Numerical Study of the Effect of the Shape and Area of Shallow Foundations on the Bearing Capacity of Sandy Soils

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Abstract: The settlement and bearing capacity of shallow foundation models with different shapes and areas on cohesionless subsoil under the applied vertical load are presented in this study. Different shapes of foundations with rectangular, square, strip, plus horizontal cross-sectional shapes are numerically studied after the validation on the laboratory model has been conducted and the constitutive soil model that simulates the behaviour of sandy soil has been chosen. The result of the validation showed that the HS model is the most suitable for the simulation of stress-deformation behaviour of sand. The effect of the shape and area are clearly visible and greatly affects the bearing capacity of the soil. The study generally compared Vesic's, Hansen's and German's bearing capacity equations and showed that Vesic's and Hansen's bearing capacity equations are best suited to the bearing capacity computed from numerical analysis by Plaxis3D. Finally, as a development of Hansen's bearing capacity equation, a new equation of plus shape foundation bearing capacity has been determined.

Keywords: bearing capacity; sand (cohesionless soil); shallow foundations; foundation shape

1 Introduction

The settlement and bearing capacity of the soil are influenced by the size, shape, and depth of the foundations as well as the loads, the physical and mechanical properties and reinforcement of the soils ([1]; [2]; [3]). Generally, foundations are classified into shallow and deep foundations. Shallow foundations are those that transfer structural loads to the soil layers at a relatively shallow depth. According to Terzaghi [4], a shallow foundation is one that is laid at a depth D_f that does not exceed the foundation's width B, or $D_f/B \leq 1$. Studies conducted later on have indicated that D_f/B for shallow foundations can be as high as 3 to 4 ([1]; [2]).

Shallow foundations are a very commonly used type of foundation system. Several bearing capacity equations proposed by different authors and adopted in different codes are available to calculate the ultimate bearing capacity of soil at foundation levels. However, different methods of evaluating bearing capacity yield different results. Subsequently, one should estimate the bearing capacity and settlement of this type of foundation on a sound basis and as close to reality as possible, which will enhance the selection, design and construction of the foundation of the structure.

Shallow foundations can be built in a variety of shapes, but the most popular and extensively utilized are circular, square, and strip foundations. However, the only thing that differentiates them from one another is their horizontal cross-sections. Extensive research has been carried out on the effect of foundation shapes on settlement and soil bearing capacity ([5]; [6]; [7]; [8]; [9]; [10]; [11]; [12]; [13]; [14]; [15]; [16]). However, they do not account for the size of the foundation, and only a few studies, such as ([17]; [18]; [19]) that focus on plus-shaped foundations, came to the conclusion that multi-edge foundations might perform better than square-shape foundations. Nonetheless, the majority of these studies on multi-edge foundations were conducted on small experimental models. Therefore, to solve these issues, and for a more realistic study of the bearing capacity of shallow foundations [20], numerical modelling using PLAXIS 3D is used to model several areas (Full scale) and shapes of foundations.

Studying the effect of the foundation shape on the bearing capacity of the foundation soil and its settlement by laboratory or field methods requires conducting a very large number of laboratory experiments. Given the difficulty of conducting these experiments, their high cost and the multiple sources of inaccuracy in the results, including achieving the same initial state in all experiments, as well as, the great effort and time required, the finite element method was used to calculate the numerical bearing capacity and the variables were studied more extensively.

The aim of this research is to study the soil bearing capacity and determine the best empirical equation of bearing capacity where the comparisons between the numerical bearing capacity, the Hansen, Vesic and German empirical equation of bearing capacity are carried out.

2 Research Materials and Methodology

The analytical method was used to achieve the objectives of this research, according to the following stages:

2.1 Laboratory Experiments

Sufficient laboratory experiments were conducted according to ASTM to determine the specifications of the sand used in the study. All laboratory experiments were carried out in the laboratory of soil mechanics and foundations at the Faculty of Civil Engineering - Tishreen University.

The granular gradient curve is shown in Fig. 1. It was found that the coefficient of uniformity (Cu) was 3.4 and the coefficient of curvature (Cz) was 0.95, so it was classified as SP (Poorly Graded Sand) according to the Unified Soil Classification System (USCS). Table 1 shows the results of laboratory experiments of sand at relative density 60%.

