An Easy Way for the Generation of Structural Topologies under Random Loads Using Cellular Automata

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Abstract: The topology optimization, is a dynamically developing research area, with numerous applications, to many research and engineering fields and despite the decades of progress, still remains one of the most important research tasks, within the area of structural and material design. The implementation of innovative, efficient and versatile optimization approaches and methods, stimulate this process. Among many research problems, where topology optimization is present, there is generation of topologies for structures under random loads. As reported in the literature, random changes in load magnitude, angle of load application, as well as its position can affect resulting topologies. The idea of the paper is to propose an easy to implement numerical approach, which allows for the prediction of resulting topologies of structures, in the case of load uncertainty. This simple, but effective technique, based on transforming random loads into deterministic problem of multiple loads, is discussed, its numerical implementation based on the idea of Cellular Automata, is described and some examples are presented to illustrate the concept. Based on obtained results, it can be concluded that the approach discussed in the paper can be a useful tool to support the research within structural topology optimization, under random loads.

Keywords: structural topology optimization; random loads; Cellular Automata

1 Introduction

The papers by Bendsoe and Kikuchi [1] and Bendsoe [2], dated back to the late 80s of the 20th Century, are broadly treated as the pioneering ones within the field of structural topology optimization. Since then, the intensive research on this subject has been conducted for decades and the results have been widely presented in engineering literature. The numerous approaches to generation of optimal topology have been presented together with appearing concepts which have been implemented in various engineering and research fields. A broad discussion on various aspects of topology optimization has been provided by many survey papers: e.g. [3-6] with the recent ones by Ribeiro et al. [7], Logo and Ismail [8].

Despite a long lasting development the topology optimization still remains one of the most important research fields within the area of structural and material design. Novel ideas and formulations emerge simultaneously with new fields of their application. The researchers community continuously works on innovative and efficient topology optimization methods and algorithms, what stimulates that progress. The spectrum of numerous solutions of topology optimization problems ranges from classic Michell structures to sophisticated contemporary engineering ones.

Among many research problems where topology optimization is present there is structural topology optimization with consideration of load uncertainties. This subject has received for the recent decades increasing interest within the design optimization community. The variety of approaches and techniques has been proposed to handle with random and uncertain loads and the comprehensive study of associated problems has been provided in numerous publications. Among them, there are problems of reliability-based topology optimization and robust topology optimization. The recent papers [9-13] and the newly published by Yin et al. [14], Shen et al. [15], Wang et al. [16], Tauzowski et al. [17], along with presentation of particular subjects, bring extensive literature review on topology optimization under uncertainty.

The idea of this paper is to present an easy to implement numerical approach which allows to predict resulting topologies of structures in the case of random loads. The simple but effective technique based on transforming random loads into deterministic problem of multiple loads is implemented. From computational point of view the applied approach is based on the concept of Cellular Automata. Cellular Automata (CA) are developed to represent behavior of complicated systems in a relatively easy way. The special local rules are implemented with a view to mimic the performance of a considered system. Then, local physical quantities are respectively updated, what allows to describe the global behavior of the system.

Since the late 1940s when von Neumann and Ulam proposed the concept of Cellular Automata this idea has been found interesting by researchers representing various fields. In the paper by Inou et al. [18] probably for the first time topology optimization has been discussed within CA approach. Since then many papers have been published on that subject. The majority of them have appeared during last two decades, see e.g. [19-22] or [23]. The efficient CA algorithm has been also proposed and then developed by Bochenek and Tajs-Zielińska [24] [25] and recently [26] [27].

The outline of the paper is as follows. In Section 2.1, the topology optimization problem is formulated with extension to multiple load case described in Section 2.2. The concept of Cellular Automata is introduced in Section 2.3, together with the detailed description of numerical algorithm built based on this idea. The illustrative introductory example discussed in Section 2.4 presents implementation of the paper concept. Next, utilizing results of the preliminary analysis, the Cellular Automaton is applied in Section 3 to solve the selected tasks of topology generation under random loads. Where available, the obtained topologies are compared with those works reported in the literature. Based on the results of performed tests, the paper ends in Section 4, with concluding remarks.

2 Problem Formulation and Numerical Treatment

2.1 Structural Topology Optimization

The idea of performing topology optimization is to generate within a specified design domain a material layout so as to meet the assumed optimality criteria. The optimized structure gains a new shape and material layout since some parts of material are relocated and others are selectively removed. This allows, for example, creating a stiffer construction with minimal amount of material. Generated this way concept solutions can be the inspiration for further efforts of engineers and designers. Over years many formulations of topology optimization problems have been proposed. The discussion on this subject one can find for example in the paper by Lewiński et al. [28]

When searching for the stiffest design, it is very often that the structure compliance is minimized, since minimal compliance results in maximal stiffness of the optimized structure. The compliance can be defined as the work done by the applied external forces, as proposed in the early papers [1] [2]. Along with the development of topology optimization, especially within the power law approach, the problem has been formulated also as minimization of the elastic strain energy stored in the deforming structure. The compliance has been defined then as twice the total strain energy, see e.g. Stolpe and Svanberg [29]. The problem of equivalence of these two forms of objective function has been discussed together with proposals of new approaches and fields of implementation. Hence, it is important that the structure is elastic, subjected to specified external forces and fixed structural support. In more complex cases, including mixed boundary conditions or various types of loading, modifications of the above formulations are required. The papers by Niu et al. [30], Zhang et al. [31] or Araujo et al. [32] may serve here as examples.

