Sensitivity Analysis of Imperfect Systems Using Almost Orthogonal Filters

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Abstract: This paper considers the application of the almost orthogonal filters in the sensitivity analysis of imperfect systems. First, we explain the concepts of dynamical systems sensitivity. Then we design almost orthogonal filters based on almost orthogonal polynomials. These filters are a generalization of the classical orthogonal filters commonly used in circuit theory, control system theory, signal processing, signal approximation and process identification. The advantage of the almost orthogonal filters is that they can be used for the modeling and analysis of systems with imperfections, i.e. imperfect technical systems. In this paper, we use a designed filter to obtain a model of an imperfect system, where the model's parameters have been determined with the help of genetic algorithm. A new approach for determining the sensitivity of imperfect systems is also given and an example of an imperfect system in the form of a hydraulic multitank system is considered.

Keywords: sensitivity analysis; imperfect systems; almost orthogonal polynomials; almost orthogonal filters; multitank system

1 Introduction

Sensitivity analysis considers the impact of parameter or disturbance changes on the change of the systems' state coordinates. In this paper, our focus is on the parametric sensitivity of the imperfect systems. Analysis of the parametric sensitivity is usually performed as a series of tests in which the operator sets different parameter values to see if and how these changes impact the system dynamic behaviour. By showing how the model behaviour responds to changes in the parameter values, sensitivity analysis is a useful tool in model design as well as in model evaluation.

Uncertainty in engineering analysis usually pertains to stochastic uncertainty, i.e., variance in product or process parameters [1-3] characterized by probability. Methods for calculating sensitivity under stochastic uncertainty are well documented. Imprecision, or the concept of uncertainty in choice, is one such form. Recently, systems with imperfections have been intensively studied [4-8]. The components, used for designing any real system, are not perfect and their parameters values are in the range of allowed (or not) tolerance. The reasons can be various: imperfect manufacturing, systems exploitation conditions (environment temperature, pressure, moisture, electromagnetic fields, variations in voltage, etc.). With respect to that fact, every real system in analogue technique is in some way imperfect. Digital systems, on the other hand, are considered to be perfect. Imperfections of their components do not impact system accuracy as a whole.

Therefore, because of the imperfections, parameters are not completely defined, i.e. they can vary in a certain range. In the case of systems modeling by some classical method, we use fixed parameters values, although it is not the case in reality. For the purpose of modeling these systems, it is possible to use orthogonal functions, i.e. orthogonal polynomials [9, 10]. Orthogonal polynomials are already used in approximation theory and numerical integration, and also in other scientific disciplines, e.g. in solving series of limitary problems in mathematics and physics and in solving some quantum mechanics problems. A very important application of orthogonal polynomials is the designing of orthogonal filters [11-15]. These filters are useful for orthogonal signal generators, least square approximations, and the practical realizations of optimal and adaptive systems. However, since the components of these systems cannot be manufactured exactly, filters made with these components are not quite orthogonal, but rather almost orthogonal. The signals obtained by these filters are almost orthogonal as well. The measure of nearness between the obtained and the regular orthogonal signals depends on the exactness of the component manufacturing. Thus, almost orthogonal filters are imperfect filters. Therefore, for designing these filters we cannot use the classical orthogonal polynomials, but rather we must use almost orthogonal [16-18]. In this paper, almost orthogonal filters have been used for the sensitivity analysis of imperfect systems. Theoretical results have been verified with performed experiments on laboratory setup, consisting of a multitank hydraulic system, and compared with similar method for sensitivity analysis.

2   Sensitivity of Dynamical Systems

Consider the linear system described by the transfer function in general form:

$$W(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \tag{1}$$

with the output:

$$y(s) = W(s) x(s) \tag{2}$$

where $x(s)$ represents the input of the system.