Table 1
The results of laboratory experiments of sand at relative density 60%

Parameter	Value	Parameter	Value
e_{max}	0.656	G (-)	2.65
e_{min}	0.402	E_{oed} (kN/m^2)	40000
γ_{unsat} (kN/m^3)	17.6 kN/m^3	ϕ (°)	34
γ_{sat} (kN/m^3)	21	c (kN/m^2)	1.1

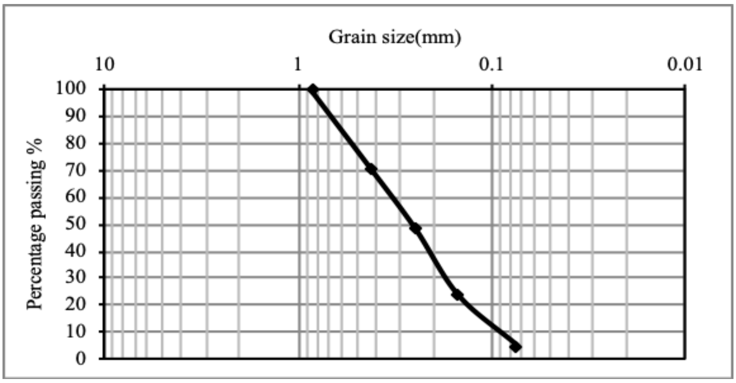

Figure 1
The granular gradient curve of sand

2.2 Determination of the Constitutive Model

The validation of the numerical model in the finite element method is considered vital through the comparison of a numerical model results and measurements. The validation procedures should take into consideration the appropriate constitutive model, geometric simplifications, and other sources of inconsistencies to obtain a satisfactory representative numerical model [21].

2.2.1 Laboratory Model

To select the constitutive model, a laboratory model was built (Fig. 2) consisting of a metal cylindrical mould with dimensions of 280 mm (diameter) x 230 mm (height) filled with sand at a relative density of 60%, and from a circular metal foundation with dimensions: 50 mm (D) x 17 mm (thick), placed on the aforementioned sandy soil. The foundation was carried with a vertical load and the corresponding settlement was recorded. The dimensions of the mould were sufficient, where the diameter of the mould is 5.6 times the diameter of the circular foundation, and the height of the mould is 4.6 times the diameter of the foundation, which are sufficient dimensions so that the deformations do not reach the side borders of the mould and therefore have no effect on the results.

Figure 2
laboratory model

2.2.2 Modelling the Laboratory Model using Plaxis 3D

The same dimensions of the laboratory model were used to determine the geometric dimensions of the numerical model, (FE-Model), where a three-dimensional model (3D-Model) was used, similar to the laboratory model, as the sides of the model allowed only vertical settlement, and the bottom of the model was fixed. The foundation had a circular shape (D=50 mm) and the foundation has been considered a rigid equivalent to a uniformly distributed settlement imposed on the soil surface. For reasons of symmetry, only a quarter of the model was modelled. Fig. 3 shows the numerical model used in calibration for the laboratory model.

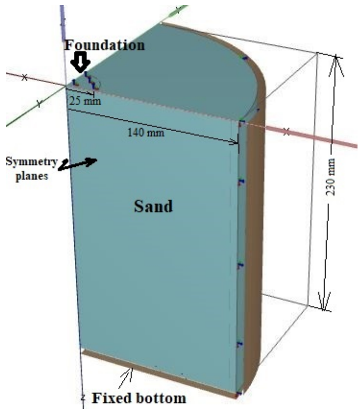

Figure 3

Numerical model used to validate the laboratory model

Two constitutive models were used in simulating the laboratory model, the Hardening soil model (HS) being one of the advanced models and the Mohr-Coulomb model (MC) being one of the most common and widely used models. Table.2 and Table.3 show the parameters of each model that are derived from laboratory experiments on the sand used in this study.

Table 2

Soil parameters used (MC- Model)

Parameter	Value	Parameter	Value
γ_{unsat} (kN/m^3)	17.6	φ (°)	34
γ_{sat} (kN/m^3)	21	c (kN/m^2)	1.1
E_{ref} (kN/m^2)	29500	Ψ (°)	3

Table 3

Soil parameters used (MC- Model)

Parameter	Value	Parameter	Value
γ_{unsat} (kN/m^3)	17.6	C (kN/m^2)	1.1
γ_{sat} (kN/m^3)	21	φ (°)	34
E_{50}^{ref} (kN/m^2)	29500	ψ (°)	3
E_{oed}^{ref} (kN/m^2)	40000	ν_{ur} (-)	0.2
E_{ur}^{ref} (kN/m^2)	120000	m (-)	0.53

Fig. 4 shows a comparison between the results of the numerical model and the results of the laboratory model, using the constitutive models mentioned earlier.

