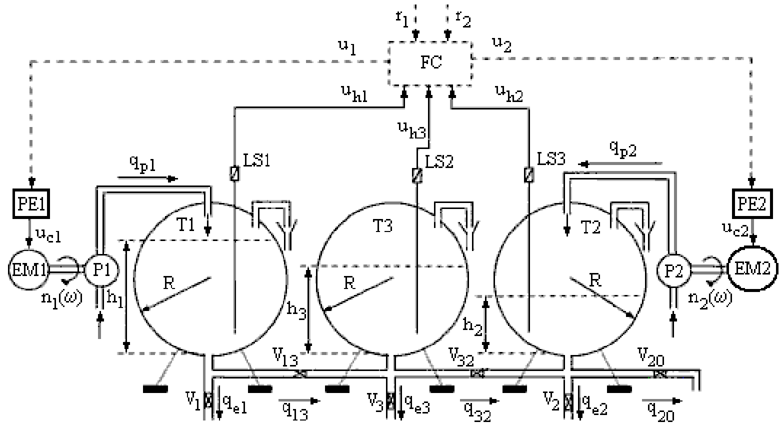

Figure 2
FCS and process structure

The simplified FCS structure is presented in Figure 3, where $\mathbf{d}=[d_{e1}=1-\mu_{e1}\quad d_{e2}=1-\mu_{e2}\quad d_{e3}=1-\mu_{e3}]^T$ is the disturbance input vector, $\mathbf{r}=[r_1\quad r_2]^T$ is the reference input vector, e_i

$$e_i = r_i - u_{hi} = r_i - k_{mi}h_i,\ i=1...2, \tag{30}$$

are the control errors grouped in the control error vector $\mathbf{e}=[e_1\quad e_2]^T$, μ_{e1}, μ_{e2} and μ_{e3} are the deterministic disturbance inputs are the positions of the valves V_1, V_2 and V_3, $0\le\mu_{e1},\mu_{e2},\mu_{e3}\le 1$, with the notations 0 for the completely close valves and 1 for the completely open valves, and k_{mi}, $i=1...3$, are the sensor gains.

Using the notation $A(h_i)=\pi h_i(2R-h_i)$, $i=1...3$, for the transversal section area of sphere (i.e., tank) i at height (liquid level) h_i, the first principle mathematical model of the process proposed in [13] is discretized, and the following disturbed discrete-time process model is used in T-S FC design:

$$\begin{aligned}
&e_1(t+1)=\frac{k_{m1}(t)}{A(u_{h1}(t)/k_{m1}(t))}[S\operatorname{sgn}(\frac{r_1-e_1(t)}{k_{m1}(t)}-\frac{u_{h3}(t)}{k_{m3}(t)})\\
&\sqrt{2g\,|\frac{r_1-e_1(t)}{k_{m1}(t)}-\frac{u_{h3}(t)}{k_{m3}(t)}|}+d_{e1}S_V\sqrt{2g\frac{r_1-e_1(t)}{k_{m1}(t)}}-k_{p1}(t)u_1(t)],\\
&e_2(t+1)=\frac{k_{m2}(t)}{A(u_{h2}(t)/k_{m2}(t))}[-S\operatorname{sgn}(\frac{u_{h3}(t)}{k_{m3}(t)}-\frac{r_2-e_2(t)}{k_{m2}(t)})\\
&\sqrt{2g\,|\frac{u_{h3}(t)}{k_{m3}(t)}-\frac{r_2-e_2(t)}{k_{m2}(t)}|}+d_{e2}S_V\sqrt{2g\frac{r_2-e_2(t)}{k_{m2}(t)}}\\
&+S_V\sqrt{2g\frac{r_2-e_2(t)}{k_{m2}(t)}}-k_{p2}(t)u_2(t)],\\
&u_{h3}(t+1)=\frac{k_{m3}(t)}{A(u_{h3}(t)/k_{m3}(t))}[S\operatorname{sgn}(\frac{u_{h1}(t)}{k_{m1}(t)}-\frac{u_{h3}(t)}{k_{m3}(t)})\sqrt{2g\,|\frac{u_{h1}(t)}{k_{m1}(t)}-\frac{u_{h3}(t)}{k_{m3}(t)}|}\\
&-S\operatorname{sgn}(\frac{u_{h3}(t)}{k_{m3}(t)}-\frac{u_{h2}(t)}{k_{m2}(t)})\sqrt{2g\,|\frac{u_{h3}(t)}{k_{m3}(t)}-\frac{u_{h2}(t)}{k_{m2}(t)}|}-d_{e3}S_V\sqrt{2g\frac{u_{h3}(t)}{k_{m3}(t)}}],\\
&h_1(t)=r_1-e_1(t),\\
&h_2(t)=r_2-e_2(t),
\end{aligned} \tag{31}$$

