Fractal Analysis of Forward Exchange Rates[1]

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Abstract: In this paper we work with nonparametric methods in modeling and analyzing the financial times series. We use the concept of fractal dimension for measuring the complexity of time series of observed financial data. The aim of this paper is to distinguish between the randomness and determinism of the financial information. We will compare the fractal analysis of the selected forward exchange rates. Fractal analysis has been introduced into financial time series by Mandelbrot and Peters. Due to the financial crisis this theory has gained new momentum. Fractal analysis indicates that conventional econometric methods are inadequate for analyzing financial time series. Adequate analysis of the financial time series allows us to predict precisely the future values and risks connected with portfolios that are influenced. We test for fractional dynamic behavior in a 1-month forward exchange rate USD into GBP and Gold Price against USD.

Keywords: fractal analysis; estimation dimension; long memory; financial time series

1 Introduction

The purpose of this paper is to show a potential presence of stochastic long memory in economic and financial time series. The long term memory property describes the high-order correlation structure of a series. The long memory existence in financial time series may be caused by investors' reactions to market information. Some investors react to information as it is received, while some investors wait for confirmation of the new information and they do not react until a trend is clearly established. Classical capital market theory assumes that the markets follow a random walk, and this means that the current prices reflect all available information and future price changes can be determined only by new information. With all prior information already reflected in prices. This means each day's price movement is unrelated to the previous day's activity. It is assumed that all investors immediately react to new information, so that the future

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is unrelated to the past or the present. Thus, all investors react to new information with equal probability. This assumption has been necessary for the application of the Central Limit Theorem to capital market analysis. But investors really do not make their decisions in this manner. Although the reaction of the investors is random, they may prefer some information and therefore the probability of their decisions is not identical; their decision is biased in some direction. Therefore the market may follow a biased random walk.

Biased random walks were studied by Hurst in 1940s. Hurst was a hydrologist studying the discharge rates of the River Nile at Aswan. He detected long memory behavior on the River Nile data. Hurst attributed this result to be a consequence of the flow rate having serial correlation. The *Hurst parameter H* displays long range dependence. Hurst [5] motivated Mandelbrot and his co–workers (Mandelbrot and van Ness [9]) to introduce fractional Gaussian noise to model long memory phenomena.

Long memory or long term dependence is observed in contemporary financial time series. There exist a number of studies that have investigated the issue of persistence in financial asset returns. Using the Hurst rescaled-range (R/S) method, Greene and Fielitz (1977) reported long memory in daily stock returns series. This result has been overturned by Lo (1991) via the development and implementation of the more appropriate modified R/S method. The absence of long memory in stock returns is also reported by Aydogan and Booth (1988), Cheung, Lai, and Lai (1993), Cheung and Lai (1995), Crato (1994), and Barkoulas and Baum (1996). Booth, Kaen, and Koveos (1982) and Cheung (1993) report long-memory evidence in spot exchange rates. Helms, Kaen, and Rosenman (1984), Cheung and Lai (1993), Fang, Lai, and Lai (1994), and Barkoulas, Labys, and Onochie (1997) report that stochastic long memory may be a feature of some spot and futures foreign currency rates and commodity prices.

The presence of fractal structure in asset returns raises a number of theoretical and empirical issues. First, as long memory represents a special form of nonlinear dynamics, it calls to question linear modeling and invites the development of nonlinear pricing models at the theoretical level to account for long memory behavior.

The rest of this paper is organized as follows. Section two introduces the stochastic processes and self–similar stochastic processes. Section three briefly describes fractional Brownian motion. Section four describes fractal dimension and fuzzy sets. Data and empirical estimates are discussed in section five. The paper ends with a summary of our results.