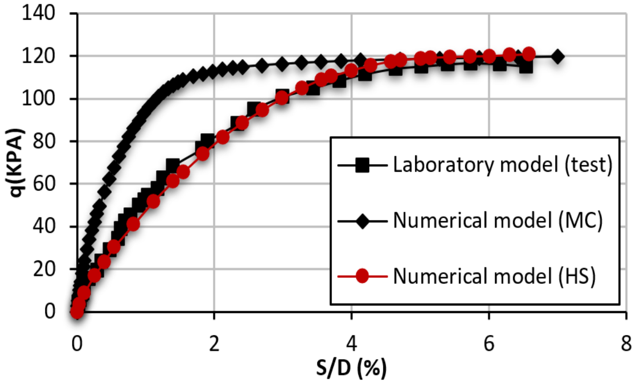

Figure 4
Comparison between the results of the numerical model and the results of the laboratory model

Where q is the applied stress on circular foundation, S/D is the nondimensional relative settlement and S is the corresponding settlement. If the settlement (S) is expressed in a nondimensional relative settlement of S/D, the load-settlement response is not affected by the scale effect [22].

In Fig. 4 a great convergence can be seen between the results of the laboratory experiments and the modelling using (HS-Model), This is consistent with the findings of [23]. It is also noted that (HS-Model) simulates soil behavior more accurately than (MC-Model) until reaching collapse. Therefore, HS is the adopted constitutive model in this research.

3 Results and Discussion

After it is verified that the constitutive model of the FEM model reflects the stress-strain behaviour with acceptable accuracy, compared to the results of the laboratory model, the numerical study is conducted in light of the research objectives through a parametric study of different shapes and areas of shallow foundations to determine the optimal relationship of the bearing capacity of sandy soil.

In this study, the effect of the foundation area and shape on the soil bearing capacity is examined. The shapes chosen for the areas 9-50-144-625 m^2 are displayed in Tables 4-5-6-7. With the use of the HS model, which simulates the behavior of the employed sandy soil, foundations are modelled on Plaxis 3D as fully solid foundations. The equivalent dimensions L1, B1 represent the dimensions of a rectangular foundation with the same area of the plus foundation, as shown in Fig. 5, where these dimensions were used to determine the bearing capacity using the empirical equations of Vesic, Hansen, and the German code as these relationships are not applicable for irregular or multi-edge foundations [16].

Table 4
shapes related to area 9 m^2

shape of foundation	Symbol	Area (A) [m^2]	Dimensions [m]	Equivalent width B1[m]	Equivalent length L1[m]	L1/B1
Circular	C1	9	D=3.385	3.385	3.385	1
Square	S1	9	L=3 B=3	3	3	1
Rectangle l/B=2	R1-2	9	L=4.24 B=2.12	2.12	4.24	2
Rectangle l/B=3	R1-3	9	L=5.196 B=1.732	1.732	5.196	3
Rectangle l/B=5	R1-5	9	L=6.71 B=1.342	1.342	6.71	5
Plus sign shape +	P1-0.2	9	b=2.236 a=0.4472	2.9	3.1	1.1
Plus sign shape +	P1-0.35	9	b=1.937 a=0.678	2.733	3.292	1.2
Plus sign shape +	P1-0.5	9	b=1.732 a=0.866	2.6	3.464	1.33
Plus sign shape +	P1-0.75	9	b=1.5 a=1.125	2.4	3.75	1.563
Plus sign shape +	P1-1	9	b=1.342 a=1.342	2.23	4.03	1.8
Plus sign shape +	P1-2	9	b=1 a=2	1.8	5	2.78

Table 5
shapes related to area 50 m^2

shape of foundation	Symbol	Area (A) [m^2]	Dimensions [m]	Equivalent width B1[m]	Equivalent length L1[m]	L1/B1
Circular	C2	50.26	D=8	8	8	1
Square	S2	50.26	L=7.09 B=7.09	7.09	7.09	1
Rectangle l/B=2	R2-2	50.26	L=10.03 B=5.01	5.01	10.03	2
Rectangle l/B=3	R2-3	50.26	L=12.28 B=4.09	4.09	12.28	3
Rectangle	R2-5	50.26	L=15.85	3.17	15.85	5

l/B=5			B=3.17			
Plus sign shape +	P2-0.2	50.26	b=5.284 a=1.057	6.76	7.435	1.1
Plus sign shape +	P2-0.35	50.26	b=4.576 a=1.602	6.46	7.78	1.2
Plus sign shape +	P2-0.5	50.26	b=4.09 a=2.047	6.145	8.18	1.33
Plus sign shape +	P2-0.75	50.26	b=3.545 a=2.659	5.67	8.862	1.563
Plus sign shape +	P2-1	50.26	b=3.171 a=3.171	5.29	9.5	1.8
Plus sign shape +	P2-2	50.26	b=2.363 a=4.727	4.25	11.82	2.78