where S_V is the inner area of outflow pipes, and k_{p1} and k_{p2} are the actuator gains. The relation between the variables in the model (31) and the variables in the discrete-time input affine MIMO state-space model (1) are

$$\mathbf{x}=[x_1\quad x_2]^T=[e_1\quad e_2]^T=\mathbf{e},\ \mathbf{y}=[y_1=h_1\quad y_2=h_2]^T. \tag{32}$$

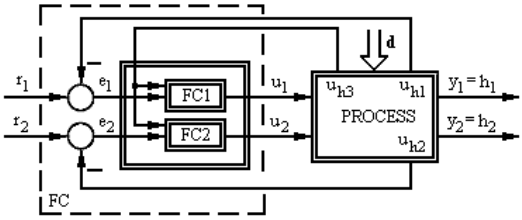

Figure 3
Simplified FCS structure

Figure 3 shows that the MIMO FC consists of two separately designed T-S FCs, FC1 and FC2. The fuzzification in FC is done using the input membership functions presented in Figure 4.

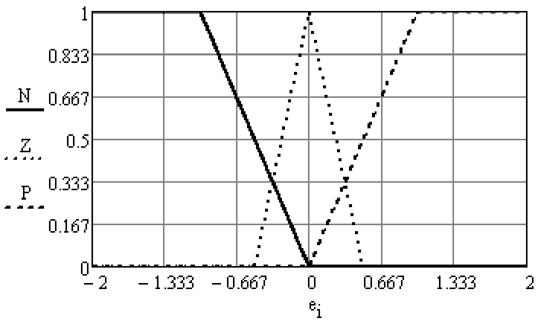

Figure 4
Input membership functions

The inference engine employs the MIN t-norm for the AND operator as specified in Section 2. The inference engine is assisted by the following complete rule base as $n_{RB} = 9$:

$$\begin{aligned}
&R^1: \text{IF } e_1 \text{ IS P AND } e_2 \text{ IS P THEN } \mathbf{u} = \mathbf{u}_1,\\
&R^2: \text{IF} e_1 \text{ IS N AND } e_2 \text{ IS N THEN } \mathbf{u} = \mathbf{u}_2,\\
&R^3: \text{IF } e_1 \text{ IS N AND } e_2 \text{ IS P THEN } \mathbf{u} = \mathbf{u}_3,\\
&R^4: \text{IF } e_1 \text{ IS P AND } e_2 \text{ IS N THEN } \mathbf{u} = \mathbf{u}_4,\\
&R^5: \text{IF } e_1 \text{ IS P AND } e_2 \text{ IS Z THEN } \mathbf{u} = \mathbf{u}_5,\\
&R^6: \text{IF } e_1 \text{ IS Z AND } e_2 \text{ IS P THEN } \mathbf{u} = \mathbf{u}_6,\\
&R^7: \text{IF } e_1 \text{ IS N AND } e_2 \text{ IS Z THEN } \mathbf{u} = \mathbf{u}_7,\\
&R^8: \text{IF } e_1 \text{ IS Z AND } e_2 \text{ IS N THEN } \mathbf{u} = \mathbf{u}_8,\\
&R^9: \text{IF } e_1 \text{ IS Z AND } e_2 \text{ IS Z THEN } \mathbf{u} = \mathbf{u}_9,
\end{aligned} \tag{33}$$

