Fuzzy Methods for Comparing Project Situations and Selecting Precedent Decisions

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Abstract: This paper characterizes project management task aspects substantiating the expediency of applying fuzzy methods for comparing project situations and selecting precedent decisions. It discusses methods for assessing the similarity of the fuzzy features of project situations, based on operations with fuzzy sets, pseudometric distances between fuzzy sets, and the fuzzy distance between fuzzy sets. The paper also describes approaches to comparing fuzzy project situations on the basis of aggregating the results of comparing individual features with the use of various convolutions or fuzzy inference algorithms, as well as by individual priority features. An example of selecting precedent project decisions relevant to project situations is given, where relative pseudometric distance between fuzzy sets is used to estimate the degree of similarity among the fuzzy features of project situations, and the modifiable Mamdani fuzzy inference algorithm is used for comparing fuzzy project situations and selecting precedent project decisions.

Keywords: project management; fuzzy project situations; fuzzy distance; fuzzy logic inference; precedent decision

1 Introduction

Case-based reasoning methods are currently used for making effective project decisions [1-3]. The characteristic features of project management tasks, which substantiate the advisability of applying these methods, are as follows:

- the “non-stationarity” of the conceptual and terminological tools, the rapidly changing structure and parameters of the project management subject area [4, 5];
- the incompleteness and insufficiency of information on the project situations to be compared, including the expert nature of information on project situation features and their heuristic representation [6];
- temporal and resource limitations imposed on the formation and selection of precedent decisions;
- the complexity of the homogeneous representation of project situations and precedent project decisions;
- different “scale” of project situations, herewith implying elaboration of similar project decisions [7];
- the complexity of estimating the similarity of the fuzzy features of project situations;
- the complexity of comparing project situations due to different compositions of the features, their different significances and degree of consistency;
- the task of selecting precedent project decisions relevant to project situations generally reduces to the task of classification and depends on the corresponding method of comparing fuzzy project situations [8].

The above mentioned causes a contradiction between the necessity to increase the degree of the reasonableness of project management decisions by means of the application of automated procedures of data accumulation and processing and a certain imperfection of the currently available decision support methods in terms of taking into account the specific features of innovative projects.

The characteristic features of project management tasks mentioned in the Introduction justify the expediency of using the representation of the features of project situations and project precedent decisions in the form of fuzzy sets and fuzzy relations.

We introduce the following nomenclature:

$Q_l = \left\{ \tilde{q}_n^{(l)} \mid n = 1, ..., N \right\}$ is the *l*-th typical fuzzy project situation ($l = 1,\ldots, L$) represented by fuzzy sets (numbers) $\tilde{q}_n^{(l)}$ of its features;

$P_k = \left\{ \tilde{p}_n^{(k)} \mid n = 1, ..., N \right\}$ is the *k*-th current project situation ($k = 1,\ldots, K$) represented by fuzzy sets (numbers) $\tilde{p}_n^{(k)}$ of its features; N is the number of features to be compared.

This way it is possible to perform fuzzy granulation and to determine the relevance between a project situation and a precedent decision.

The paper presents the analysis of fuzzy methods for comparing project situations for selecting relevant precedent decisions.

2 Assessment of Similarity between the Corresponding Features of Project Situations to be Compared

The method for assessing the degree of similarity between the respective features of the project situations to be compared must meet the following requirements:

1) The method must not only indicate which feature is more/less significant, but also enable one to judge about the significance of the difference in the features.

2) The method must preserve the adequacy of similarity assessment: for fuzzy values of features with disjoint supports; for coinciding fuzzy values of features; for crisp values of features.

3) The method must take into account the form of membership functions for fuzzy values of features with disjoint supports. Herewith, it is desirable that high values of the α-levels of features being compared have a greater effect on the comparison result.

4) The method must enable one to compare fuzzy values of features different in both the width of the basic range and the form of the membership functions.