Table 6
shapes related to area 144 m^2

shape of foundation	Symbol	Area (A) [m^2]	Dimensions [m]	Equivalent width B1[m]	Equivalent length L1[m]	L1/B1
Circular	C3	144	D=13.54	13.54	13.54	1
Square	S3	144	L=12 B=12	12	12	1
Rectangle l/B=2	R3-2	144	L=16.98 B=8.48	8.48	16.98	2
Rectangle l/B=3	R3-3	144	L=20.79 B=6.93	6.93	20.79	3
Rectangle l/B=5	R3-5	144	L=26.8 B=5.37	5.37	26.8	5
Plus sign shape +	P3-0.2	144	b=8.94 a=1.79	11.5	12.52	1.1
Plus sign shape +	P3-0.35	144	b=7.746 a=2.711	10.94	13.168	1.2
Plus sign shape +	P3-0.5	144	b=6.93 a=3.46	10.37	13.86	1.33
Plus sign shape +	P3-0.75	144	b=6 a=4.5	9.6	15	1.563
Plus sign shape +	P3-1	144	b=5.39 a=5.37	8.93	16.12	1.8
Plus sign shape +	P3-2	144	b=4 a=8	7.2	20	2.78

Table 7
shapes related to area 625 m^2

shape of foundation	Symbol	Area (A) [m^2]	Dimensions [m]	Equivalent width B[m]	Equivalent length L[m]	L/B
Circular	C4	625	D=28.21	28.21	28.21	1
Square	S4	625	L=25 B=25	25	25	1
Rectangle l/B=2	R4-2	625	L=35.36 B=17.67	17.67	35.36	2
Rectangle l/B=3	R4-3	625	L=43.3 B=14.4	14.4	43.3	3
Rectangle l/B=5	R4-5	625	L=55.9 B=11.2	11.2	55.9	5
Plus sign shape +	P4-0.2	625	b=18.633 a=3.727	23.96	26.09	1.1
Plus sign shape +	P4-0.35	625	b=16.137 a=5.648	22.782	27.434	1.2
Plus sign shape +	P4-0.5	625	b=14.434 a=7.22	21.664	28.85	1.33
Plus sign shape +	P4-0.75	625	b=12.5 a=9.375	20	31.25	1.563
Plus sign shape +	P4-1	625	b=11.18 a=11.18	18.63	33.54	1.8
Plus sign shape +	P4-2	625	b=8.33 a=16.66	14.98	41.713	2.78

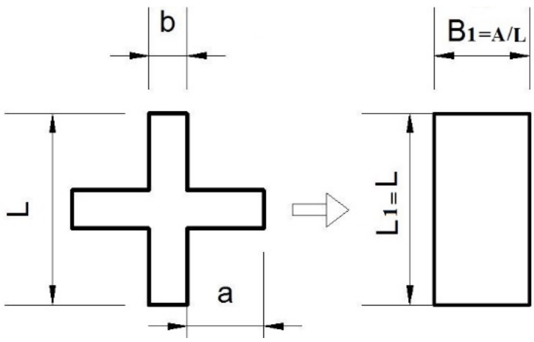

Figure 5
Symbols and equivalent dimensions of the Plus Foundation

3.1 Comparison of the Numerical and Empirical Equations of Ultimate Bearing Capacity

3.1.1 Square, Circle and Rectangular Foundations

As can be seen from Figures 6, 7, 8, and 9 for areas 9, 50, 144, 625 m^2 respectively, the Vesic equation achieves the best fit for square and circle shapes while the Hansen equation achieves the best fit for rectangular shapes. The reason being that Vesic uses in his equation the same Nc and Nq terms of the Hansen relation but $N\gamma$ is different.

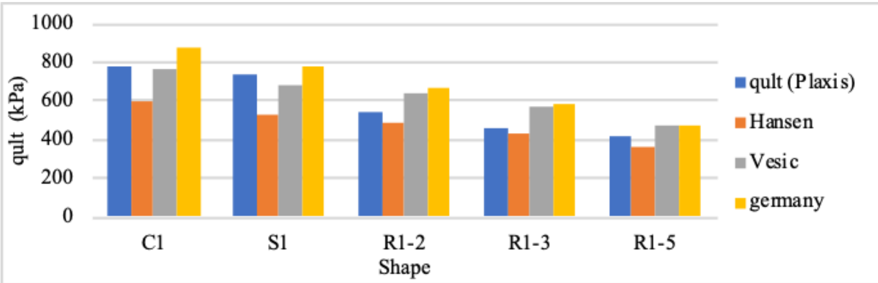

Figure 6

Comparison of Plaxis' calculation of bearing capacity and an empirical equation of bearing capacity for a 9-m^2 area