where $\mathbf{u} = [u_1 \quad u_2]^T$, and the rule consequents $\mathbf{u}_k = [u_1^i \quad u_2^i]^T$, $i = 1...9$, are determined as follows on the basis of Theorem 1. More inputs can be considered, but this leads to the complication of the FCS structure and of the design, and rule base reduction techniques should be used [28]-[31]. The Lyapunov function candidate

$$V : R^2 \to R,\ V(\mathbf{e}) = 0.5e_1^2 + 0.5e_2^2 \tag{34}$$

is chosen in order to design stable FCSs for this MIMO process. For $\mathbf{d} = \mathbf{0}$ the time derivative of $V(\mathbf{e})$ along the trajectory of (31), referred to as $\dot{V}(\mathbf{e})$, is

$$\begin{aligned} V(t+1) &= e_1(t)e_1(t+1) + e_2e_2(t+1) = \frac{e_1(t)k_{m1}(t)}{A(u_{h1}(t)/k_{m1}(t))} \\ &[S\mathrm{sgn}(\frac{r_1 - e_1(t)}{k_{m1}(t)} - \frac{u_{h3}(t)}{k_{m3}(t)})\sqrt{2g\,|\frac{r_1 - e_1(t)}{k_{m1}(t)} - \frac{u_{h3}(t)}{k_{m3}(t)}|} - k_{p1}(t)u_1(t)] \\ &+ \frac{e_2(t)k_{m2}(t)}{A(u_{h2}(t)/k_{m2}(t))}[-S\,\mathrm{sgn}(\frac{u_{h3}(t)}{k_{m3}(t)} - \frac{r_2 - e_2(t)}{k_{m2}(t)})\sqrt{2g\,|\frac{u_{h3}(t)}{k_{m3}(t)} - \frac{r_2 - e_2(t)}{k_{m2}(t)}|} \\ &+ S_V\sqrt{2g\,\frac{r_2 - e_2(t)}{k_{m2}(t)}} - k_{p2}(t)u_2(t)]. \end{aligned} \tag{35}$$

The control laws in the rule consequents of MIMO FC are designed to fulfil the condition (9) in Theorem 1, which leads to

$$V_i(t+1) = F(\mathbf{e}(t)) + \mathbf{g}^T(\mathbf{e}(t))\mathbf{u}_i(\mathbf{e}(t)) \le 0,\ i = 1...9. \tag{36}$$

The condition (36) is important because it supports the formulation of the rule base of MIMO FC summarized in Table 1 and proved in Appendix 1. But the controller design depends on the process, and different expressions of Lyapunov function candidates can be used in other applications [32]-[41].

Concluding, Theorem 1 is verified. Therefore the T-S FCS designed in this section is stable.

The values of process parameters considered in this case study are

$$\begin{aligned} &S = 0.005\,\mathrm{m}^2,\ S_V = 0.005\,\mathrm{m}^2,\ R = 1\,\mathrm{m},\ g = 9.8\,\mathrm{m/s}^2, \\ &k_{p1} = k_{p2} = 0.094\,\mathrm{m}^3/(\mathrm{V\,s}),\ k_{m1} = k_{m2} = 1\,\mathrm{V/m}\,, \end{aligned} \tag{37}$$

and the sampling period was set to $T_s = 0.01\,\mathrm{s}$.

Three digital simulation scenarios were considered in order to illustrate the stable behaviour of our T-S FCS scenario 1 (reference inputs $r_1 = 1.5\,\mathrm{m}$ and $r_2 = 1.5\,\mathrm{m}$, and initial conditions $h_1(0) = 0.1\,\mathrm{m}$, $h_2(0) = 1.9\,\mathrm{m}$ and $h_3(0) = 1.5\,\mathrm{m}$ applied to T-S FCS), scenario 2 ($r_1 = 0.5\,\mathrm{m}$, $r_2 = 1.5\,\mathrm{m}$, $h_1(0) = 0.1\,\mathrm{m}$, $h_2(0) = 1.1\,\mathrm{m}$ and

$h_3(0)=1\,\text{m}$ applied to T-S FCS) and scenario 3 ($r_1=0.5\,\text{m}$, $r_2=1.5\,\text{m}$, $h_1(0)=1\,\text{m}$, $h_2(0)=0.1\,\text{m}$ and $h_3(0)=1\,\text{m}$ applied to T-S FCS). The trapezoidal function defined in [13] models the variations of deterministic disturbance inputs.