Based on the formulated requirements, to assess the similarity between the corresponding features of project situations to be compared, methods based on the following approaches can be applied:

- operations with fuzzy sets (disjunctive sum, bounded difference, disjoint sum, etc.) [9-14];
- pseudometric distance between fuzzy sets (Hamming distance, Euclidean distance, etc.) [15-20];
- ranking indexes for fuzzy sets (numbers) [21];
- logical indexes for comparing fuzzy sets (numbers) [22].

2.1 Application of Operations with Fuzzy Sets

Fuzzy set operations defined in general form through t-norms and s-norms can be used for assessing the similarity of fuzzy values of features. In what follows, we offer the most widespread examples of such operations.

1) The difference $\left(\tilde{q}_n^{(l)} - \tilde{p}_n^{(k)}\right)$ with the membership function

$$\mu_{\left(\tilde{q}_n^{(l)} - \tilde{p}_n^{(k)}\right)}(x) = \min\left(\mu_{\tilde{q}_n^{(l)}}(x), \left(1 - \mu_{\tilde{p}_n^{(k)}}(x)\right)\right),\ \forall x \in X, \quad (1)$$

where X is a universal set on which the fuzzy sets $\tilde{q}_n^{(l)}$ and $\tilde{p}_n^{(k)}$ are specified.

2) The bounded difference $\left(\tilde{q}_n^{(l)} \theta \tilde{p}_n^{(k)}\right)$ with the membership function

$$\mu_{\left(\tilde{q}_n^{(l)} \theta \tilde{p}_n^{(k)}\right)}(x) = \max\left(0,\ \mu_{\tilde{q}_n^{(l)}}(x) - \mu_{\tilde{p}_n^{(k)}}(x)\right),\ \forall x \in X. \quad (2)$$

3) The disjunctive sum $\left(\tilde{q}_n^{(l)} \oplus \tilde{p}_n^{(k)}\right)$ with the membership function

$$\mu_{\left(\tilde{q}_n^{(l)} \oplus \tilde{p}_n^{(k)}\right)}(x) = \max\left(\min\left(\mu_{\tilde{q}_n^{(l)}}(x), \left(1 - \mu_{\tilde{p}_n^{(k)}}(x)\right)\right), \min\left(\left(1 - \mu_{\tilde{q}_n^{(l)}}(x)\right), \mu_{\tilde{p}_n^{(k)}}(x)\right)\right), \quad (3)$$
$$\forall x \in X.$$

4) The disjoint sum $\left(\tilde{q}_n^{(l)} \Delta\, \tilde{p}_n^{(k)}\right)$ with a membership function:

$$\mu_{\left(\tilde{q}_n^{(l)} \Delta\, \tilde{p}_n^{(k)}\right)}(x) = \left|\mu_{\tilde{q}_n^{(l)}}(x) - \mu_{\tilde{p}_n^{(k)}}(x)\right|,\ \forall x \in X\ . \quad (4)$$

The choice of this or that operation with fuzzy sets leads to different results of assessing the similarity of fuzzy features. Such a choice is justified by the identified conditions of comparing project situations and by the system of preferences of the decision making person.

Example. For $\tilde{q}_n^{(l)}$ = {0.1/x_1, 0.5/x_2, 1.0/x_3, 0.7/x_4, 0.3/x_5} and $\tilde{p}_n^{(k)}$ = {0.2/x_1, 0.4/x_2, 0.6/x_3, 0.8/x_4, 1.0/x_5}, the operation results are as follows:
$\left(\tilde{q}_n^{(l)} - \tilde{p}_n^{(k)}\right)$ = {0.1/x_1, 0.5/x_2, 0.4/x_3, 0.2/x_4, 0.0/x_5}; $\left(\tilde{q}_n^{(l)} \theta \tilde{p}_n^{(k)}\right)$ = {0.0/x_1, 0.1/x_2, 0.4/x_3, 0.0/x_4, 0.0/x_5}; $\left(\tilde{q}_n^{(l)} \oplus \tilde{p}_n^{(k)}\right)$ = {0.2/x_1, 0.5/x_2, 0.4/x_3, 0.3/x_4, 0.7/x_5}; $\left(\tilde{q}_n^{(l)} \Delta\, \tilde{p}_n^{(k)}\right)$ = {0.1x_1/, 0.1/x_2, 0.4/x_3, 0.1/x_4, 0.7/x_5}.