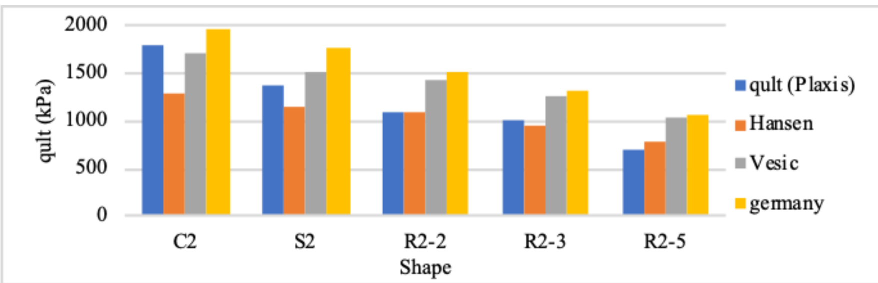

Figure 7

Comparison of Plaxis' calculation of bearing capacity and an empirical equation of bearing capacity for a 50-m^2 area

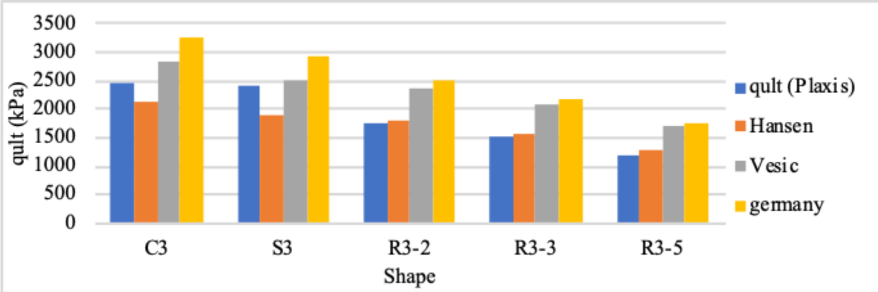

Figure 8
Comparison of Plaxis' calculation of bearing capacity and an empirical equation of bearing capacity for a 144-m^2 area

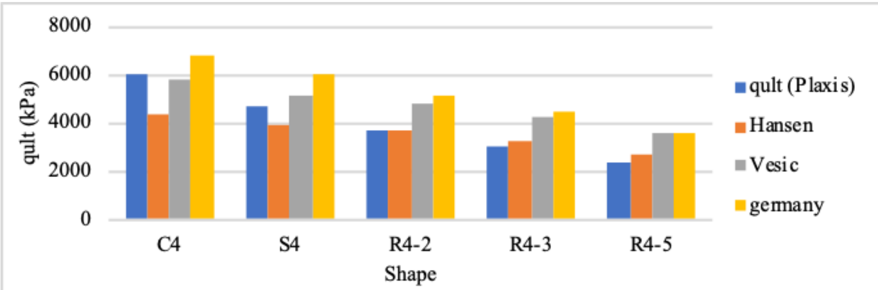

Figure 9
Comparison of Plaxis' calculation of bearing capacity and an empirical equation of bearing capacity for a 625-m^2 area

3.1.2 Plus Shape Foundations

The relationships between the ultimate bearing capacity (qult) and a/b where the dimensions a and b are clarified in figure 5. It is noted that the best value of a/b is about 0.5 (Fig. 10). This is consistent with the laboratory study of Ghazavi and Mirzaeifar [17], because at this value the best blocking occurs and the soil between edges behaves as if it was part of the foundation and moves down upon loading as a single unit.

Figures 11, 12, 13 and 14 show a comparison between the numerical results and empirical equations of ultimate bearing capacity of different areas 9, 50, 144 and 625m^2 respectively for plus shape foundations using the equivalent dimension of a rectangular foundation where L is the length of the enveloped square shape of the plus shape foundation and B1=A/L where A is the Area of the plus shape foundation as shown in Fig. 5.

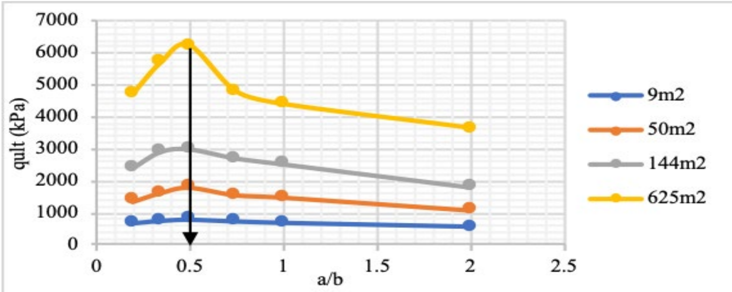

Figure 10

The relationships between ultimate bearing capacity (qult) and a/b

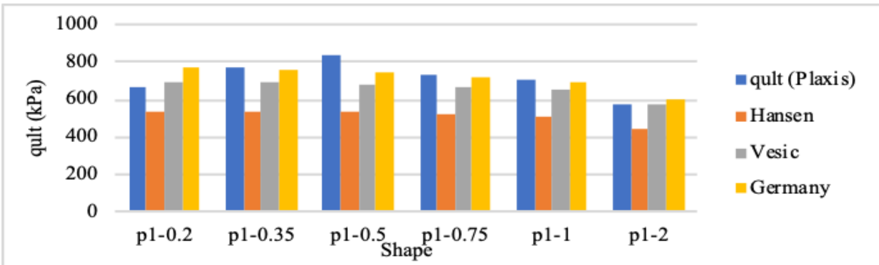

Figure 11

Comparison of Plaxis' calculation of bearing capacity and an empirical equations of bearing capacity for a 9-m^2 area of plus foundation