Equation (1) has $(n+m+2)$ parameters a_i ($i=0,1,\ldots,n$), b_j ($j=0,1,\ldots,m$) [19]. So it is possible to define $(n+m+2)$ sensitivity functions in s-domain as follows:

$$u_{a_i}(s) = \frac{\partial y(s)}{\partial a_i} \quad i = 0,1,\ldots,n$$

$$u_{b_i}(s) = \frac{\partial y(s)}{\partial b_i} \quad i = 0,1,\ldots,m \tag{3}$$

In accordance we have:

$$u_{a_i}(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{\left(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0\right)\left(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0\right)} \frac{s^i}{} x(s) \tag{4}$$

For parameters b_i, sensitivity functions can be also obtained:

$$u_{b_i}(s) = \frac{s^i}{\left(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0\right)} x(s) \tag{5}$$

In the case of the system sensitivity in steady state, we can use [20-24]:

$$y_i(\infty) = \lim_{s \to 0} s W_s(s) x(s) \tag{6}$$

$$u_{a_i} = \frac{\Delta y_i(\infty)}{\Delta a_i}$$

$$u_{b_i} = \frac{\Delta y_i(\infty)}{\Delta b_i} \tag{7}$$

3   Almost Orthogonal Filters

To analyze the sensitivity of the imperfect systems, we need to have the best possible model of the given system. For that purpose we will use almost orthogonal Legendre type polynomials [1, 4]. It has already been demonstrated how relation (1) can be turned into an orthogonal filter [12-14]. Then this filter can be used for systems modeling. This modeling method achieves greater accuracy with a lesser number of variable parameters used [11, 12].

The filter generates almost orthogonal functions $\varphi_k^{(\varepsilon)}(t)$ [1, 8], which can be used for designing the imperfect systems models and for the least square approximation, using the following relation:

$$y_M(t) \approx \sum_{k=0}^{n} c_k \varphi_k^{(\varepsilon)}(t) \tag{8}$$

An adjustable model of imperfect system is given in Fig. 1. Labels in the figure have the following meanings: $\delta(t)$ is the Dirac impulse function, $h(t)$ is the Heaviside step function, functions $\varphi_i(t)$ are inverse Laplace transforms of the functions $\Phi_i(t)$, and $\varphi_n^{(\varepsilon)}(t)$ represent Legendre type almost orthogonal functions. This is the sequence of almost orthogonal exponential functions over interval $(0, \infty)$ with weight function $w(t) = e^{-t}$.
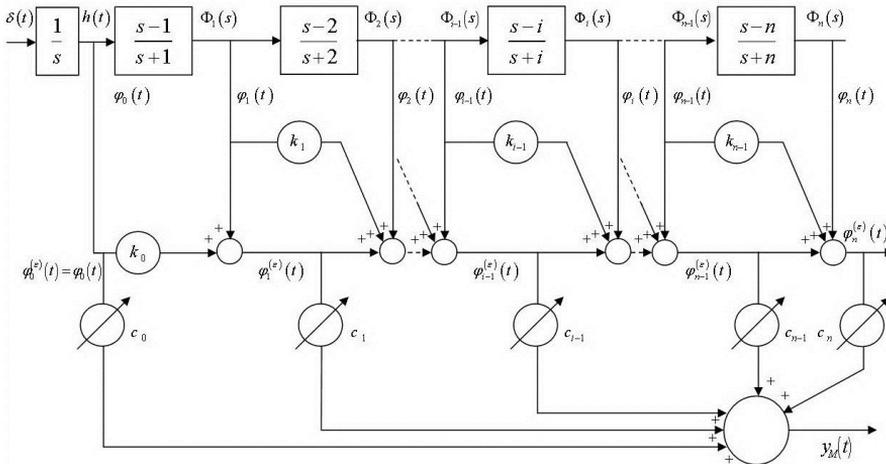

Figure 1
Adjustable model of imperfect system

The transfer function of the system model, described by almost orthogonal filter (see Fig. 1), has the following form [1]:

$$W_n^{(\varepsilon)}(s) = \frac{b_m(\varepsilon)s^m + b_{m-1}(\varepsilon)s^{m-1} + \cdots + b_1(\varepsilon)s + b_0(\varepsilon)}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \; m < n \tag{9}$$

Coefficients $b_m(\varepsilon)$ have complex dependence on parameter ε, and we can write this in the following way: $b_m\big(c_n\big(k_n\big(r_n(\varepsilon)\big)\big)\big)$, where $r_n(\varepsilon)$ is coefficient defined in [1]. The coupled transfer function of this system is:

$$W_s^{(\varepsilon)}(s) = \frac{\displaystyle\sum_{j=0}^{m} b_j(\varepsilon)s^j}{\displaystyle\sum_{i=0}^{n} a_i s^i + \sum_{j=0}^{m} b_j(\varepsilon)s^j} \tag{10}$$