2.2 Application of Pseudometric Distances between Fuzzy Sets

The main types of pseudometric distances for assessing the degree of similarity of analogous features of project situations, represented by the fuzzy sets $\tilde{q}_n^{(l)}$ and $\tilde{p}_n^{(k)}$, are the Hamming and Euclidean distances between fuzzy sets [23, 24].

The relative Hamming distance between fuzzy sets is as follows:

$$d_H\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \frac{1}{n}\sum_{i=1}^{n}\left|\mu_{\tilde{q}_n^{(l)}}(x_i) - \mu_{\tilde{p}_n^{(k)}}(x_i)\right|,\ x_i \in X. \quad (5)$$

The relative Euclidean distance between fuzzy sets is

$$d_E\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \frac{1}{\sqrt{n}}\sqrt{\sum_{i=1}^{n}\left(\mu_{\tilde{q}_n^{(l)}}(x_i) - \mu_{\tilde{p}_n^{(k)}}(x_i)\right)^2},\ x_i \in X. \tag{6}$$

<u>Example</u>. For $\tilde{q}_n^{(l)}$ = {0.3/x_1, 1.0/x_2, 0.4/x_3, 0.0/x_4} and $\tilde{p}_n^{(k)}$ = {0.2/x_1, 0.5/x_2, 1.0/x_3, 0.0/x_4}: $d_H\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = 0.3$ and $d_E\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = 0.395$.

The assessment resulting from the application of pseudometric distances does not require any defuzzification. On the one hand, this facilitates subsequent aggregation of the results of feature-by-feature comparison (as distinct from the application of the previously described operations with fuzzy sets); on the other hand, this is characterized by a lower possibility of taking into account conditions for determining the relevance of the project situations to be compared and the system of preferences of the decision making person.

The pseudometric distances discussed above are treated conventionally, while the application of L. A. Zadeh's generalization principle enables us to treat distance as a fuzzy set as follows [25, 26]:

$$\forall \delta \in \Re^+ \quad \tilde{d}\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \max_{\delta \in \tilde{d}\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right)}\left(\min\left(\mu_{\tilde{q}_n^{(l)}}(x_i), \mu_{\tilde{p}_n^{(k)}}(x_i)\right)\right),\ \forall x_i \in X. \tag{7}$$

where $\Re^+$ – the set of non-negative numbers.

<u>Example</u>. For $\tilde{q}_n^{(l)}$ = {0.1/x_1, 0.5/x_2, 1.0/x_3, 0.7/x_4, 0.3/x_5} and $\tilde{p}_n^{(k)}$ = {0.2/b_1, 0.4/b_2, 0.6/b_3, 0.8/b_4, 1.0/b_5}: $\tilde{d}\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right)$ = {0.7/δ_1, 0.8/δ_2, 1.0/δ_3, 0.5/δ_4, 0.2/δ_5}.

2.3 Application of Ranking Indexes for Fuzzy Sets (Numbers)

In this section we give examples of the most widespread indexes for ranking fuzzy sets (numbers).