Table 1
Rule base of MIMO FC

R^i	Premise		Consequent	
	e_1	e_2	u_1	u_2
R^1	P	P	$2S\sqrt{gR}/k_{p1}$	$(2S\sqrt{gR}+S_V\sqrt{2g\frac{r_2-e_2}{k_{m2}}})/k_{p2}$
R^2	N	N	$-2S\sqrt{gR}/k_{p1}$	$(-2S\sqrt{gR}+S_V\sqrt{2g\frac{r_2-e_2}{k_{m2}}})/k_{p2}$
R^3	N	P	$-2S\sqrt{gR}/k_{p1}$	$(2S\sqrt{gR}+S_V\sqrt{2g\frac{r_2-e_2}{k_{m2}}})/k_{p2}$
R^4	P	N	$2S\sqrt{gR}/k_{p1}$	$(-2S\sqrt{gR}+S_V\sqrt{2g\frac{r_2-e_2}{k_{m2}}})/k_{p2}$
R^5	P	Z	$2S\sqrt{gR}/k_{p1}$	$(e_2-S\,\text{sgn}(\frac{u_{h3}}{k_{m3}}-\frac{r_2-e_2}{k_{m2}})\sqrt{2g\lvert\frac{u_{h3}}{k_{m3}}-\frac{r_2-e_2}{k_{m2}}\rvert}$ $+S_V\sqrt{2g\frac{r_2-e_2}{k_{m2}}})/k_{p2}$
R^6	Z	P	$(e_1+S\,\text{sgn}(\frac{r_1-e_1}{k_{m1}}-\frac{u_{h3}}{k_{m3}})$ $\sqrt{2g\lvert\frac{r_1-e_1}{k_{m1}}-\frac{u_{h3}}{k_{m3}}\rvert})/k_{p1}$	$(2S\sqrt{gR}+S_V\sqrt{2g\frac{r_2-e_2}{k_{m2}}})/k_{p2}$
R^7	N	Z	$-2S\sqrt{gR}/k_{p1}$	$(e_2-S\,\text{sgn}(\frac{u_{h3}}{k_{m3}}-\frac{r_2-e_2}{k_{m2}})\sqrt{2g\lvert\frac{u_{h3}}{k_{m3}}-\frac{r_2-e_2}{k_{m2}}\rvert}$ $+S_V\sqrt{2g\frac{r_2-e_2}{k_{m2}}})/k_{p2}$
R^8	Z	N	$(e_1+S\,\text{sgn}(\frac{r_1-e_1}{k_{m1}}-\frac{u_{h3}}{k_{m3}})$ $\sqrt{2g\lvert\frac{r_1-e_1}{k_{m1}}-\frac{u_{h3}}{k_{m3}}\rvert})/k_{p1}$	$(-2S\sqrt{gR}+S_V\sqrt{2g\frac{r_2-e_2}{k_{m2}}})/k_{p2}$
R^9	Z	Z	$(e_1+S\,\text{sgn}(\frac{r_1-e_1}{k_{m1}}-\frac{u_{h3}}{k_{m3}})$ $\sqrt{2g\lvert\frac{r_1-e_1}{k_{m1}}-\frac{u_{h3}}{k_{m3}}\rvert})/k_{p1}$	$(e_2-S\,\text{sgn}(\frac{u_{h3}}{k_{m3}}-\frac{r_2-e_2}{k_{m2}})\sqrt{2g\lvert\frac{u_{h3}}{k_{m3}}-\frac{r_2-e_2}{k_{m2}}\rvert}$ $+S_V\sqrt{2g\frac{r_2-e_2}{k_{m2}}})/k_{p2}$

The digital simulation results obtained for the simulation scenarios 1, 2 and 3 are presented in Figures 5, 6 and 7, respectively. These results highlight the stable behaviour of the T-S FCS for different inputs and initial conditions.