Now, sensitivity functions related to parameters a_i and b_i are the following:

$$u_{a_i}(s) = \frac{\displaystyle\sum_{j=0}^{m} b_j(\varepsilon)s^j}{\left(\displaystyle\sum_{i=0}^{n} a_i s^i + \sum_{j=0}^{m} b_j(\varepsilon)s^j\right)^2} s^i x(s) \tag{11}$$

$$u_{b_i}(s) = \frac{\displaystyle\sum_{i=0}^{n} a_i s^i}{\left(\displaystyle\sum_{i=0}^{n} a_i s^i + \sum_{j=0}^{m} b_j(\varepsilon)s^j\right)^2} s^i x(s) \tag{12}$$

4   Case Study – Description

For the purpose of sensitivity analysis of imperfect systems model, we will use a multitank system shown in Fig. 2. The multitank system [25] (Fig. 2) comprises a number of separate tanks fitted with drain valves. The separate tank mounted in the base of the set-up acts as a water reservoir for the system. Some of the tanks have a constant cross section, while others are spherical or conical, and so have a variable cross section. This creates the main nonlinearities of the system. A variable speed pump is used to fill the upper tank. The liquid flows out of the tanks due to gravity. The tank valves act as flow resistors. The area ratio of the valves is controlled and can be used to vary the outflow characteristic. Each tank is equipped with a level sensor based on hydraulic pressure measurement.

Figure 2
The multitank system by Inteco

The multitank system relates to liquid level control problems commonly occurring in industrial storage tanks. For example, steel producing companies around the world have repeatedly confirmed that substantial benefits are gained from accurate mould level control in continuous bloom casting. Mould level oscillations tend to stir foreign particles and flux powder into molten metal, resulting in surface defects in the final product. The multitank system has been designed to operate with an external, PC-based digital controller. The control computer communicates with the level sensors, valves and pump by a dedicated I/O board and the power interface. The I/O board is controlled by the real-time software which operates in MATLAB®/Simulink RTW/RTWT® rapid prototyping environment.

The multitank system given in Fig. 2 can be described using the well-known "mass balance" equations:

$$\frac{dH_1}{dt} = \frac{1}{\beta_1(H_1)}q - \frac{1}{\beta_1(H_1)}C_1 H_1^{\alpha_1}$$

$$\frac{dH_2}{dt} = \frac{1}{\beta_2(H_2)}C_1 H_1^{\alpha_1} - \frac{1}{\beta_2(H_2)}C_2 H_2^{\alpha_2} \qquad (13)$$

$$\frac{dH_3}{dt} = \frac{1}{\beta_3(H_3)}C_2 H_2^{\alpha_2} - \frac{1}{\beta_3(H_3)}C_3 H_3^{\alpha_3}$$

where q represents the inflow to the upper tank, H_i is the fluid level in the i-th tank (i=1, 2, 3), C_i is the resistance of the output orifice of i-th tank, α_i represents the

flow coefficient for the i-th tank. $\beta_i\left(H_1\right)$ represents the cross sectional area of i-th tank at the level H_i. These values for the single tanks are the following: $\beta_i\left(H_i\right)=aw$ is the constant cross sectional area of the upper tank; $\beta_2\left(H_2\right)=cw+\dfrac{H_2}{H_{2\max}}bw$ is the variable cross sectional area for the middle tank, and $\beta_3\left(H_3\right)=w\sqrt{R^2-\left(R-H_3\right)^2}$ is the variable cross sectional area of the lower tank.

The specified parameter values are the following:

$a=0.25m,\ b=0.345m,\ c=0.1m,\ w=0.035m,\ R=0.364m$,

and $H_{1\max}=H_{2\max}=H_{3\max}=0.35m$.