1) The fuzzy set (number) ranking index based on the fuzzy preference relation:

$$I_1\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \sup_{(x_1, x_2) \in Supp\left(\tilde{q}_n^{(l)}\right) \times Supp\left(\tilde{p}_n^{(k)}\right)} \min\left(\mu_{\tilde{q}_n^{(l)}}(x_1), \mu_{\tilde{p}_n^{(k)}}(x_2), \mu_Q(x_1, x_2)\right). \tag{8}$$

This index uses the fuzzy preference relation Q on $\Re^2$, e.g. with the membership function

$$\mu_Q(x_1, x_2) = \begin{cases} 1, & x_1 \geq x_2, \\ 0, & x_1 < x_2. \end{cases} \tag{9}$$

The ranking of $\tilde{q}_n^{(l)}$ and $\tilde{p}_n^{(k)}$ is performed according to the rule

$$I_1\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) \geq I_1\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) \Rightarrow \tilde{q}_n^{(l)} \geq \tilde{p}_n^{(k)}. \tag{10}$$

2) The fuzzy set (number) ranking index based on comparing their mean values:

$$I_2\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) \geq m\left(\tilde{q}_n^{(l)}\right) - m\left(\tilde{p}_n^{(k)}\right), \tag{11}$$

where $m\left(\tilde{q}_n^{(l)}\right), m\left(\tilde{p}_n^{(k)}\right)$ is the mean values of the fuzzy numbers $\tilde{q}_n^{(l)}$ and $\tilde{p}_n^{(k)}$, respectively.

The sign and value of the index $I_2\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right)$ are indicative of what fuzzy number is greater and how much.

3) The index of ranking the fuzzy numbers $\tilde{q}_n^{(l)}$ and $\tilde{p}_n^{(k)}$, based on the membership function of the fuzzy number $\tilde{D} = \frac{\tilde{q}_n^{(l)}}{\tilde{q}_n^{(l)} + \tilde{p}_n^{(k)}}$ [22]:

$$I_3\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \int_0^{0.5} (1 - \mu_{\tilde{D}}(z))dz + \int_{0.5}^{1} \mu_{\tilde{D}}(z)dz, \tag{12}$$

$$\mu_{\tilde{D}}(z) = \sup_{z = x_1/(x_1+x_2)} \min(\mu_{\tilde{q}_n^{(l)}}(x_1), \mu_{\tilde{p}_n^{(k)}}(x_2)). \tag{13}$$

$$I_3\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) \geq 0.5 \Rightarrow A \geq B. \tag{14}$$

4) The fuzzy number ranking indexes proposed by D. Dubois and H. Prade, and based on seeking the highest/lowest value of the membership function among pairs of elements of fuzzy number supports [21]:

$$I_4^1\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \sup_{\substack{(x_1,x_2) \in Supp\left(\tilde{q}_n^{(l)}\right) \times Supp\left(\tilde{p}_n^{(k)}\right) \\ x_1 \geq x_2}} \min\left(\mu_{\tilde{q}_n^{(l)}}(x_1), \mu_{\tilde{p}_n^{(k)}}(x_2)\right), \tag{15}$$

$$I_4^2\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \sup_{x_1 \in Supp\left(\tilde{q}_n^{(l)}\right)} \inf_{\substack{x_2 \in Supp\left(\tilde{p}_n^{(k)}\right) \\ x_2 \geq x_1}} \min\left(\mu_{\tilde{q}_n^{(l)}}(x_1), 1 - \mu_{\tilde{p}_n^{(k)}}(x_2)\right), \tag{16}$$

$$I_4^3\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \inf_{x_1 \in Supp\left(\tilde{q}_n^{(l)}\right)} \sup_{\substack{x_2 \in Supp\left(\tilde{p}_n^{(k)}\right) \\ x_2 \leq x_1}} \max\left(1 - \mu_{\tilde{q}_n^{(l)}}(x_1), \mu_{\tilde{p}_n^{(k)}}(x_2)\right), \tag{17}$$

$$I_4^4\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = 1 - \sup_{\substack{(x_1,x_2) \in Supp\left(\tilde{q}_n^{(l)}\right) \times Supp\left(\tilde{p}_n^{(k)}\right) \\ x_1 \leq x_2}} \min\left(\mu_{\tilde{q}_n^{(l)}}(x_1), \mu_{\tilde{p}_n^{(k)}}(x_2)\right), \tag{18}$$

$$I_4^i\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) \geq I_4^i\left(\tilde{p}_n^{(k)}, \tilde{q}_n^{(l)}\right) \Rightarrow \tilde{q}_n^{(l)} \geq \tilde{p}_n^{(k)}, i \in \{1, ..., 4\}. \tag{19}$$