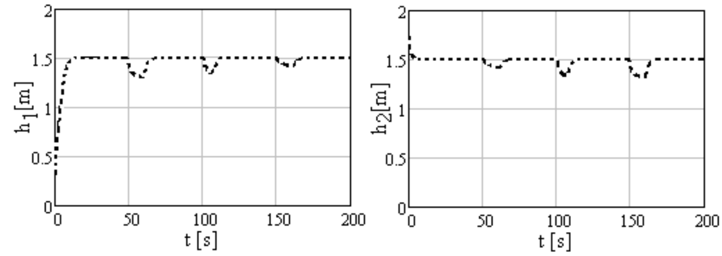

Figure 5

Digital simulation results (scenario 1)

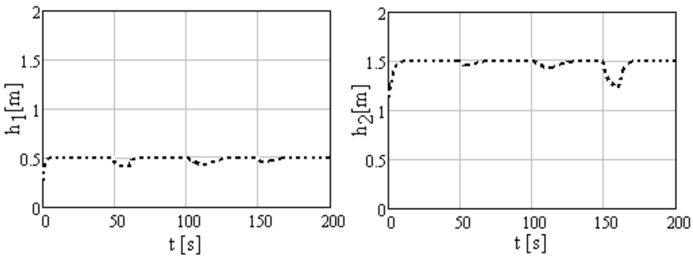

Figure 6

Digital simulation results (scenario 2)

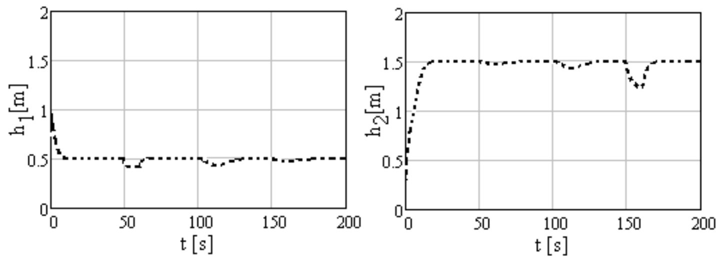

Figure 7

Digital simulation results (scenario 3)

Conclusions

A new stability approach to nonlinear MIMO process characterized by discrete-time input affine state-space models has been proposed. The approach has been applied to the stable design of a T-S FC for the level control of spherical three tank systems. Future research will be focused on the refinement of the stability analysis theorem in order to become less dependent on the process.

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Appendix 1. Proof of stability condition

Theorem 1 is applied as follows in order to formulate the rule base of the MIMO FC for the spherical three tank system using the FCS structure given in Fig. 3. Since the design of the rule consequents is based on controller regions in the input space of the MIMO FC and on inequalities. Let the universe of discourse be $X = [-2,2] \times [-2,2]$, and $\mathbf{e} = \mathbf{0} \in X$. The Lyapunov function candidate defined in (37) is considered, and it is a continuously differentiable positive function on X.

The control laws in the rule consequents of MIMO FC are designed in order to fulfil (36). Therefore the following analysis is done for all rules.

Rule R^1. e_1 IS P and e_2 IS P. So $X_1^A = [0,2] \times [0,2]$. Accepting these conditions, equation (36) is transformed into

$$\dot{V}_1(\mathbf{e}) \le 0,\ \forall \mathbf{e} \in X_1^A. \tag{38}$$

We choose u_1^1 and u_2^1 and we introduce the control laws in Table 1. So

$$\dot{V}_1(\mathbf{e}) = \frac{e_1 k_{m1}}{A(u_{h1}/k_{m1})}[S\mathrm{sgn}(\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}})\sqrt{2g\,|\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}}|} - 2S\sqrt{gR}]$$
$$+ \frac{e_2 k_{m2}}{A(u_{h2}/k_{m2})}[-S\,\mathrm{sgn}(\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}})\sqrt{2g\,|\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}}|} - 2S\sqrt{gR}]. \quad (39)$$

The following inequalities hold:

$$S\mathrm{sgn}(\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}})\sqrt{2g\,|\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}}|} < 2S\sqrt{gR}, \quad (40)$$

$$-S\,\mathrm{sgn}(\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}})\sqrt{2g\,|\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}}|} < 2S\sqrt{gR}, \quad (41)$$

because the terms in the modulus in the left-hand sides of (40) and (41) are in fact levels and they fulfil the constraints (29). Equations (39)–(41) lead to (38).