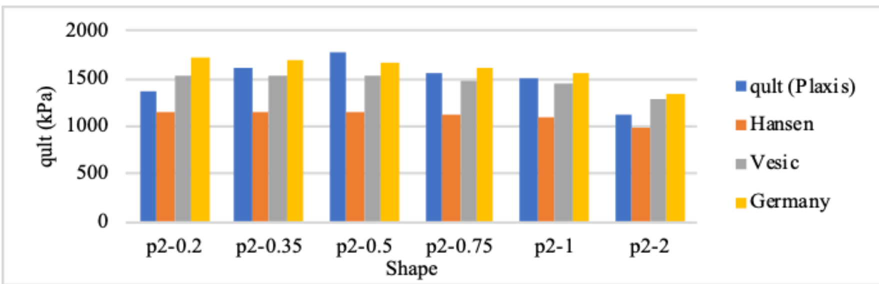

Figure 12

Comparison of Plaxis' calculation of bearing capacity and empirical equations of bearing capacity for a 50-m^2 area of plus foundation

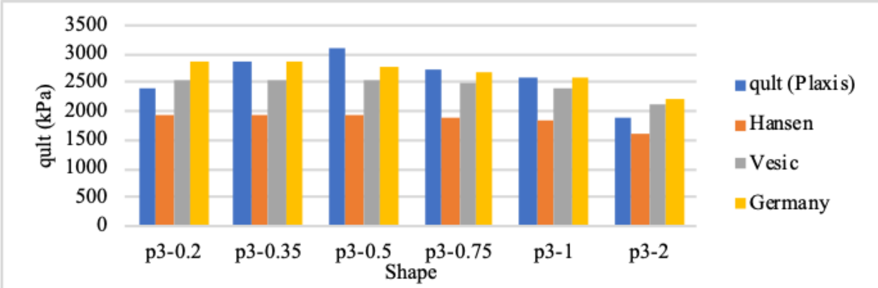

Figure 13

Comparison of Plaxis' calculation of bearing capacity and empirical equations of bearing capacity for a 144-m^2 area of plus foundation

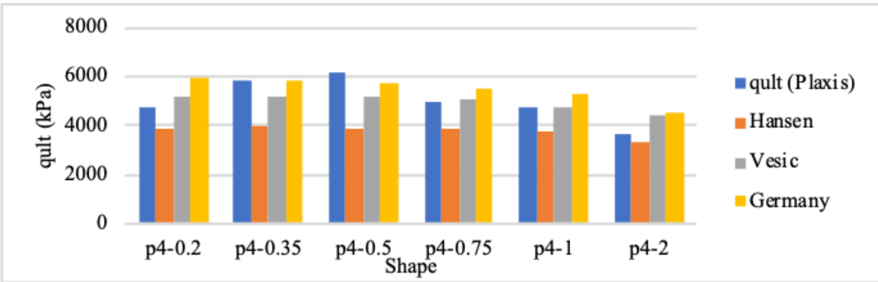

Figure 14

Comparison of Plaxis' calculation of bearing capacity and empirical equations of bearing capacity for a 625-m^2 area of plus foundation

It is noted from Figures 11, 12, 13 and 14 that the Vesic equation fits the numerical bearing capacity in most cases especially when a/b>0.5 but they do not take the effect of blocking well when a/b≤0.5. To solve this issue the Hansen equation is chosen since it gives the best results when it is multiplied by the blocking coefficient which is determined using the solver feature in Excel. Equation (1) is the new equation of bearing capacity of the plus shape foundation that is modified from (Chazavi and Hadiani [19]) equation.

$$q_{ult,Plus} = q_{ult,Hansen} \times e^{\left(\frac{1.8*10^{-6}\left(\frac{B_1}{L_1}\right)^{7.3}}{4.6*10^{-7}+\left(\frac{B_1}{L_1}\right)^{24.2}}\right)^{0.21}} \qquad (1)$$

Where $q_{ult,Hansen}$: is the ultimate bearing capacity calculated by the Hansen equation using the equivalent dimension of the rectangular foundation as clarified in Fig. 5. Figures 15, 16, 17 and 18 show a comparison between numerical, empirical equations and modified equations (1) of the ultimate bearing capacity of different areas 9, 50, 144 and 625 m^2 respectively.