The finite element based strategy for structural topology optimization has gained strong attention of researchers and engineers and considerable progress within this area has been observed. This paper follows the structural topology optimization problem formulated in widely recognized paper by Sigmund [33]. The objective function and constraints are defined within the finite element approach. The objective is to minimize compliance c represented by Eq. (1). The available material volume fraction κ is defined and treated as the constraint imposed on structure volume V in the optimization process, Eq. (2):

Minimize…

$$c(\mathbf{d}) = \mathbf{u}^T \mathbf{k} \mathbf{u} = \sum_{n=1}^{N} d_n^p \mathbf{u}_n^T \mathbf{k}_n \mathbf{u}_n \tag{1}$$

subject to:

$$V(\mathbf{d}) = \kappa V_0 \tag{2}$$

$$\mathbf{k}\mathbf{u} = \mathbf{f} \tag{3}$$

$$0 < d_{\min} \le d_n \le 1 \tag{4}$$

The quantity $\mathbf{u}_n$ represents displacement vector whereas $\mathbf{k}_n$ stands for the stiffness matrix. Both are defined for N elements. The design variable d_n which represents the material relative density is assigned to each element. In Eq. (3) $\mathbf{k}$ is the global stiffness matrix, $\mathbf{u}$ stands for the global displacement vector and $\mathbf{f}$ represents vector of forces. Singularity of d_n is avoided due to the simple bounds imposed in Eq. (4) on the design variables with $d_{\min}$ as a non-zero minimal value of relative density.

The SIMP - solid isotropic material with penalization, see e.g. Bendsoe and Sigmund [34], in the form of power law is adapted as the material representation. For each finite element the modulus of elasticity E_n is a function of the design variable d_n:

$$E_n = d_n^p E_0 \qquad \rho_n = d_n \rho_0 \tag{5}$$

In Eq. (5) p (typically p=3) is responsible for penalization of intermediate densities what allows controlling the design process and leads to obtaining black-and-white resulting structures. The quantities E_0 and ρ_0 stand for modulus of elasticity and material density, both defined for a solid material. The topology generation process leads to a redistribution of material within design domain, which results in removing parts unnecessary from design criterion point of view.

2.2 Topology Optimization for Multiple Load Case and Paper Concept

As described in the papers by Bendsoe [35], Bendsoe and Sigmund [34], Sigmund [33] the compliance minimization problem formulated in the previous section can be extended to multiple load case. Similarly to the formulation of multi-objective optimization problem, the case of multiple load case can be implemented to the formulation of the topology optimization problem by using the weighted sum of objectives/compliances subjected to all considered load cases. The topology optimization algorithm has to be therefore only slightly modified to cope with this problem, what was illustrated in [33] by the two load case example, see Fig. 1, which is recalled below.

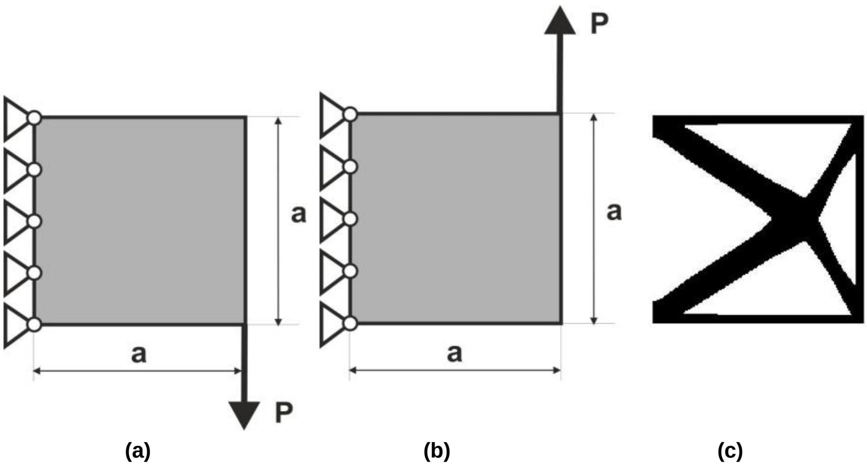


Figure 1

(a) The square structure: support and two loads (b) The resulting topology for the two load case

Dealing with the above problem, the equilibrium equations are solved for both load cases and the objective is defined as the sum of compliances referring to each case. The numerical implementation, as described in detail in [33], requires only modification of a few lines of numerical code, namely insertion of the sum of compliances which replaces the single one.