Rewrite the right sides of (13) in the form $F(x, q)=[F_1, F_2, F_3]$, where:

$$F_1\left(q,H_1\right)=\frac{1}{\beta_1\left(H_1\right)}q-\frac{1}{\beta_1\left(H_1\right)}C_1 H_1^{\alpha_1}$$

$$F_2\left(H_1,H_2\right)=\frac{1}{\beta_2\left(H_2\right)}C_1 H_1^{\alpha_1}-\frac{1}{\beta_2\left(H_2\right)}C_2 H_2^{\alpha_2} \tag{14}$$

$$F_3\left(H_2,H_3\right)=\frac{1}{\beta_3\left(H_3\right)}C_2 H_2^{\alpha_2}-\frac{1}{\beta_3\left(H_3\right)}C_3 H_3^{\alpha_3}$$

For the model (13), for fixed $q=q_0$ we can define an equilibrium state (steady-state points) given by $q_0=C_1 H_{10}^{\alpha_1}=C_2 H_{20}^{\alpha_2}=C_3 H_{30}^{\alpha_3}$.

The linearized model is obtained by the Taylor expansion of (14) around the assumed equilibrium state:

$$\frac{dh}{dt}=J_H h+J_q u \tag{15}$$

where: $h=H-H_0$ is the modified state vector (deviation from the equilibrium state H_0), $u=q-q_0$ is deviation of the control, relative to q_0, J_p and J_q are Jacobians of the function (14): $J_H=\left[\dfrac{\partial F\left(H,q\right)}{\partial H}\right]_{H=H_0,q=q_0}$, $J_q=\left[\dfrac{\partial F\left(H,q\right)}{\partial q}\right]_{H=H_0,q=q_0}$ i.e.:

$$J_H = \begin{bmatrix} \dfrac{-C_1\alpha_1}{\left(H_{10}\right)^{1-\alpha_1}\beta_1\left(H_{10}\right)} & 0 & 0 \\[3ex] \dfrac{C_1\alpha_1}{\left(H_{10}\right)^{1-\alpha_1}\beta_2\left(H_{20}\right)} & \dfrac{-C_2\alpha_2}{\left(H_{20}\right)^{1-\alpha_2}\beta_2\left(H_{20}\right)} & 0 \\[3ex] 0 & \dfrac{C_2\alpha_2}{\left(H_{20}\right)^{1-\alpha_2}\beta_3\left(H_{30}\right)} & \dfrac{C_3\alpha_3}{\left(H_{30}\right)^{1-\alpha_3}\beta_3\left(H_{30}\right)} \end{bmatrix},$$

$$J_q = \begin{bmatrix} \dfrac{1}{\beta_1\left(H_{10}\right)} \\[2ex] 0 \\[1ex] 0 \end{bmatrix}$$

(16)

This linear model (16) can be used for the sensitivity analysis, for the stability analysis, and for the design of local controllers of the pump-controlled system.

5   Case Study – Almost Orthogonal Modeling

The multitank (imperfect system) model can be obtained in two ways [1]. The first method is to use (8) with direct appliance of genetic algorithm [26, 27] to the adjustment of the parameters c_i with respect to the minimization of the mean squared error:

$$J = \frac{1}{T}\int_0^T \left(y_S - y_M\right)^2 dt$$

(17)

where y_S is the output of unknown system and y_M is the model output. Genetic algorithm is an optimization technique based on the simulation of the phenomena taking place in the evolution of the species and adapting it to an optimization problem. They have demonstrated very good performances as global optimizers in many types of applications [1, 12, 28-30].

After obtaining the optimal parameters, Laplace transform is applied to the output signal. The model of the imperfect system can be directly obtained by dividing the output $Y(s)$ with the input $X(s)$.

The second method is to assume the form of the transfer function and then to adjust the function parameters in order to minimize the criteria function. In the case of imperfect systems, these coefficients will be dependent on ε. To obtain the model of the multitank system, we will use the almost orthogonal filter in Fig. 1,

which has three sections. The only known data about the system is the measured output - tank liquid level $H_2(t)$ for a given step input, shown in Fig. 3.
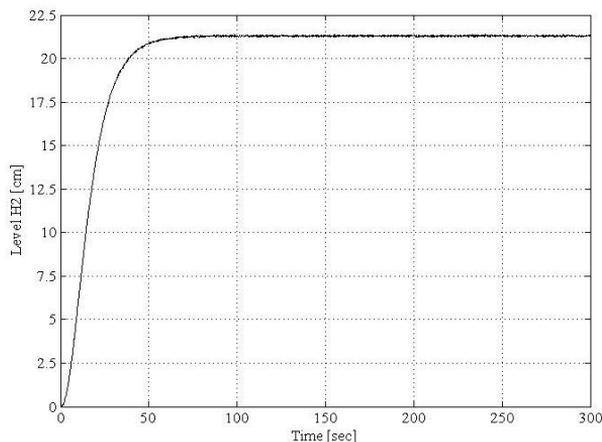