The ranking indexes $I_1\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right)$, $I_4^1\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right), ..., I_4^4\left(\tilde{p}_n^{(k)}, \tilde{q}_n^{(l)}\right)$ ignore the form of the membership functions of $\tilde{q}_n^{(l)}$ and $\tilde{p}_n^{(k)}$.

The ranking index $I_2\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right)$ takes into account the form of the membership functions, but its values are not normalized, and this complicates interpretation of assessment results.

The values of the ranking index $I_3\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right)$ are normalized; however, it should be used for comparing non-negative fuzzy numbers or with allowance for the shift of the membership functions of fuzzy numbers being compared.

Note that the mean value $m\left(I_3\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right)\right)$ of the index $I_3\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right)$ has a clearer quantitative interpretation than the latter.

2.4 Application of Logical Indexes for Comparing Fuzzy Sets (Numbers)

The approach based on logical operations is applicable to assessing the similarity of the fuzzy values of project situation features. Herewith, the values of feature membership functions are treated as the truth degrees of a statement, and base set elements, in their turn, are taken into account in the determination of the truth/falsity of this statement. Therefore, the task is to determine the logical interrelation, i.e., whether the truth of the statement about the membership of the element in one fuzzy number entails the truth of a similar statement with respect to another fuzzy number.

The typical representation of logical indexes for comparing fuzzy numbers [22] is as follows:

$$ml\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \min_{x \in \Re} f\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right). \tag{20}$$

The following operations are most often used as $f\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right)$:

- fuzzy inclusion of the fuzzy number $\tilde{q}_n^{(l)}$ into the fuzzy number $\tilde{p}_n^{(k)}$, with the membership function

  $$\mu_f(x) = \mu_{\tilde{q}_n^{(l)}}(x) \to \mu_{\tilde{p}_n^{(k)}}(x), \forall x \in \Re, \tag{21}$$

  where $\to$ is fuzzy implication operation;
- fuzzy equality (equivalence) of the fuzzy numbers $\tilde{q}_n^{(l)}$ and $\tilde{p}_n^{(k)}$, with the membership function

$$\mu_f(x) = \left(\mu_{\tilde{q}_n^{(l)}}(x) \to \mu_{\tilde{p}_n^{(k)}}(x)\right) \mathrm{T} \left(\mu_{\tilde{p}_n^{(k)}}(x) \to \mu_{\tilde{q}_n^{(l)}}(x)\right), \forall x \in \Re, \tag{22}$$

where $\mathrm{T} \to$ is t-norm operation, e.g., min.

The fuzzy inclusion operation is used when the falling of the fuzzy feature $\tilde{q}_n^{(l)}$ of the number into a class described by the reference fuzzy feature $\tilde{p}_n^{(k)}$ is sufficient. The fuzzy equality is typical for cases when it is required to determine the maximum coincidence of fuzzy features.

The result of comparing fuzzy numbers is much dependent on selecting the implementation of a fuzzy implication operation. Thus, the Larsen and Mamdani fuzzy implication operations do not suit the goals discussed since the result of the pointwise integration of fuzzy numbers is nonzero only if the supports of both fuzzy numbers coincide with the base set.

The Lukasiewicz, Gödel, Kleene–Dienes, and Kleene–Dienes–Lukasiewicz fuzzy implication operations yield equally correct results for the case of the full inclusion of the $\tilde{q}_n^{(l)}$ support into the set of modal $\tilde{p}_n^{(k)}$ values. However, in the case of complete equality between $\tilde{q}_n^{(l)}$ and $\tilde{q}_n^{(l)}$, the Kleene–Dienes and Kleene–Dienes–Lukasiewicz implication operations underestimate the degree of their compliance.