The idea of the present paper is to adapt the above approach to deal with random loads. These are treated here as sets of loads for which topology is generated. The values, positions or angles of application are generated from particular random distributions.

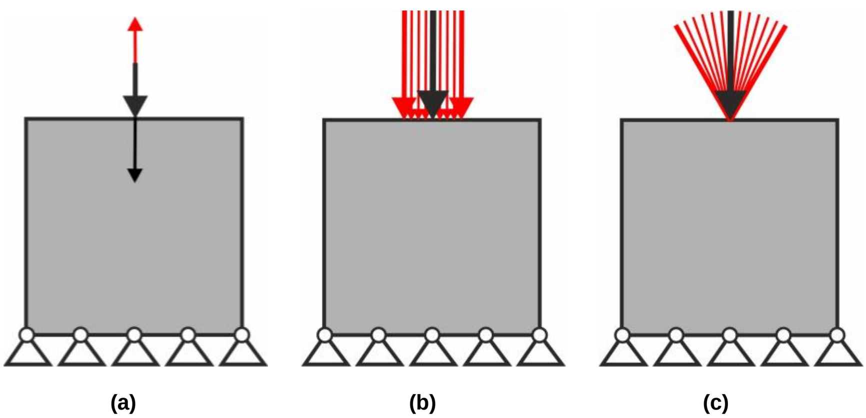


Figure 2

(a) Random magnitude of load (b) Random load position (c) Random angle of load application

Allowing for random changes of load magnitude, angle or position of load application, see Fig. 2, the formulations (1)-(4) has to be slightly modified. In what follows, the objective is now represented by the sum of compliances, Eq. (5), calculated for each load case

$$\sum_{i=1}^{L} c^{(i)} \tag{5}$$

whereas $\mathbf{u}^{(i)}$ and $\mathbf{f}^{(i)}$ refer to displacement and force vectors representing each load case.

The sets of loads are randomly generated according to the following:

$$P^{(i)} = P + r\Delta_P \text{, or } \quad P^{(i)} = P + (2r-1)\Delta_P \tag{6}$$

for random load value,

$$x_P^{(i)} = x_P + r\Delta_x \text{, or } \quad x_P^{(i)} = x_P + (2r-1)\Delta_x \tag{7}$$

for random load position with load value unchanged, and

$$\alpha^{(i)} = r\Delta_\alpha \;\text{ or }\; \alpha^{(i)} = (2r-1)\Delta_\alpha \;\text{ and }\; P_1^{(i)} = P\cos\alpha^{(i)} \quad P_2^{(i)} = P\sin\alpha^{(i)} \tag{8}$$

for random angle of load application.

In Eqs. (6-8) the Δ_P, Δ_x, Δ_α represent admisible change of load magnitude, position and angle of application, respectively. The random value r is taken from uniform distribution or from the normal one. In numerical implementation the *rand* and *randn* Matlab functions have been applied.

2.3 Cellular Automata Rules for Topology Optimization

The effectiveness of topology optimization process is determined by selecting a proper method of topology generation. Heuristic optimization techniques are gaining popularity among researchers because they are easy for numerical implementation, do not require gradient information, and one can easily combine this type of algorithm with any finite element structural analysis code.

In this paper an efficient heuristic approach based on the concept of Cellular Automata is proposed. The implementation of CA requires decomposition of the design domain usually into uniform lattice of cells which are usually equivalent to finite elements while performing analysis and topology optimization. It is assumed that the interaction between cells takes place only within the neighboring cells and is governed by local rules. The rules are identical for all cells and are applied simultaneously to each of them.

In this paper, a heuristic local update rule [27] is implemented utilizing the Jacobi update scheme, where updating is based on the states of the surrounding cells determined in the previous iteration, see Eq. (9):

$$d_n^{new} = d_n + [F(n) + \sum_{k=1}^{M} F(k)] \frac{m}{M+1} \tag{9}$$

In Eq. (9), m denotes move limit (e.g. m=0.2). The values of F for a central cell (n) and for M neighboring ones (k) are calculated based on local compliance values. In what follows, the structural analysis is performed first and based on obtained results the values of local compliances are calculated for all cells/elements. Then, compliances are sorted in ascending order and those having the lowest and the highest values are identified. As the next step, N_1, N_2 are selected and values of $F(n)$ are assigned according to Eq. (10):

$$F(n) = \begin{cases} -C_\alpha & \text{if } n < N_1 \\ f(n) & \text{if } N_1 \le n \le N_2 \\ -C_\alpha & \text{if } n > N_2 \end{cases} \tag{10}$$