Figure 3
Step response of unknown hydraulic system

The transfer function of unknown imperfect system (the multitank system) can be obtained by applying inverse Laplace transform:

$$W_3^{(\varepsilon)}(s) = \frac{b_3(\varepsilon)s^3 + b_2(\varepsilon)s^2 + b_1(\varepsilon)s + b_0(\varepsilon)}{s^3 + 6s^2 + 11s + 6} \qquad (18)$$

with the parameters $b_i(\varepsilon)$, which directly depends on ε i.e., $b_i(\varepsilon) = f(c_i(\varepsilon))$, i=0,1,2,3 [1]. $c_i(\varepsilon)$ are coefficients from (8).

The transfer function directly depends on ε. Parameter ε is an uncertain quantity which describes the imperfection of the system. Variations of ε contain cumulative impacts of all imperfect elements, model uncertainties, and measurement noise on the system output. The range of variations can be determined by conducting several experiments. Hence, it is expected that the responses obtained from different experiments are mutually different. The responses are within certain boundaries, which depend on parameter ε i.e., on the real system components quality. So, $W_3^{(\varepsilon)}(s)$ represents the model of imperfect system, obtained by the almost orthogonal polynomials. This general model describes all the possible models whose parameters are in the range $\pm\varepsilon$ relative to the idealized system model. In our case, the experimental value obtained for ε is equal to 0.01.

The optimal values of the adjustable parameters c_0, c_1, c_2 and c_3, needed for the best model of the unknown imperfect system, are determined by using genetic algorithm. Genetic algorithm used in simulation has the following parameters: an

initial population of 150, a number of generations 300, a stochastic uniform selection, reproduction with 12 elite individuals, and Gaussian mutation with shrinking and scattered crossover. The chromosome has a structure which consists of four parameters encoded as real numbers: c_0, c_1, c_2, c_3. The goal of the simulation was to make a mean squared error as small as possible for a chosen input, i.e., to obtain the best model of the unknown system in the sense of mean squared error. So, relation (17) was used as the fitness function for the genetic algorithm. The experiment time was 300 seconds.

6   Case Study – Sensitivity Analysis

Applying the previously described procedure, the following parameters values are obtained: c_0=1.14095, c_1=-2.08069, c_2=1.29674, and c_3=-0.2287. To perform sensitivity analysis of the obtained imperfect systems model, the first parameter c_0 was changed in the limits ±10%, ±5% and ±1% from the optimal value obtained by genetic algorithm, while the other parameters kept their values. For each change of parameter c_0, the output in steady state was measured. Based on the equations given in Section 2, the sensitivity value related to parameter c_0 was calculated.

Table I

c_1=-2.08069, c_2=1.29674, c_3=-0.2287			
Tolerance	c_0	$\Delta y(\infty)$	$\left|u_{c_0}\right|$
+10%	1.255045	-0.004118	0.036094
+5%	1.197997	-0.002041	0.035777
+1%	1.152359	-0.000412	0.036112
-1%	1.129541	0.000413	0.036199
-5%	1.083903	0.002085	0.036549
-10%	1.026855	0.004216	0.036951

Table II

c_0=1.14095, c_2=1.29674, c_3=-0.2287			
Tolerance	c_1	$\Delta y(\infty)$	$\left|u_{c_1}\right|$
+10%	-1.872621	0.007451	0.035814
+5%	-1.976655	0.003382	0.032517
+1%	-2.059883	0.002142	0.020512
-1%	-2.059884	0.002137	0.020464
-5%	-2.184721	-0.003363	0.032327
-10%	-2.288759	-0.006888	0.033104