The logical index presented below is devoid of this limitation [27]:

$$ml\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right) = \max_{x \in \Re} \min f\left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)}\right). \tag{23}$$

3 Approaches to Comparing Fuzzy Project Situations

To compare fuzzy project situations for selecting relevant decisions, it is required to aggregate the results of comparing the fuzzy features of these situations. The aggregation of the results of comparing the fuzzy features of situations is generally based on one of the following approaches:

- reduction of the multicriterion assessment task to the one-criterion one based on the aggregation of the results of comparing individual features with the use of various convolutions (additive, multiplicative, maximin, minimax, etc.) or fuzzy inference algorithms (by Mamdani, Larsen, Takagi-Sugeno, Tsukamoto, etc.);
- by priority features, the other being considered as additional, whose comparison results must meet the established rules.

In the comparison of project situations, the former approach prevails, i.e., that based on the aggregation of the results of comparing individual features. Besides, sometimes the task of aggregating the results of comparing the features of project situations is solved "automatically". Problems arise in aggregation depending on various comparison conditions; namely, if

- different scales are used to compare different features;
- it is necessary to take into account different personal significances of the features;
- it is required do take into account the effect of consistency (including correlation and interplay) of the features on the overall result of comparing the situations;
- the project situations to be compared are characterized by a complex "structure" of feature aggregation.

Depending on these and some other conditions, the following strategies for aggregating the results of comparing the features of project situations are possible:

- the overall result of comparing project situations is represented as a hierarchy of partial results of feature comparison;
- the overall result of comparing project situations is formed under conditions of the equivalence of feature comparison results for the following instances:
  - "simultaneous" achievement of all the partial feature comparison results,
  - achievement of one of the partial feature comparison results,
  - compromise (intermediate) achievement of partial feature comparison results (e.g., achievement of individual partial results of feature comparison),
  - hybrid strategies targeted at the selection (identification) of convolution operations depending on the obtained values of partial feature comparison results [28];
- the overall result of comparing project situations is based on recursive aggregation of partial feature comparison results;
- the overall result of comparing project situations is formed under conditions of the inequivalence of partial feature comparison results for the following instances:
  - achievement of the required threshold values of partial feature comparison results,

   - different weights for partial feature comparison results and taking them into account in subsequent aggregation, for example, using ordered weighted averaging (OWA) operators [29],
   - a hierarchical AND/OR tree structure of feature aggregation.
- the overall result of comparing project situations is based on various quantifiers (including fuzzy) for the convolution of feature comparison results, e.g., in terms of the consistency of most of the features, in terms of the inconsistency of at least one feature.

4 An Example of Selecting Precedent Project Decisions Relevant to Fuzzy Project Situations

We use the relative Euclidean pseudometric distance between fuzzy sets to assess the similarity of the fuzzy features of project situations, and we use the modified Mamdani fuzzy inference algorithm for comparing fuzzy project situations and selecting precedent project decisions [30, 31].

In view of these conditions, the fuzzy model of comparing fuzzy project situations and selecting precedent project decisions can be represented as

$$\tilde{R}_\Sigma = \bigcup_{p=1,\dots,P} \left(\min(\min_{n=1,\dots,N} \left(d_E \left(\tilde{q}_n^{(l)}, \tilde{p}_n^{(k)} \right), \tilde{R}_p \right) \right), \quad (24)$$

where $\tilde{R}_\Sigma$ is an output fuzzy variable, whose value corresponds to the precedent project decision being selected; P is the number of fuzzy model rules.

Figure 1 illustrates the example of comparing fuzzy project situations and selecting precedent project decisions.