As to the intermediate interval $N_1 \le n \le N_2$ a monotonically increasing function representing elements compliances is selected and then its values are assign to the design elements, respectively. Here, the linear function in Eq. (11) is built to fulfill the conditions $f(N_1) = -C_\alpha$ and $f(N_2) = C_\alpha$, thus:

$$f(n) = 2C_\alpha \frac{n}{N_2 - N_1} - C_\alpha \frac{N_2 + N_1}{N_2 - N_1} \tag{11}$$

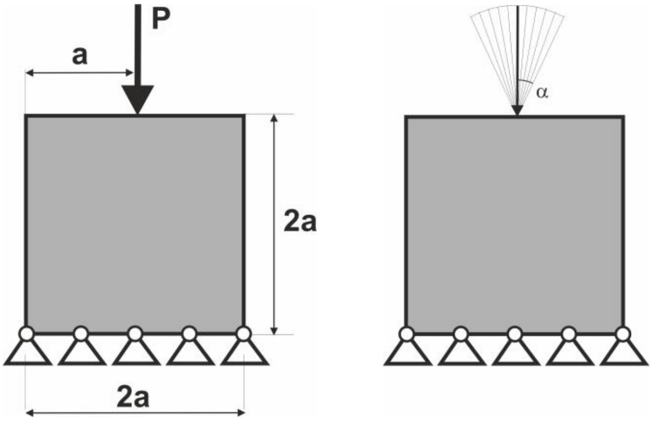
The quantity C_α is a user-specified parameter, usually equal to 1.

The numerical algorithm was built in order to implement the above proposed design rule. As for the optimization procedure, the sequential approach was adapted, meaning that for each iteration, the structural analysis performed for the optimized element is followed by a local updating process. Simultaneously, a global volume constraint is applied for a specified volume fraction. As a result, the generated topologies preserve a specified volume fraction of a solid material during the optimization process.

2.4 Introductory Example

The rectangular structure, discussed by many authors, shown in the Fig. 3 has been chosen as the introductory example.

Volume fraction has been selected as 0.2. The regular mesh of 2500 (50×50) elements has been implemented to perform structural analysis and topology optimization, for E=1, P=1, ν=0.3. The Moore type neighborhood has been applied. Generation of topology has been performed for a single deterministic load as well as for the increasing number of random ones. In the subsequent figures the resulting structures are presented.

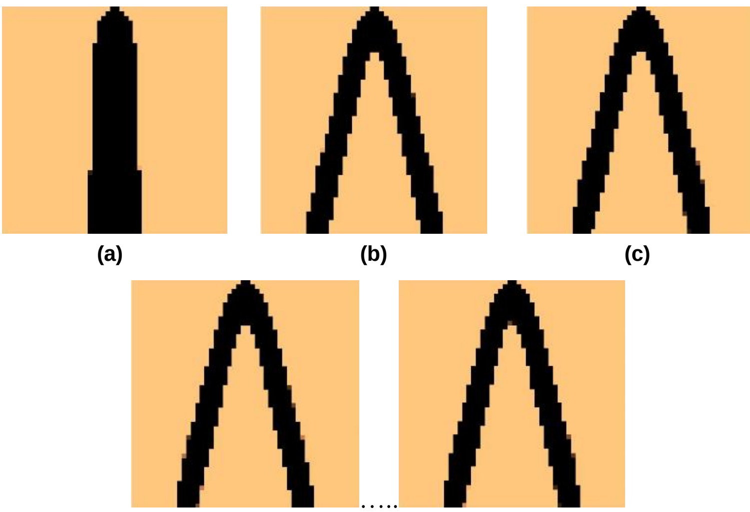


(a) (b)

Figure 3

(a) The square structure under single (deterministic) load (b) The structure under random loads

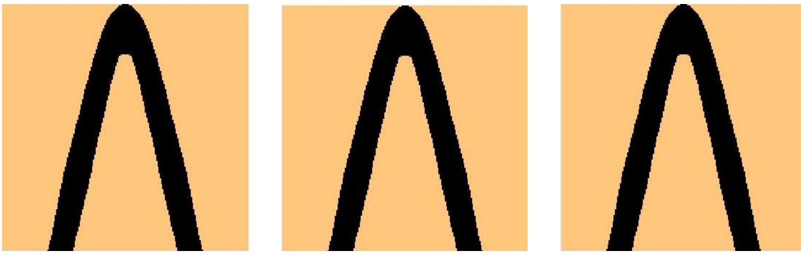
It can be seen from the results shown in the Fig. 4 that there is no significant difference between topologies generated under increasing number of loads. Moreover, the average values of compliances are close each other.