Table III

Tolerance	c_2	$\Delta y(\infty)$	$\left|u_{c_2}\right|$
c_0=1.14095, c_1=-2.08069, c_3=-0.2287			
+10%	1.426414	-0.003801	0.029312
+5%	1.361577	-0.001772	0.027331
+1%	1.309707	-0.000221	0.017099
-1%	1.283773	0.000202	0.015601
-5%	1.231903	0.001705	0.026309
-10%	1.167066	0.003678	0.028366

Table IV

Tolerance	c_3	$\Delta y(\infty)$	$\left|u_{c_3}\right|$
c_0=1.14095, c_1=-2.08069, c_2=1.29674			
+10%	-0.205831	0.000315	0.027585
+5%	-0.217261	0.000285	0.024962
+1%	-0.226413	0.000026	0.011368
-1%	-0.230987	0.000031	0.013229
-5%	-0.240135	0.000263	0.023025
-10%	-0.251572	0.000683	0.029864

The results are given in Table I, where $\Delta y(\infty)$ represents a deviation of the response in steady state and $\left|u_{c_0}\right|$ represents system sensitivity in steady state related to parameter c_0. We repeat this procedure for the other parameters c_1, c_2 and c_3 and the results are given in Tables II, III and IV respectively. The results demonstrated that the imperfect systems model is most sensitive to parameter c_0, and least sensitive to parameter c_3 (see Fig. 4). This result can be used in reality, when it is necessary to parametrically adjust the desired output value. In our case it is the best to use adjustable parameter c_0, because the output is the most sensitive to this parameter. If it is not possible to adjust the steady state output with only one parameter, it is necessary to make adjustments with two parameters c_0 and c_1, and so on. This also means that the model is most sensitive to parameter b_0, and the least sensitive to parameter b_3 with the highest index.

The results obtained by the developed method for sensitivity analysis using the almost orthogonal filter have been compared with those obtained by the nominal range sensitivity method [31], a known method for sensitivity analysis. Nominal range sensitivity analysis evaluates the effect on model outputs exerted by individual inputs, varying only one of the model inputs across its entire range of plausible values, while holding all other inputs at their nominal or base-case values.
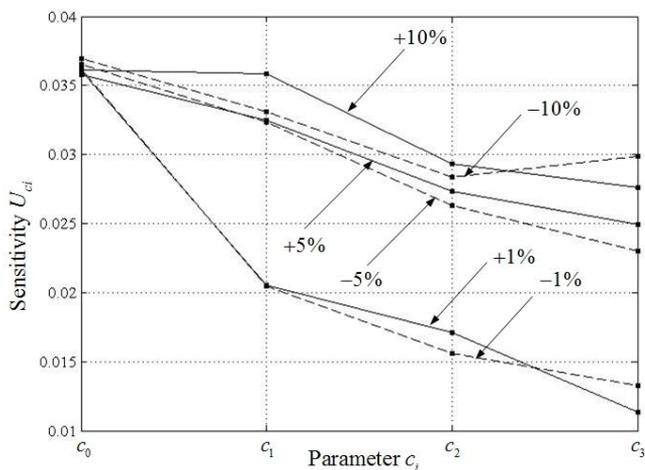
Figure 4
Graphic dependence $\left|u_{c_i}\right| = f\left(c_i\right)$

The results are given in Table V and Fig. 5. We can see that the results are similar to those shown in Fig. 4, with dependencies moved to the lower sensitivity values. The drawback of this method is that it does not include the effect of interactions or correlated inputs. The method is also time-consuming and demands a nominal range for each input.