Rule R^2. e_1 IS N and e_2 IS N. So $X_2^A = [-2,0] \times [-2,0]$. Accepting these conditions, equation (36) is transformed into

$$\dot{V}_2(\mathbf{e}) \le 0, \ \forall \mathbf{e} \in X_2^A. \quad (42)$$

We choose u_1^2 and u_2^2, and we introduce the resulted control laws (that belong to the rule consequents) in Table 1. Therefore

$$\dot{V}_2(\mathbf{e}) = \frac{e_1 k_{m1}}{A(u_{h1}/k_{m1})}[S\mathrm{sgn}(\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}})\sqrt{2g\,|\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}}|} + 2S\sqrt{gR}]$$
$$+ \frac{e_2 k_{m2}}{A(u_{h2}/k_{m2})}[-S\,\mathrm{sgn}(\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}})\sqrt{2g\,|\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}}|} + 2S\sqrt{gR}]. \quad (43)$$

Equations (40) and (41) hold for this rule, too. Equations (40), (41) and (43) lead to the fulfilment of the condition (42).

Rule R^3. e_1 IS N and e_2 IS P. So $X_3^A = [-2,0] \times [2,0]$. Therefore the condition (36) is transformed into

$$\dot{V}_3(\mathbf{e}) \le 0, \forall \mathbf{e} \in X_3^A. \quad (44)$$

We choose the expressions of the control laws u_1^3 and u_2^3, and these rule consequents are introduced in Table 1. Therefore

$$\dot{V}_3(\mathbf{e}) = \frac{e_1 k_{m1}}{A(u_{h1}/k_{m1})}[S\mathrm{sgn}(\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}})\sqrt{2g\,|\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}}|} + 2S\sqrt{gR}]$$
$$+ \frac{e_2 k_{m2}}{A(u_{h2}/k_{m2})}[-S\,\mathrm{sgn}(\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}})\sqrt{2g\,|\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}}|} - 2S\sqrt{gR}]. \quad (45)$$

Equations (40), (41) and (45) lead to the fulfilment of (44).

Rule R^4. e_1 IS P and e_2 IS N. So $X_4^A = [0,2] \times [-2,0]$. Therefore the condition (36) is transformed into

$$\dot{V}_4(\mathbf{e}) \le 0, \forall \mathbf{e} \in X_4^A. \quad (46)$$

We choose u_1^4 and u_2^4, and these rule consequents are introduced in Table 1. So

$$\dot{V}_4(\mathbf{e}) = \frac{e_1 k_{m1}}{A(u_{h1}/k_{m1})}[S\mathrm{sgn}(\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}})\sqrt{2g\,|\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}}|} - 2S\sqrt{gR}] \\ + \frac{e_2 k_{m2}}{A(u_{h2}/k_{m2})}[-S\,\mathrm{sgn}(\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}})\sqrt{2g\,|\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}}|} + 2S\sqrt{gR}], \quad (47)$$

and equations (40), (41) and (47) lead to the fulfilment of (46).