(d) (e)

Figure 4

The square structure 50×50 cells. Topologies obtained for: (a) Deterministic load (b) Random 100 loads (c) Random 1000 loads (d) Random 10000 loads (e) Random 100000 loads. Uniform distribution $\Delta_\alpha = \pi/18$

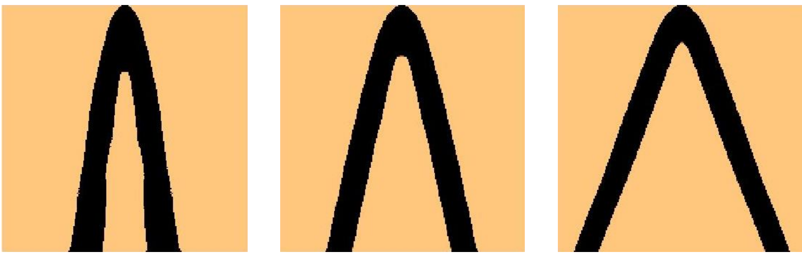


(a) (b) (c)

Figure 5

The square structure 100×100 cells. Topologies obtained for: (a) Random 100 loads (b) Random 1000 loads (c) Random 10000 loads. Uniform distribution $\Delta_\alpha = \pi/18$

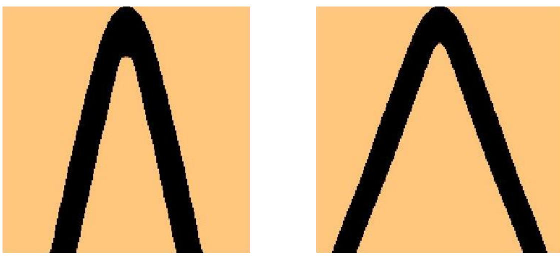
The calculations have been repeated for the square structure which has been discretized with mesh of 100×100 cells. The topologies obtained for 100, 1000 and 10000 applied random loads are shown in the Fig. 5.



(a) (b) (c)

Figure 6

The square structure 200×200 cells. Topologies obtained for: (a) Random 1000 loads, $\Delta_\alpha = \pi/36$ (b) Random 1000 loads, $\Delta_\alpha = \pi/18$. (b) Random 1000 loads. Uniform distribution $\Delta_\alpha = \pi/9$



(a) (b)

Figure 7

The square structure 200×200 cells. Topologies obtained for: (a) Random 1000 loads, $\Delta_\alpha = \pi/18$ (b) Random 1000 loads, $\Delta_\alpha = \pi/9$. Normal distribution.

Another comparison regards topologies generated for various Δ_α. The considered structure has been discretized with mesh of 200×200 cells. The topologies obtained for $\Delta_\alpha = \pi/36, \pi/18, \pi/9$ and 1000 applied random loads are shown in the Fig. 6.

The topologies obtained for $\Delta_\alpha = \pi/18, \pi/9$ and 1000 applied random loads from normal distribution are shown in the Fig. 7.

As can be seen from the results of the test example discussed in this section the implemented strategy which mimics random loads allows to generate topologies in an easier way.

3 Results of Topology Generation under Random Loads

In order to illustrate the paper concept more thorough some numerical test examples have been selected. Below presented are the results of numerical calculations. The attention has been focused mainly on the case of random angle of load application.

3.1 The Michell Structure

The structure shown in the Fig. 8 has been selected as the test example. The angle of applied load, see Fig. 8b, is treated as the random value which is generated according to uniform or normal distribution.

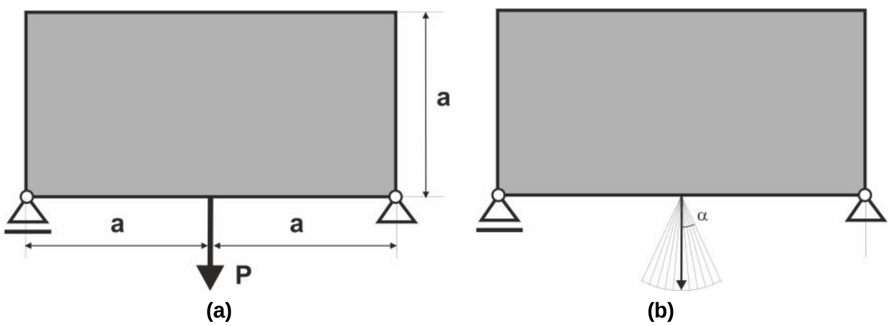
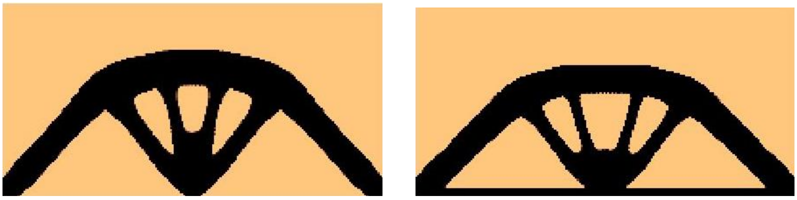


Figure 8
The rectangular Michell structure: (a) load and support (b) random angle of applied load