2 Stochastic Processes and Self-Similar Stochastic Processes

Given an observed time series, a question which is of interest is whether the data were generated by a dynamical system of finite dimension or whether the system is stochastic. In many observed time series it is not clear what the fundamental underlying process is that drives the system. However, in real processes we observe certain aspects that are the evidence of an underlying, more complex process. We can observe processes that display power-law scaling and long range dependence. The main problem is, if a given time series is related to an underlying more substantial process, whether it is possible to determine whether the underlying process is driven by a deterministic set of equations or a stochastic system, or whether the process is self-similar. This opens the question of what is the difference between a deterministic and a stochastic process and whether is it possible to make this distinction based on empirical observations.

Cutler [2] defined what is meant by stochastic and deterministic time series. Lamperti [6] introduced the definition of a self-similar stochastic process. Mandelbrot [8] introduced Fractional Brownian Motion. We give a brief summary in this section.

Definition 1

The time series $\{X(t_n): n=1,2,\ldots\}$ is said to be *strictly stationary* if for any finite collection $t_1, t_2, \ldots, t_n$ and for all τ,

$$Pr\{X(t_1)<x_1, \ldots, X(t_n)<x_n\} = Pr\{X(t_1+\tau)<x_1, \ldots, X(t_n+\tau)<x_n\}.$$

Definition 2

A mapping $g: \chi\rightarrow\psi$, between the metric spaces χ and ψ with metrics ρ_1 and ρ_2 respectively, is said to satisfy a *Lipschitz condition* if, for all $x_1, x_2 \in \chi$,

$$\rho_2(g(x_1), g(x_2)) \leq k\rho_1(x_1, x_2),$$

where k is a constant. If in addition, g is one to one and g^{-1} also satisfies a Lipschitz condition on its domain, then g is bi-Lipschitz.

Definition 3

Let $\{X(t_n): n=1,2,\ldots\}$ be a strictly stationary time series with values in ψ. The *predictive dimension,* denoted by ζ, is defined as the smallest $n\geq 1$ such that there exists a mapping $\Psi: \psi^n\rightarrow\psi$ such that

$$X(t_n) = \Psi[X(t_1), \ldots, X(t_{n-1})],$$

with probability 1. If no function Ψ exists for all $n\geq 1$, then $\zeta=\infty$.

Cutler formulated a theorem that says that a strictly stationary process with known predictor function Ψ and finite predictive dimension ζ can be predicted as a function of the previous ζ observations. Subsequently this is used for defining a stochastic and deterministic time series.

Theorem

Let $\{X(t_n): n=1,2,\ldots\}$ be a strictly stationary time series with finite predictive dimension ζ and predictor function Ψ. Then, for all integers $m\geq 0$,

$$Y(t_{m+1+\zeta})=\Psi(Y(t_{m+1}), Y(t_{m+2}), \ldots, Y(t_{m+\zeta})),$$

with probability 1.

Definition 4

A strictly stationary time series $\{X(t_n): n=1,2,\ldots\}$ is said to be *deterministic* if $\zeta<\infty$ and *stochastic* if $\zeta=\infty$, where ζ is the predictive dimension.

The following discussion will be concerned with Lamperti's [6] idea of scaling in a process $X(t)$. Firstly we introduce the notion of *equality of finite dimensional distributions.*

Definition 5

Let $X_1(t)$ and $X_2(t)$ be two stochastic processes. We will say that these processes have the same finite dimensional distributions if, for any $n\geq 1$ and $t_1, t_2, \ldots, t_n$

$$(X_1(t_1), X_1(t_1), \ldots, X_1(t_n)) \overset{d}{=} (X_2(t_1), X_2(t_1), \ldots, X_2(t_n)) \text{ or } (X_1(t)) \overset{d}{=} (X_2(t)),$$

where $\overset{d}{=}$ denotes equality of probability distributions [4].

Definition 6

d–dimensional process $X(t)$ is a *semi–stable process,* if it obeys a simple continuity condition and, for $s>0$, the relationship

$$\{X(st)\} \overset{d}{=} \{b(s)\ (X(t)+c(s))\}$$

holds, where $b(s)$ is a positive function and $c(s)\in R^{\boldsymbol{d}}$.