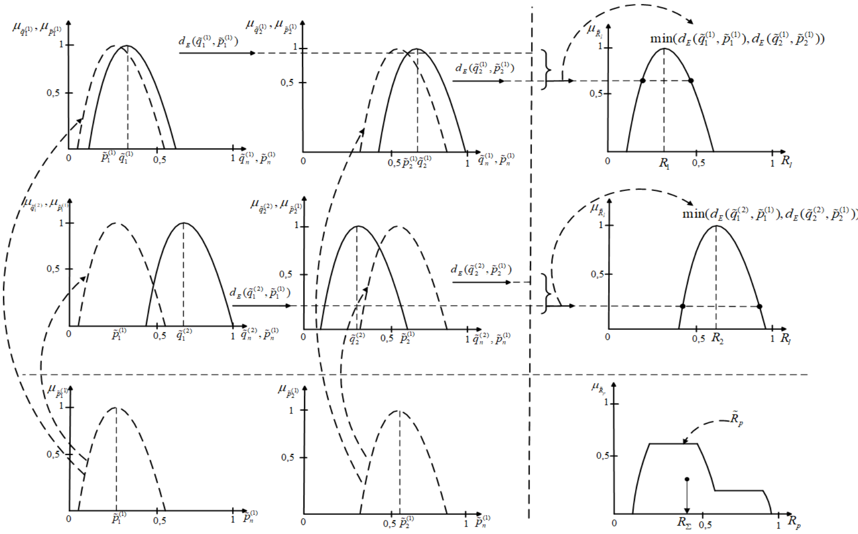

Figure 1
An example of comparing fuzzy project situations and selecting precedent project decisions

5 Software Implementation

To solve the task of comparing fuzzy project situations and selecting precedent project decisions, the *Loginf* program module has been developed in *Python*. The *matplotlib* library was used for result visualization.

Figure 2 shows the listing of the program of comparing fuzzy project situations and selecting precedent project decisions (for 2 features and 2 rules).

```
from matplotlib import pyplot as plt
import numpy as np
import loginf
base_set = np.arange(0, 1.01, 0.01) # Base Set
# RULE 1:
promise11 = loginf.FuzzyGaussian(base_set, 0, 0.14) # Promise 1
promise12 = loginf.FuzzyGaussian(base_set, 1, 0.14) # Promise 2
consequent1 = loginf.FuzzyGaussian(base_set, 0.6, 0.1) # Consequent 1
rule1 = loginf.FuzzyRule(consequent1, promise11, promise12) # Rule 1
# RULE 2:
promise21 = loginf.FuzzyGaussian(base_set, 1, 0.14) # Promise 1
promise22 = loginf.FuzzyGaussian(base_set, 0, 0.14) # Promise 2
consequent2 = loginf.FuzzyGaussian(base_set, 0.37, 0.1) # Consequent 2
rule2 = loginf.FuzzyRule(consequent2, promise21, promise22) # Rule 2
# TEST INPUT SETS:
# Input fuzzy sets:
input_set1 = loginf.FuzzyGaussian(base_set, 0.38, 0.05)
input_set2 = loginf.FuzzyGaussian(base_set, 0.68, 0.1)
```

```
# RULE OUTPUTS:
rule_output1 = rule1.consequent_output(input_set1, input_set2, plot = True)
rule_output2 = rule2.consequent_output(input_set1, input_set2, plot = True)
result = rule_output1.union(rule_output2) # Union output set
center = result.center_of_gravity() # Centroid
```

Figure 2

The program of comparing fuzzy project situations and selecting precedent project decisions

The visualization of solving the task of comparing fuzzy project situations and selecting a precedent project decision is exemplified in Figure 3.