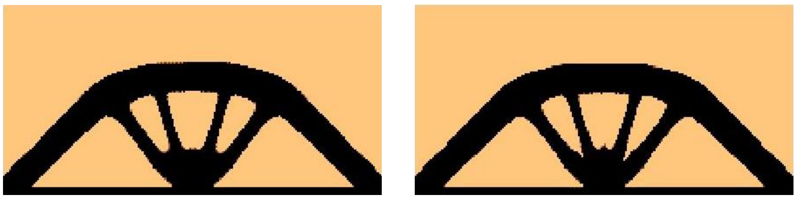
Volume fraction has been selected as 0.3. The regular mesh of 20000 (200×100) elements has been implemented to perform structural analysis and topology optimization, for E=1, P=1, ν=0.3. The Moore type neighborhood has been applied. The topology generation has been performed for a single deterministic load as well as for selected random ones. In the Fig. 9a the resulting structure obtained for the single load case is presented.



(a) **(b)**

Figure 9

The Michell structure 200×100 cells. (a) Topology obtained for the deterministic load. (b) Random 100 loads. Uniform distribution $\Delta_{\alpha} = \pi/6$



(a) **(b)**

Figure 10

The Michell structure 200×100 cells. Topologies obtained for: (a) Random 1000 loads (b) Random 10000 loads. Uniform distribution $\Delta_{\alpha} = \pi/6$

The topologies generated under random loads are shown in the Figs. 9b, 10. The number of loads has been selected as 100, 1000 and 10000, and random values have been generated from uniform distribution for $\Delta_{\alpha} = \pi/6$.

The topology obtained for $\Delta_{\alpha} = \pi/6$ and 1000 applied random loads taken from normal distribution are shown in the Fig. 11.

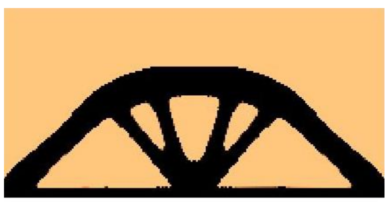


Figure 11

The Michell structure 200×100 cells. Topology obtained for random 1000 loads. Normal distribution $\Delta_{\alpha} = \pi/6$

For the structure under random load the additional line connecting supports has been generated. The obtained results can be compared with the ones reported in [36] [16].

3.2 The Tower Structure

The second example is the tower structure shown in the Fig. 12. The angle of applied load, see Fig. 12b, is treated as the random value which is generated according to uniform or normal distribution.

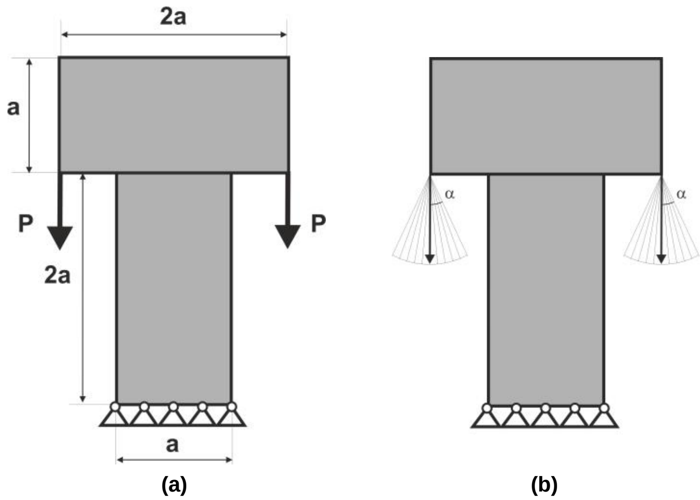


Figure 12
The tower structure (a) Load and support (b) Random angle of applied load

Volume fraction has been selected as 0.25. The regular mesh of 38400 elements has been implemented. The data are the same as for other examples. The Moore type neighborhood has been applied. The topology generation has been performed for a single deterministic load as well as for selected random ones.

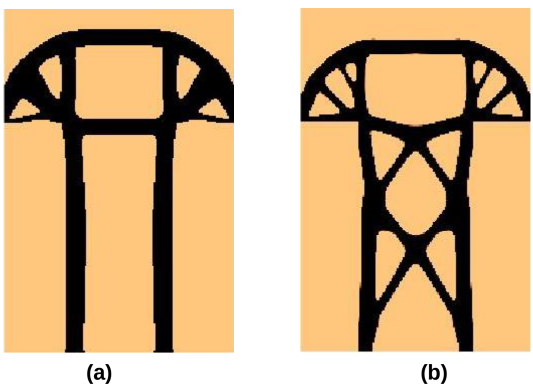


Figure 13
The tower structure 240×160 cells. (a) Single load case, deterministic solution (b) Random 100 loads. Uniform distribution $\Delta_\alpha = \pi/4$