Lamperti [6] showed that if $X(t)$ is a proper semi-stable process and $X(0)=0$, then $c(s)=0$ and $b(s)=s^H$ where H is a positive constant. That is,

$$\{X(st)\} \overset{d}{=} \{s^H X(t)\}. \tag{1}$$

Definition 7

The increments of a random function $\{X(t): -\infty<t<\infty\}$ are said to be self–similar with parameter H if for any $s>0$ and any τ

$$\{X(st+\tau) - X(\tau)\} \overset{d}{=} \{s^H (X(t+\tau) - X(\tau))\}.$$

If the increments of $X(t)$ are self–similar and $X(0)=0$, then $X(t)$ is also self–similar (see equation (1)). If $X(t)$ has self–similar and stationary increments and is mean square continuous, then it can be shown that $0 \le H < 1$.

The covariance structure is derived from scaling law as follows [4]:

Let $X(t)$ be process with stationary self–similar increments. Then the *covariance function* is

$$E[(X(t+\tau+1)-X(t+\tau))(X(t+1)-X(t))] = \frac{1}{2}\sigma_H^2 \left\{ |\tau+1|^{2H} + |\tau-1|^{2H} - 2|\tau|^{2H} \right\},$$

where $\sigma_H^2 = E[(X(t+1)-X(t))^2]$ for all t.

The process $X(t)$ is said to be *isotropic* if

$$\{X(t) - X(s)\} \stackrel{d}{=} \{X(|t-s|)\}. \tag{2}$$

3 Fractional Brownian Motion

A *Gaussian process* is uniquely determined by its auto covariance function. *Fractional Brownian Motion* is a unique Gaussian self-similar process that we will denote as $B_H(t)$ [4]. The increments of fractional Brownian motion are referred to as *fractional Gaussian noise*. If $B_H(0)=0$, then the process $B_H(t)$ is isotropic (see equation (2)).

When $H=0.50$, $B_H(t)$ is simply *Brownian motion*. The system is independently distributed. When H differed from 0.50, the observations are not independent. Each observation carried a "memory" of all the events that preceded it. What happens today influences the future. Where we are now is a result of where we have been in the past. Time is important. The impact of the present on the future can be expressed as a correlation:

$$C=2^{(2H-1)}-1, \tag{3}$$

where C is the correlation measure and H is the Hurst exponent. The time series is random, and events are random and uncorrelated. The present does not influence the future. Its probability density function can be a normal curve, but it does not have to be.

When $H>0.50$ the autocorrelations are positive and have a power-law decay, hence long range dependence. If $0.50 \le H < 1.00$, the time series have a persistent or trend–reinforcing character. If the time series was up (down) in the last period, then the chances are that it will continue to be positive (negative) in the next period. Trend is apparent. The strength of the trend-reinforcing behavior, or persistence, increases as H approaches 1.0. The strength of the bias depends on how far H is above 0.50. The closer H is to 0.5, the noisier it will be, and the less defined its trends will be. Persistent series are called by Mandelbrot as fractional Brownian motion, or biased random walk.

When $H<0.50$ the correlation are negative and have a rapid decay. For $0 \leq H<0.50$ the time series is antipersistent, or ergodic. If the time series was up in the previous period, it is more likely to be down in the next period. Conversely, if it was down before, it is more likely to be up in the next period. The strength of this antipersistent behavior depends on how close H is to zero. The closer it is to zero, the closer C in equation (3) moves toward –0.50, or negative correlation. This time series is more volatile than a random series.

The Hurst coefficient H characterizes long-memory dependence. Self-similarity of the time series is characterized by fractal dimension. Fractal dimension expresses the regularity of series and states how similarity scales up when such a time series is observed over a longer time interval. The self-similarity could be also regarded as a measure of geometrical complexity of an object under discussion.

In principle, fractal dimension and Hurst coefficient are independent of each other: fractal dimension is a local property, and long-memory dependence is a global characteristic. Nevertheless, the two notions are closely linked. For self-affine processes, the local properties are reflected in the global ones, resulting in the relationship $D+H=2$ between fractal dimension, D, and Hurst coefficient, H.