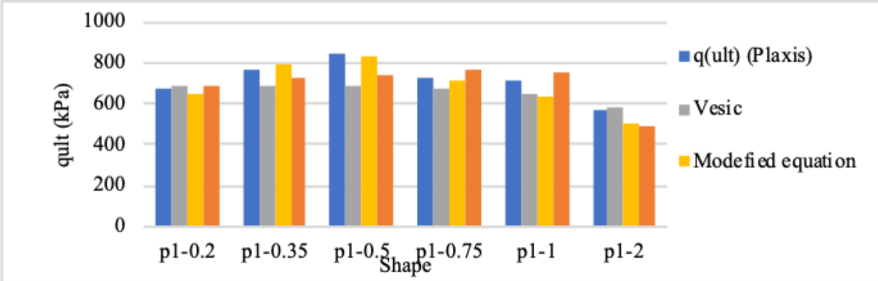

Figure 15
Comparison of numerical bearing capacity, empirical equations, and a modified equation for a 9-m^2 area

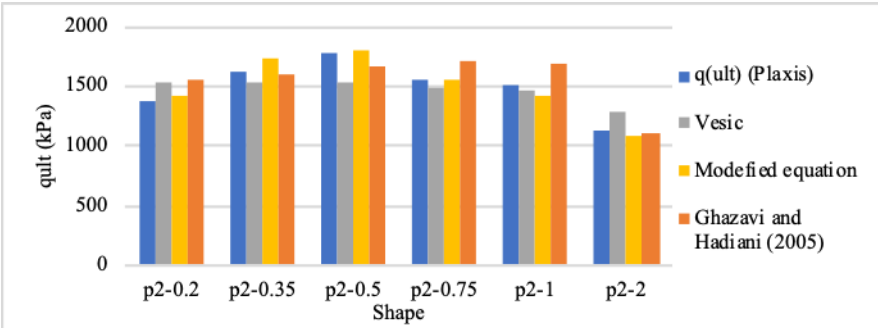

Figure 16
Comparison of numerical bearing capacity, empirical equations, and a modified equation for a 50-m^2 area

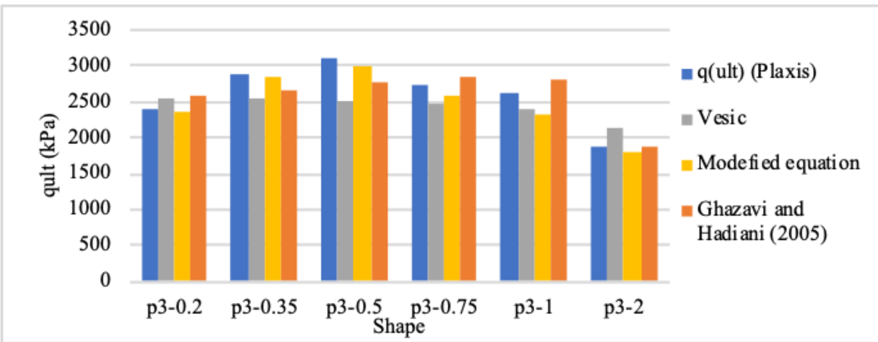

Figure 17
Comparison of numerical bearing capacity, empirical equations, and a modified equation for a 144-m^2 area

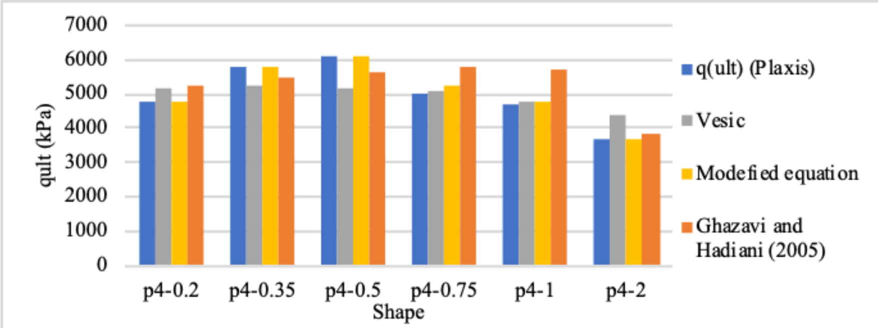

Figure 18

Comparison of numerical bearing capacity, empirical equations, and a modified equation for a 625-m^2 area

It is clear from Figures 15, 16, 17 and 18 that the modified equation achieves the best consistency with numerical results for different areas of the plus shape foundation.