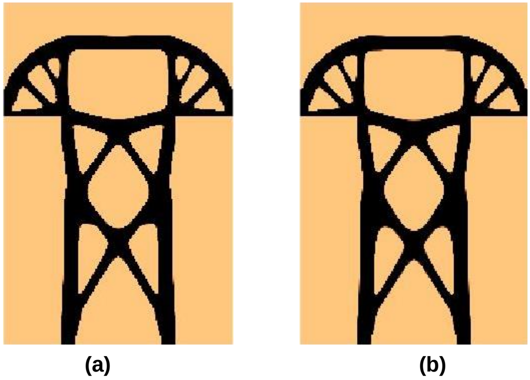


Figure 14
The tower structure. Topologies obtained for: (a) Random 1000 loads (b) Random 10000 loads. Uniform distribution $\Delta_\alpha = \pi/4$

In the Fig. 13a the resulting structure obtained for the single load case is presented, whereas the Figs. 13b, 14 show topologies found for uniform random loads

The topology obtained for $\Delta_\alpha = \pi/9$ and 1000 applied random loads taken from normal distribution are shown in the Fig. 15.

For the structure under random load the additional stiffening within column part has been generated. The above solution can be compared with the one presented in [37] and [38], where similar changes in topology layout are observed when comparing deterministic and randomized results.

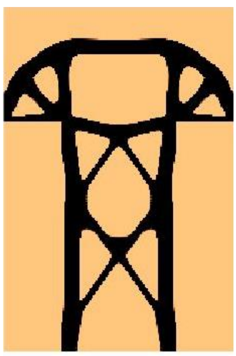


Figure 15
The tower structure. Topology obtained for random 1000 loads. Normal distribution $\Delta_\alpha = \pi/9$

3.3 The Foot Structure

The structure shown in the Fig. 16 has been selected as the next test example.

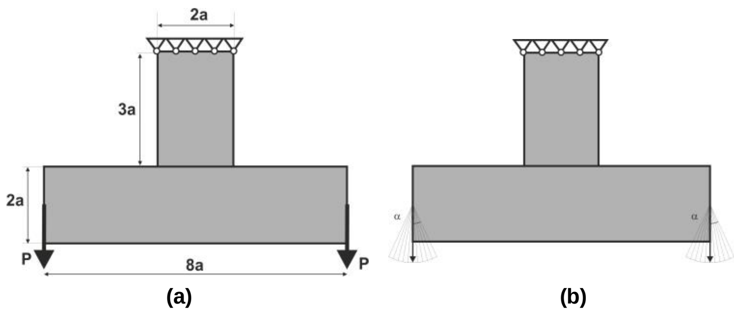


Figure 16
The foot structure. (a) Load and support (b) Random angle of applied load

The angle of applied load, see Fig. 16b, is treated as the random value which is generated according to uniform or normal distribution. Volume fraction has been selected as 0.25. The regular mesh of 25000 elements has been implemented. The Moore type neighborhood has been applied. The topology generation has been performed for a single deterministic load as well as for selected random ones. In the Fig. 17a the resulting structure obtained for the single load case is presented.

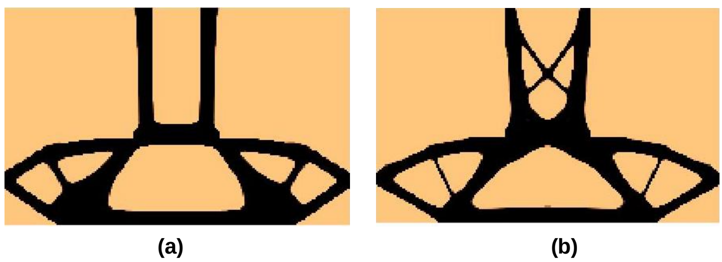


Figure 17
The foot structure. 200×125 cells. (a) Single load case, deterministic solution (b) Random 100 loads. Uniform distribution $\Delta_\alpha = \pi/4$

The topologies generated for random loads are shown in the Figs. 17b, 18. The number of loads has been selected as 100, 1000 and 10000, random values have been generated from uniform distribution and $\Delta_\alpha = \pi/4$.

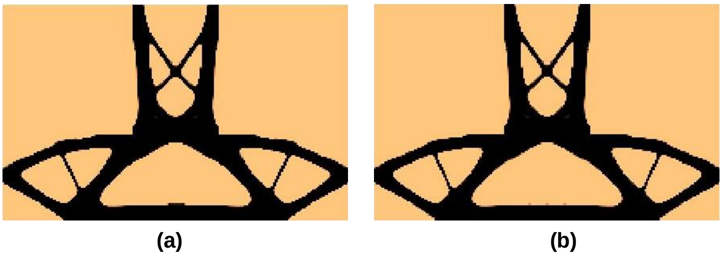


Figure 18

The foot structure. Topologies obtained for: (a) Random 1000 loads (b) Random 10000 loads. Uniform distribution $\Delta_\alpha = \pi/4$

The topology obtained for $\Delta_\alpha = \pi/9$ and 1000 applied random loads taken from normal distribution are shown in the Fig. 19.