The determination of the fractal dimension is inherently associated with set-based constructs. The generic box dimension [8] measures in which way the number of occupied boxes (those including the elements of the time series) increases when the size of the box decreases. The other common techniques of fractal determination uses a so-called correlation dimension in which a count of elements concerns a family of spheres constructed around each data point. What is common to the existing techniques (in spite of evident technical differences) is that all of them exploit sets regarded as information granules that allow us to see only a certain part of the phenomenon. The changes in the size of the information granules imply how large a part we are taking into consideration. Information granulation is an example of abstraction. There are numerous facets of the granular information processing, and there are a variety of formal frameworks in which such information granulation takes place. These include, for instance, set theory, fuzzy sets, random sets, rough sets and many others [7].

4 Fractal Dimension and Fuzzy Sets

In this section, we review the main constructs of fractal dimension and then proceed with their generalization in terms of information granules expressed in the language of fuzzy sets [7].

Consider a time series $\{X(t_n): n=1,2,\ldots, N\}$, where $X(t_n) \in R$ and t_n denote discrete time moments in which the values of this time series are recorded. In this sense, we are provided with a collection of two-dimensional elements $S=\{ X(t_n): n=1,2,\ldots, N\}$. The structural complexity of S is measured by a fractal dimension $\hat{D}$ defined by the following limit

$$\hat{D} = \lim_{\varepsilon \to 0} \frac{\log(N(\varepsilon))}{\log(\varepsilon)}, \tag{4}$$

where $N(\varepsilon)$ is a number of boxes of size ε used to cover the object (here the given time series). In essence, the above relationship relates to the power law stating that $N = \varepsilon^{\hat{D}}$. In practice, the fractal dimension has to be estimated with the use of some experimental data. A collection of ''c'' experiments concerns a determination of the number of boxes $N(\varepsilon)$ for a given value of the size of the box. Then experimental pairs $(\varepsilon_j, N(\varepsilon_j))$, $j= 1, 2, \ldots; c$ are used to determine parameters of the linear model. It can be easily shown (from (3)) that in a double logarithmic model of the form

$$\log N(\varepsilon) = D\log(\varepsilon) + C \tag{5}$$

the fractal dimension D appears as a slope of the computed regression line. The regression model itself is constructed through a minimization of the well known performance index Q treated as a sum of squared errors

$$Q = \sum_{k=1}^{c} (\log N(\varepsilon_k) - D\log(\varepsilon_k) - c)^2 . \tag{6}$$

The most intuitive approach to the determination of the fractal dimension uses the box method [8]. Another method uses a sphere of radius ε. Total number of points covered by the spheres $N(\varepsilon)$ is equal to

$$N(\varepsilon) = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{N} \Omega_{ij}(\varepsilon) \tag{7}$$

where $\Omega_{ij}(\varepsilon)$ is a sphere defined as follows

$$\Omega_{ij}(\varepsilon) = \begin{cases} 1 \text{ if } \sqrt{(t_i - t_j)^2 + (x_i - x_j)^2} \le \varepsilon \\ 0 \text{ otherwise} \end{cases} . \tag{8}$$

Pedrycz and Bargiela [7] used fuzzy set $A_{ij}(\varepsilon)$ to compute of the fractal dimension:

$$N(\varepsilon) = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \Omega_{ij}(\varepsilon) A_{ij}(\varepsilon) \text{, where} \tag{9}$$

$$A_{ij}(\varepsilon) = \exp\left(\frac{-(x_i - x_j)^2}{\varepsilon^2} \right). \tag{10}$$

For the time series we can use:

$$\Omega_i(\varepsilon) = \sum_{i=1}^{\varepsilon} (x_i - M_\varepsilon) \tag{11}$$

where $\Omega_i(\varepsilon)$ is cumulative deviation over ε period, M_ε is an average x_i over period of length of ε.

$$N(\varepsilon) = \max(\Omega_i(\varepsilon)) - \min(\Omega_i(\varepsilon)). \tag{12}$$

This approach is known as *R/S* analysis ([7, 10, 11]), where usually $N(\varepsilon)$ is denoted as R/S and ε as *n*.