It is worth mentioning that the plus shape foundation at a/b=0.5 achieves the highest bearing capacity compared to other shapes that have the same area, as is shown in Fig. 19, which depicts the stress-settlement curves for foundations with an area of 144 m^2. Fig. 20 shows the improvement percentages in bearing capacity at a/b=0.5 in comparison to square shape foundations. The percentage increases with an increase in the area of the foundation; for example, with the area of 50m^2 the ultimate bearing capacity increases by 25%.

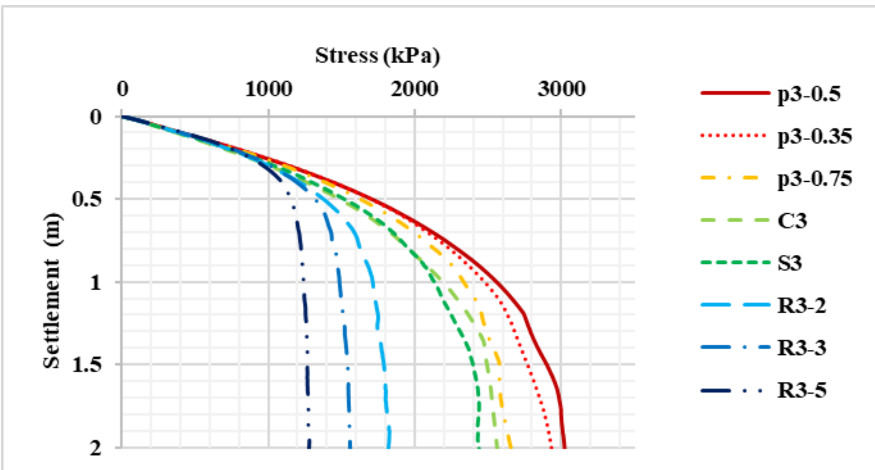

Figure 19

Stress-settlement relationship curves

From these curves in Fig. 19, we can deduce that the shear bearing capacity is the highest for the plus shape foundation at the ratio a/b = 0.5, whereas the shear bearing capacity is the least for the strip foundation.

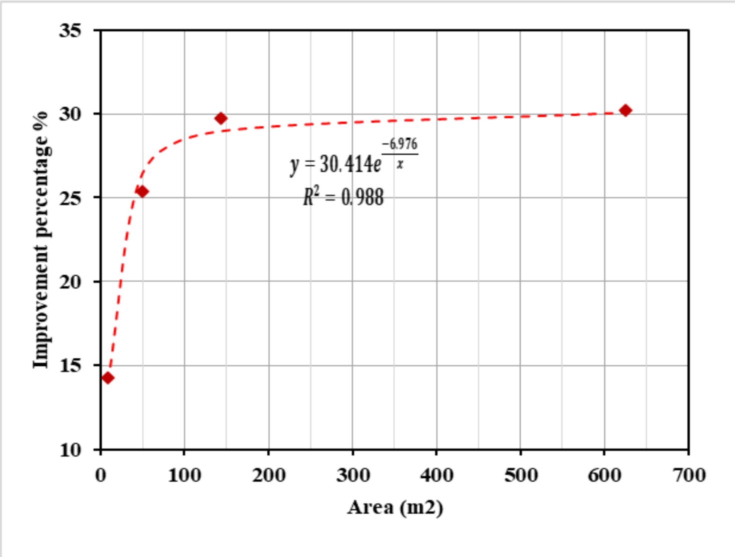

Figure 20
Improvement percentages in bearing capacity at a/b=0.5 as comparison with square shape foundation

Conclusions

According to the outcomes of the numerical analysis performed using the Plaxis 3D software, which uses the finite element method FEM for various foundation shapes and with various areas of foundation on sandy soil, the following conclusions were made:

- The Hansen equation provides the closest value to the numerical analysis of rectangular shapes, while the Vesic equation is the best fitting for circular and square foundations for all areas.
- The plus foundation outperforms square and circular foundations in terms of performance and provides the best shear bearing capacity at the ratio a/b = 0.5.
- Finding the blocking coefficient led to the development of a new equation (Equation 1) for the bearing capacity of plus foundations.
- The improvement percentages in bearing capacity at a/b=0.5 in comparison to the square shape foundation increase from 15% to 30% with an increase in the area of the foundation from 9 m^2 to 625 m^2, respectively.

The following recommendations may be taken into account for further research on the topic:

- ✓ Studying new foundation shapes, such as H, T and U-shaped foundations.
- ✓ Investigating how the bearing capacity of sand is affected by alterations in the elastic modulus and internal friction angle.
- ✓ Investigating the bearing capacity effects of foundation depth, soil layering, load inclination, decentralization, land surface slope, foundation base inclination, and groundwater presence.

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