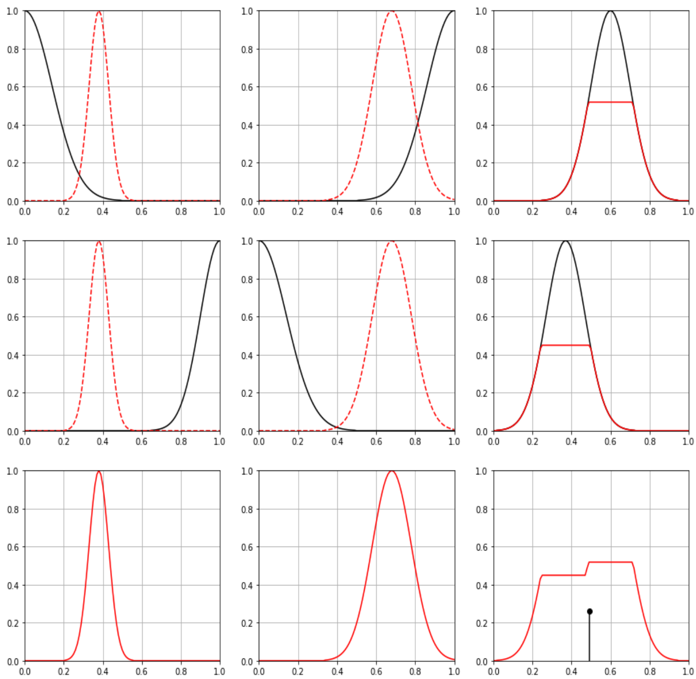

Figure 3

An example of using the *Loginf* program module

The main classes of this module are presented in Figure 4.

The following classes are used in the implementation of the *Loginf* module:

- GaussianFunction and FuzzyGaussian are used to specify fuzzy set membership functions;
- FuzzySet is intended for calculating pseudometric distances between the fuzzy features of precedent situations, performing operations with fuzzy sets (numbers), and defuzzifying the values of the fuzzy output variable;
- FuzzyRule implements the fuzzy inference algorithm.

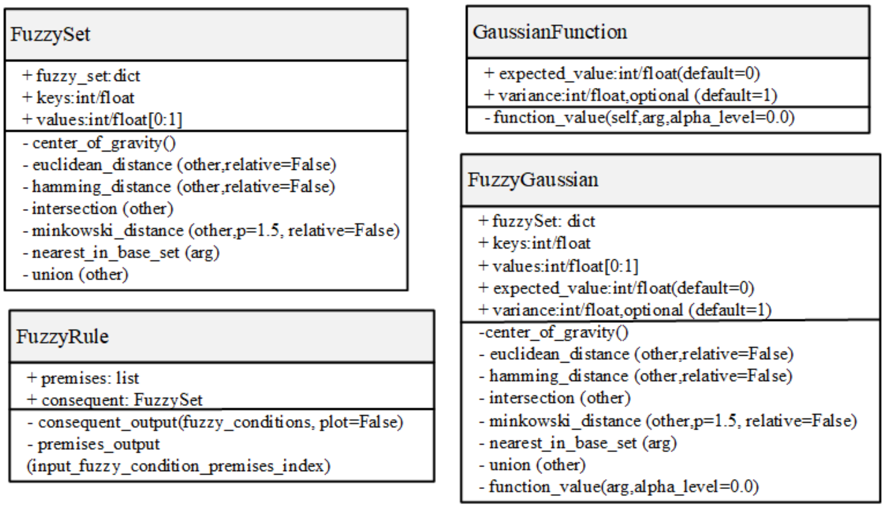

Figure 4

The main classes of the *Loginf* module

Conclusion

Methods for comparing fuzzy project situations have been analyzed and systematized.

Methods for assessing the similarity of the fuzzy features of project situations, based on operations with fuzzy sets, pseudometric distances between fuzzy sets, and the fuzzy distance between fuzzy sets have been discussed.

The paper has described approaches to comparing fuzzy project situations on the basis of transition from the multicriterion assessment task to the one-criterion one due to the aggregation of the results of comparing individual features with the use of various convolutions or fuzzy inference algorithms, as well as by individual priority features.

A program module has been developed and an example of selecting precedent project decisions relevant to project situations is given, where the relative pseudometric distance between fuzzy sets is used to assess the similarity of the fuzzy features of project situations, and the modified Mamdani fuzzy inference algorithm is used for comparing fuzzy project situations and selecting precedent project decisions.

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