Rule R^5. e_1 IS P and e_2 IS Z. So $X_5^A = [0,2] \times [-0.5,0.5]$. Therefore the condition (28) is transformed into

$$\dot{V}_5(\mathbf{e}) \le 0, \forall \mathbf{e} \in X_5^A. \quad (48)$$

We choose the forms of u_1^5 and u_2^5, and these control laws are introduced as rule consequents in Table 1. Therefore

$$\dot{V}_5(\mathbf{e}) = \frac{e_1 k_{m1}}{A(u_{h1}/k_{m1})}[S\mathrm{sgn}(\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}})\sqrt{2g\,|\frac{r_1 - e_1}{k_{m1}} - \frac{u_{h3}}{k_{m3}}|} - 2S\sqrt{gR}] \\ - \frac{e_2^2 k_{m2}}{A(u_{h2}/k_{m2})}. \quad (49)$$

Therefore equations (40) and (49) lead to the fulfilment of (48).

Rule R^6. e_1 IS Z and e_2 IS P. So $X_6^A = [-0.5,0.5] \times [0,2]$ and the condition (36) is transformed into

$$\dot{V}_6(\mathbf{e}) \le 0, \forall \mathbf{e} \in X_6^A. \quad (50)$$

We choose u_1^6 and u_2^6, and these rule consequents are introduced in Table 1. So

$$\dot{V}_6(\mathbf{e}) = -\frac{e_1^2 k_{m1}}{A(u_{h1}/k_{m1})} \\ + \frac{e_2 k_{m2}}{A(u_{h2}/k_{m2})}[-S\,\mathrm{sgn}(\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}})\sqrt{2g\,|\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}}|} - 2S\sqrt{gR}]. \quad (51)$$

Equations (41) and (51) lead to the fulfilment of (50).

Rule R^7. e_1 IS N and e_2 IS Z. So $X_7^A = [-2,0]\times[-0.5,0.5]$ and the condition (36) is transformed into

$$\dot{V}_7(\mathbf{e}) \le 0, \forall \mathbf{e} \in X_7^A. \tag{52}$$

We choose u_1^8 and u_2^8, and these control laws (that belong to the rule consequents) are introduced in Table 1. Therefore

$$\dot{V}_8(\mathbf{e}) = -\frac{e_1^2 k_{m1}}{A(u_{h1}/k_{m1})} + \frac{e_2 k_{m2}}{A(u_{h2}/k_{m2})}[-S\,\mathrm{sgn}(\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}})\sqrt{2g\,|\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}}|} + 2S\sqrt{gR}], \tag{53}$$

and equations (41) and (53) lead to the fulfilment of (52).

Rule R^8. e_1 IS Z and e_2 IS N. So $X_8^A = [-0.5,0.5]\times[0,2]$ and the condition (36) is transformed into

$$\dot{V}_8(\mathbf{e}) \le 0, \forall \mathbf{e} \in X_8^A. \tag{54}$$

We choose the forms of u_1^8 and u_2^8, and these control rules are introduced as rule consequents in Table 1. Therefore

$$\dot{V}_8(\mathbf{e}) = -\frac{e_1^2 k_{m1}}{A(u_{h1}/k_{m1})} + \frac{e_2 k_{m2}}{A(u_{h2}/k_{m2})}[-S\,\mathrm{sgn}(\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}})\sqrt{2g\,|\frac{u_{h3}}{k_{m3}} - \frac{r_2 - e_2}{k_{m2}}|} + 2S\sqrt{gR}], \tag{55}$$

and equations (41) and (55) lead to the fulfilment of (54).

Rule R^9. e_1 IS Z and e_2 IS Z. So $X_9^A = [-0.5,0.5]\times[-0.5,0.5]$ and the condition (36) is transformed into

$$\dot{V}_9(\mathbf{e}) \le 0, \forall \mathbf{e} \in X_9^A. \tag{56}$$

We choose u_1^9 and u_2^9, and these rule consequents are introduced in Table 1. So

$$\dot{V}_9(\mathbf{e}) = -\frac{e_1^2 k_{m1}}{A(u_{h1}/k_{m1})} - \frac{e_2^2 k_{m2}}{A(u_{h2}/k_{m2})} < 0 \cdot \tag{57}$$

Therefore equation (57) guarantees the fulfilment of (56).

Concluding, the formulation of the rule base of the MIMO FC for the spherical three tank system (Table 1) was done such that to fulfil the condition (36). This condition is fulfilled because we proved equivalent conditions for all rules.