The detailed computations of the fractal dimension are described, for example, in [7, 10] and they are realized on the basis of the regression model (5).

Mandelbrot used *R/S* analysis which was developed by Hurst [5]. Mandelbrot, Taqqu and Wallis demonstrated the superiority of *R/S* analysis over more conventional methods of determining long-range dependence, such as analyzing autocorrelations, variance ratios and spectral decompositions, in their several papers. In this paper our analysis will be based on the study described in Peters [10] or Robinson [11]. In this paper we compute Hurst coeficient *H* and his expected value $E(H)$ using modified R/S analysis[2] and we will verify null hypothezis: *The time series is random walk*.

To verify this hypothesis, we calculate expected value of the adjusted range[3] $E(R/S_n)$ and its variance[4] $Var(E(R/S_n))$.

[2] The *R/S* statistics is modified so that its statistical behavior is invariant over a general class of short memory processes, but deviates for long-memory processes. ([11], p. 91)

[3] This formula was derrived by Anis and Lloyd ([10], p. 71)

[4] Variance was calculated by Feller ([10], p. 66)

$$E(R/S_n) = \frac{n-0.5}{n} \cdot \left(n \cdot \frac{\pi}{2} \right)^{-0.5} \sum_{r=1}^{n-1} \sqrt{\frac{(n-r)}{r}} \tag{13}$$

$$Var(E(R/S_n)) = \left(\frac{\pi^2}{6} - \frac{\pi}{2} \right) \cdot n\ . \tag{14}$$

Using the results of equation (13) we can generate the expected values of the Hurst exponent. The expected Hurst exponent will vary depending on the values of n we use to run the regression. Any range will be appropriate as long as the system under study and the $E(R/S_n)$ series cover to the same values of n. For financial purpose, we will begin with n=10. The final value of n will depend on the system under study.

R/S values are random variables, normally distributed and therefore we would expect that the values of H would also be normally distributed ([10], p. 72):

$$Var(H_n) = \frac{1}{T}, \tag{15}$$

where T is total number of observations in the sample. Note that the $Var(H_n)$ does not depend on n or H, but it depends on the total sample size T. Now t-statistics will be used to verify the significance of the null hypothesis.

If Hurst exponent H is approximately equal to its expected value $E(H)$, it means that the time series is independent and random during the analysed period (the Hurst exponent is insignificant). If the Hurst exponent H is greater (smaller) than its expected value $E(H)$, the time series is persistent (antipersistent) (the Hurst exponent is significant). If the series exhibits a persistent character, then the time series has long memory and the ratios R/S_n will be increasing. If the ratios R/S_n will be decreasing the time series will be antipersistent. The "breaks" may signalize a periodic or nonperiodic component in the time series with some finite frequency. We calculated the V–statistics to estimate precisely where this break occurs [10]:

$$V_n = (R/S)_n \big/ \sqrt{n} \tag{16}$$

5 Data and Empirical Results

The data set consists of daily forward 1-month exchange rate USD into GBP and Gold Price against USD from 02/01/1979 to 04/11/2010 for a total 8050 daily observations. These were obtained from Bank of England[5].