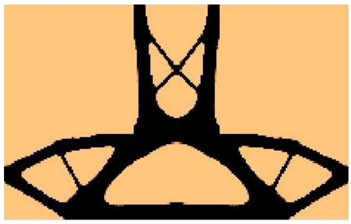


Figure 19

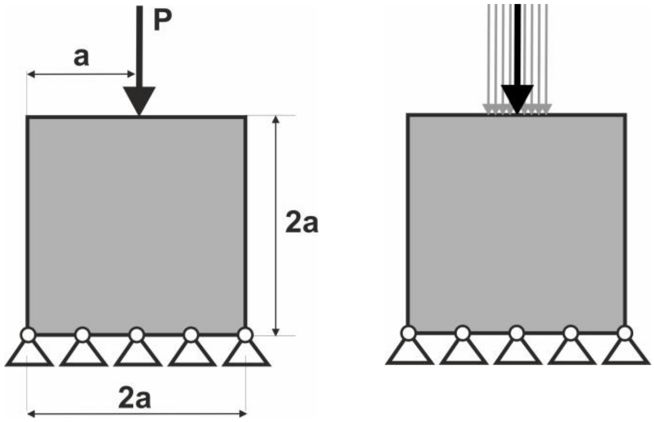
The foot structure. Topology obtained for random 1000 loads. Normal distribution $\Delta_\alpha = \pi/9$

For the structure under random load the additional stiffening within middle part of the structure has been generated. The results of this section can be compared with the one presented in [37] and [39].

3.4 The Structures under Random Load Position or Magnitude

As stated within introduction the load magnitude and its position can be also random variables. The numerical approach of this paper allows considering such cases therefore two additional, simple examples are presented to slightly broaden the discussion of this section.

As to the random load position, the structure used as introductory example is revisited, see Fig. 20. This time horizontal position of load can vary within a specified range defined by Δ_x.



(a) (b)

Figure 20
(a) The square structure under single (deterministic) load (b) The structure under random load position

(a) (b)

Figure 21
The square structure. (a) Single load case, deterministic solution (b) Topology obtained for random 1000 loads and Δ_x =0.2a. Uniform distribution

The topology has been generated and the result is presented in the Fig. 21b. One can see that it is different from these obtained under random angle of load application.

The final example regards illustration of generation of topology in the case of load magnitude changing at random. These changes usually do not affect significantly resulting topologies, however if the load orientation can vary results can be different from deterministic ones. The test example discussed in [40] is here revisited and resulting topologies are presented in the Fig. 22. The obtained structures comply with the ones reported in [40].

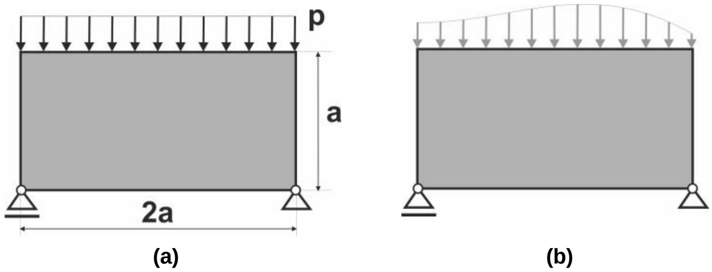


Figure 22
The rectangular structure. (a) Load and support (b) Random load magnitude

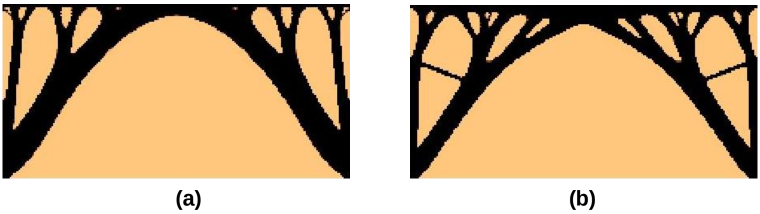


Figure 23
The rectangular structure. (a) Single load case, deterministic solution (b) Topology obtained for random 1000 loads. Normal distribution, mean of load magnitude equals 0.

Conclusions

In this paper, an easy to implement, numerical approach, which allows for the prediction of resulting topologies of structures, in the case of random loads, has been introduced and implemented. Based on obtained results, of selected test examples, it can be concluded that reasonably low number of load cases, can mimic load acting at random and is enough to obtain final topologies. The topologies generated under random loads, represent the same layout changes, compared to other deterministic solutions, as those reported in the literature. It seems that the approach discussed in this paper, can be a useful tool, supporting the research within structural topology optimization under random loads.

The benefit of the proposed approach based on a CA generator, besides the possibility of obtaining fine optimal topologies, is also that mesh dependency, and the “grey areas” can be eliminated without using any additional filtering. Moreover, the presented algorithm is versatile, which allows for its easy combination with any structural analysis solver, built on the finite element method (FEM). The presented approach can also be applied to 3D problems.

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