Table V

Tolerance	$\left|u_{c_0}\right|$	$\left|u_{c_1}\right|$	$\left|u_{c_2}\right|$	$\left|u_{c_3}\right|$
+10%	0.033678	0.032718	0.028477	0.026577
+5%	0.031261	0.030171	0.026964	0.022911
+1%	0.030147	0.027221	0.014335	0.011377
-1%	0.028883	0.026508	0.013844	0.012561
-5%	0.031455	0.030054	0.026022	0.024163
-10%	0.032149	0.030115	0.028401	0.027476
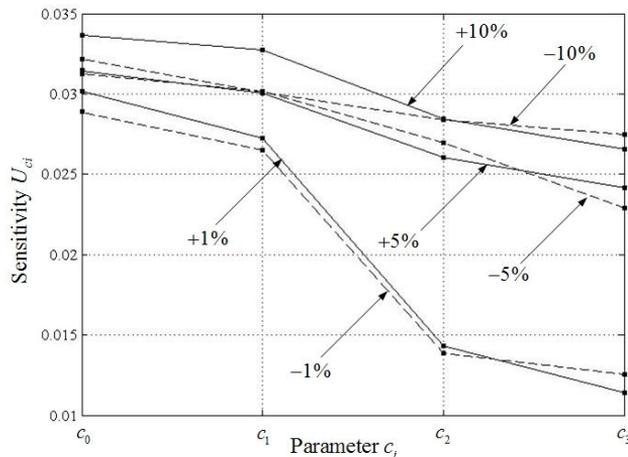
Figure 5

Graphic dependence $\left|u_{c_i}\right| = f\left(c_i\right)$

Conclusions

In this paper, the concept of the almost orthogonal polynomials is applied in the sensitivity analysis of imperfect systems. First, we designed almost orthogonal filters as generators of almost orthogonal functions. These filters can be used for the modeling, identification, simulation, and analysis of different dynamical systems as well as for the designing of adaptive systems. In this paper, an almost orthogonal filter has been used to obtain a model of an imperfect system, where the models parameters have been determined using genetic algorithm. The necessary mathematical relations for the proposed approach for determining sensitivity of imperfect systems are also given. Experiments with a multitank hydraulic system were performed to validate the theoretical results and to demonstrate that the method described in the paper is suitable for the sensitivity analysis of imperfect systems. The results have been compared with another known method for sensitivity analysis.

Acknowledgement

References

[1]   M. Milojković, S. Nikolić, B. Danković, D. Antić, Z. Jovanović: Modelling of Dynamical Systems Based on Almost Orthogonal Polynomials, Mathematical and Computer Modelling of Dynamical Systems, Vol. 16, No. 2, pp. 133-144, 2010

[2]   B. Danković, P. Rajković, S. Marinković: On a Class of Almost Orthogonal Polynomials, in *Lecture Notes in Computer Science* 5434, S. Margenov, L. G. Vulkov, and J. Wasniewski, Eds., Springer-Verlag, Berlin, 2009, pp. 241-248

[3]   Y. C. Schorling, T. Most, C. Bucher: Stability Analysis for Imperfect Systems with Random Loading, Proceedings of the 8th International Conference on Structural Safety and Reliability, Newport Beach, California, USA, June 17.-22, 2001, pp. 1-9

[4]   B. Danković, S. Nikolić, M. Milojković, Z. Jovanović: A Class of Almost Orthogonal Filters, Journal of Circuits, Systems, and Computers, Vol. 18, No. 5, pp. 923-931, 2009

[5]   T. Most, C. Bucher, Y. C. Schorling: Dynamic Stability Analysis of Nonlinear Structures with Geometrical Imperfections under Random Loading, Journal of Sound and Vibration, Vol. 276, pp. 381-400, 2004

[6]   H. Chen, L. Li: Semisupervised Multicategory Classification with Imperfect Model, IEEE Transactions Neural Networks, Vol. 20, No. 10, pp. 1594-1603, 2009

[7]   G. M. Coghill, A. Srinivasan, R. D. King: Qualitative System Identification from Imperfect Data, Journal of Artificial Intelligence Research, Vol. 32, pp. 825-877, 2008

[8]   B. Danković, Z. Jovanović, S. Nikolić, D. Mitić: Modelling of Imperfect System Based on Almost Orthogonal Polynomials, Proceedings of the 9th International Conference on Telecommunications in Modern Satellite, Cable and Broadcasting Services, TELSIKS 2009, Niš, Serbia, October 7-9, 2009, Vol. 2, pp. 514-517

[9]   G. Szegö: Orthogonal Polynomials, American Mathematical Society, Colloquium Publications, 23, Providence, 1975

[10]   Ya. L. Geronimus: Polynomials Orthogonal on a Circle and Interval. Fiz. Mat. Lit., Moscow, 1958