[5] http://www.bankofengland.co.uk

We begin by applying *R*/*S* analysis to the 1-month forward exchange rates USD into GBP. During the period 02/01/1979–04/11/2010, the Hurst coefficient *H* of the 1-month forward exchange rate USD into GBP is equal to 0.5702. The expected Hurst exponent is equal to *E*(*H*)=0.5407. The variance of *E*(*H*) is 1/*T*=1/8050, for Gaussian random variables. The standard deviation of *E*(*H*) is 0.0111. The Hurst exponent for the daily 1-month forward exchange rates USD into GBP is 2.6513 standard deviations away from its expected value. This is highly significant result at the 95% level. The time series has persistent character. Also plotted is $E(R/S_n)$ (dashed line) as a comparison against the null hypothesis that the system is an independent process (Figure 1). There is clearly a systematic deviation from the expected values. However, breaks in R/S graph (see Figure 1) appear. To estimate precisely where this break occurs, we calculated *V*–statistics (Figure 1). V-statistics clearly stops groving at n=50, n=322, n=575 or n=805 observations. These "breaks" may be signal of a periodic or nonperiodic component in the time series. We will run regression to estimate the Hurst exponent for R/S_n values in the next subperiods: $n<50$, $50\leq n \leq 4025$, $10<n<322$, $322\leq n \leq 4025$, $10<n<575$, $575\leq n \leq 4025$, $10<n<805$ and $805\leq n \leq 4025$. Table 1 and Table 2 show the regression results. During periods for $n<50$, $10<n<322$ and $10<n<575$ the time series has random character. The Hurst exponents are insignificant. During periods for $50\leq n \leq 4025$, $322\leq n \leq 4025$ and $575\leq n \leq 4025$ the time series has persistent character. The Hurst exponent is significant. It means that ancient history had random character and recent history has a long memory effect. During periods for $50\leq n \leq 805$ the time series has a persistent character, but during period $805\leq n \leq 4025$ the time series has an antipersistent character and the Hurst exponent is significant. We have found that 1-month forward exchange rate USD into GBP has 4 nonperiodic cycles. The longest is a 805-day cycle, or about 3 years. The shortest is a 50-day cycle, or about 10 weeks.

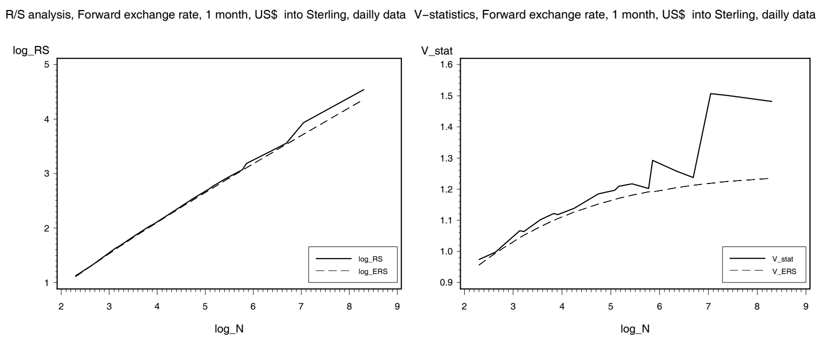

Figure 1

R/S analysis and V statistics of the daily log return of USD into GBP, (1979-2010)

H=0.5702, *E*(*H*)= 0.5407

Table 1
Regression results, 1-month forward FX rate USD vs GBP, estimation of the Hurst exponent, (1979-2010, daily data)

1-month forward	R/S	E(R/S)	R/S	E(R/S)	R/S	E(R/S)	R/S	E(R/S)
FX rate GBPvsUSD	10<n<50		50≤ n ≤4025		10<n<322		322≤ n≤4025	
Intercept	-0.236	-0.241	-0.200	0.041	-0.149	-0.152	-0.247	0.093
Hurst exponent	0.592	0.589	0.574	0.522	0.564	0.561	0.581	0.515
Standard Error	0.009	0.006	0.048	0.008	0.019	0.016	0.072	0.003
R squared	0.999	1.000	0.995	1.000	0.999	0.999	0.984	0.999
Number of obs.	7		12		13		6	
Significance	0.260		4.701		0.323		5.993	

Table 2
Regression results, 1-month forward FX rate USD vs GBP, estimation of the Hurst exponent, (1979-2010, daily data)