[11]   D. Antić, B. Danković, M. Milojković, S. Nikolić: Dynamical Systems Modeling Based on Legendre Orthogonal Function, TEHNIKA-Elektrotehnika, Vol. 58, No. 5, pp. 1-6, 2009

[12]   B. Danković, D. Antić, Z. Jovanović, S. Nikolić, M. Milojković: Systems Modeling Based on Rational Functions, Scientific Bulletin of UPT,

Transactions on Automatic Control and Computer Science, Vol. 54(68), No. 4, pp. 149-154, 2009

[13]   P. Heuberger, P. Van den Hof, B. Wahlberg: Modelling and Identification with Rational Orthogonal Basis Functions. Springer-Verlag, London, 2005

[14]   S. Nikolić, D. Antić, B. Danković, M. Milojković, Z. Jovanović, S. Perić: Orthogonal Functions Applied in Antenna Positioning, Advances in Electrical and Computer Engineering, Vol. 10, No. 4, pp. 35-42, 2010

[15]   P. C. McCarthy, J. E. Sayre, B. L. R. Shawyer: Generalized Legendre Polynomials, Journal of Mathematical Analysis and Applications, Vol. 177, pp. 530-537, 1993

[16]   A. Beny, R. H. Torres: Almost Orthogonality and a Class of Bounded Bilinear Pseudodifferential Operator, Mathematical Research Letters, Vol. 11, pp. 1-11, 2004

[17]   I. Ben-Yaacov, F. Wagner: On Almost Orthogonality in Simple Theories, Journal of Symbolic Logic, Vol. 69, pp. 398-408, 2004

[18]   B. Danković, P. Rajković: On a Class of Almost Orthogonal Polynomials, Proceedings of the 4th International Conference on Numerical Analysis and Application, Lozenetz, Bulgaria, 2008, pp. 241-248

[19]   B. Danković, D. Antić, Z. Jovanović, D. Mitić: On a Correlation between Sensitivity and Identificability of Dynamical Systems, Proceedings of the IEEE International Joint Conferences on Computational Cybernetics and Technical Informatics, ICCC-CONTI 2010, Timisoara, Romania, May 27-29, 2010, pp. 361-366

[20]   L. Breierova, M. Choudhari: An Introduction to Sensitivity Analysis. MIT System Dynamics in Education Project, 1996

[21]   R. Tomović: Sensitivity Analysis of Dynamic Systems. McGraw-Hill, New York, 1983

[22]   R. Gumovski: Sensitivity of the Control Systems. Nauka, Moscow, 1993, (in Russian)

[23]   R. Pintelon, J. Schoukens: System Identification. IEEE Press, New York, 2001

[24]   R. E. Precup, S. Preitl: Stability and Sensitivity Analysis of Fuzzy Control Systems. Mechatronics Applications, Acta Polytechnica Hungarica, Vol. 3, No. 1, pp. 61-76, 2006

[25]   Inteco, Modular Servo System-User's Manual (2008) Available at www.inteco.com.pl

[26]   J. H. Holland: Adaptation in Natural and Artificial Systems. University of Michigan Press, Ann Arbor, 1975

[27] M. Mitchel: An Introduction to Genetic Algorithms. MIT Press, Cambridge, 1996

[28] D. Mitić, D. Antić, S. Nikolić, M. Milojković: Identification of the Multitank System Using Genetic Algorithm, Proceedings of the XLV International Scientific Conference on Information, Communication and Energy Systems and Technologies, ICEST 2010, Ohrid, Macedonia, June 23-26, 2010, Vol. 1, pp. 453-456

[29] F. Durovsky, L. Zboray, Ž. Ferkova: Computation of Rolling Stand Parameters by Genetic Algorithm, Acta Polytechnica Hungarica, Vol. 5, No. 2, pp. 59-70, 2008

[30] B. Danković, D. Antić, Z. Jovanović, S. Nikolić, M. Milojković: Systems Modeling Based on Legendre Polynomials, Proceedings of the 5th International Symposium on Applied Computational Intelligence and Informatics, SACI 2009, Timisoara, Romania, May 28.-29, 2009, pp. 241-246

[31] A. C. Cullen, H. C. Frey: Probabilistic Techniques in Exposure Assessment. Plenum Press, New York, 1999