1-month forward	R/S	E(R/S)	R/S	E(R/S)	R/S	E(R/S)	R/S	E(R/S)
FX rate GBP vs USD	10<n<575		575≤ n≤4025		10<n<805		805≤ n≤4025	
Intercept	-0.146	-0.131	-0.235	0.118	0.558	-0.119	0.508	0.126
Hurst exponent	0.563	0.554	0.580	0.511	-0.126	0.551	0.486	0.510
Standard Error	0.021	0.019	0.095	0.001	0.025	0.020	0.001	0.001
R squared	0.999	0.999	0.964	1.000	1.000	0.999	1.000	1.000
Number of obs.	15		4		16		3	
Significance			6.191		-60.715		-2.153	

R/*S* analysis of Gold Price against USD from 02/01/1979 to 04/11/2010 exhibits random behavior. The Hurst coefficient H is equal to 0.547, $E(H)$ = 0.540 and it is insignificant (see Figure 2). Table 3 summarizes the regression results. However we found 2 breaks on *R*/*S* plot (respectively in *V*-statistics plot, see Figure 2) for n=161 and n=322. During periods 10<n<161 and 10≤n≤322 the time series has random character, but during periods 161<n<4025 and 322≤ n ≤4025 the time series has persistent character. The presence of the persistent value of H confirms that Gold prices against USD have fractal structure in recent history. We found one periodic cycle with length 161 (or approximately 32 weeks).

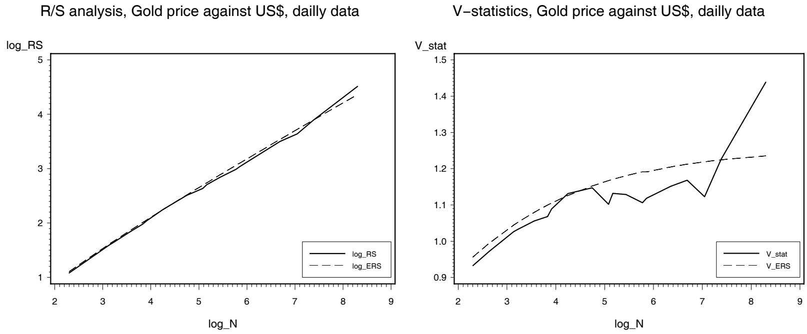

Figure 2
R/S analysis and V statistics of the daily log return of Gold prices against USD, (1979-2010)
H=0.547, $E(H)$ = 0.540

Table 3
Regression results, Gold vs GBP, estimation of the Hurst exponent, (1979-2010, daily data)

Gold versus GBP	R/S	E(R/S)	R/S	E(R/S)	R/S	E(R/S)	R/S	E(R/S)
	$10<n<161$		$161\leq n\leq 4025$		$10<n<322$		$322\leq n\leq 4025$	
Intercept	-0.2523	-0.2040	0.2446	0.0672	-0.1633	-0.1644	-0.4465	0.0894
Hurst exponent	0.5854	0.5763	0.5631	0.5181	0.5582	0.5639	0.5910	0.5150
Standard Error	0.0113	0.0109	0.0482	0.0041	0.0268	0.0146	0.0469	0.0027
R squared	0.9994	0.9994	0.9941	0.9999	0.9979	0.9934	0.9934	0.9999
Number of observation	9		10		12		7	
Significance	0.8165		4.0375		-0.5114		6.8189	

Conclusion

In this paper, we propose a fractal analysis of the selected financial time series. In both causes, we found fractal structure. Nonperiodic cycles for forward exchange rate affirm evidence that the currency markets may be nonlinear systems. Currency markets are characterized by abrupt changes traceable to central bank intervention attempts by governments to control the value of each respective currency.

Periodic cycle in the time series Gold prices against USD may be related to the economic cycle. The cycle length measures how long it takes for a single period's influence to reduce to immeasurable amounts. In statistical terms, it is the decorrelation time of the series. In terms of nonlinear dynamics, memory effect is lost when this time expires.

Information obtained by fractal analysis can be used as the basis for momentum analysis and other forms of technical analysis. The second use is in choosing periods for model development, particularly for back testing.

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