Fuzzy Model-based Design of a Transparent Controller for a Time Delayed Bilateral Teleoperation System Through State Convergence

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Abstract: Transparency optimized state convergence scheme is a simple, elegant and easy to design bilateral control algorithm for n^{th}-order linear teleoperation systems modeled on state space. It requires the solution of 3n+2 design equations to determine the control gains when the time delay in the communication channel is small. The controller thus obtained allows the slave system to follow the master system's motion in a desired dynamic way while providing the human operator with a high degree of environment's feeling during the steady state operation. This paper employs the transparency optimized state convergence scheme to design a bilateral controller for a class of nonlinear teleoperation systems which can be approximated by Takagi-Sugeno (TS) fuzzy models. To close the feedback loop around master and slave systems, a suitable parallel distributed compensation (PDC) type TS fuzzy controller is selected which allows to use the state convergence procedure in a true sense. In this way, all the benefits of the state convergence scheme have been kept intact while extending its applicability to nonlinear teleoperation systems at the same time. Further, the proposed fuzzy state convergence controller is more general as the existing linear state convergence controller can be derived from it. MATLAB simulations on a one-degree-of-freedom (DoF) nonlinear teleoperation system are included to show the efficacy of the proposed fuzzy transparent state convergence controller.

Keywords: state convergence; bilateral teleoperation systems; TS fuzzy models; MATLAB/Simulink

1 Introduction

Teleoperation refers to the distant control of a process. The main components of a teleoperation system are the human operator, master system, communication channel, slave system and the environment. The system works under the control of a human operator who generates the desired motion for the slave system by driving the master system and these motion commands are then sent over the communication channel to the slave system which executes the required task on the remote environment. This system is known as unilateral. However, if the slave system also sends a task-related (usually force) feedback to the master system then the teleoperation system is said to be bilateraly controlled. The popularity of such bilateral systems can be judged from their deployment in diverse processes ranging from miniaturized surgical tasks to large scale industrial systems [1].

The control design of bilateral systems is challenging mainly due to the presence of time delay in the communication channel and the performance is further deteriorated due to the other form of systems' uncertainties. The main objectives for the bilateral controller are to guarantee the stability in the presence of time delays and to provide a high degree of transparency (position and force tracking) [2]-[4]. Passivity based control algorithm has been widely used in bilateral systems owing to its robustness to time delays [5]-[7]. However, the passivity controller does not provide good tracking performance. To improve its tracking performance, a number of modifications to the passivity controller have also been proposed in the literature [8]. Other than passivity, adaptive control theory has also been used to design bilateral controllers [9]. Sliding mode [10] and H-infinity control [11] theories have also played an important role in robustifying the bilateral controllers. The use of intelligent techniques such as fuzzy logic and neural networks has also been explored in designing bilateral controllers [12]-[14]. The approximation property of these systems is central to the design of bilateral controllers. Another way to use fuzzy logic is model based fuzzy approach which has proved to be an effective tool in the control of nonlinear systems [15]-[22] and its application to bilateral teleoperation systems has also been reported [23]-[25].

State convergence belongs to the class of non-passive schemes and provides a simple and elegant method to design the bilateral controller [26]. The distinct feature of the method is the possibility of achieving desired dynamic behavior of the teleoperation system. This method has been shown to control both linear and nonlinear teleoperation systems in the absence and presence of time delays [27]. While good position tracking and the desired dynamic behavior was achieved with the originally proposed state convergence method, human perception of the remote environment was not good. To overcome this limitation, a modification to the original state convergence scheme has recently been proposed which has helped in improving the transparency of the teleoperation system while maintaining stability and the desired dynamic behavior [28]. The modified scheme is named as transparency optimized state convergence method and will be used in this study.

This paper proposes to employ transparency optimized state convergence method in controlling a nonlinear teleoperation system which can be approximated by a class of TS fuzzy systems having common input and output matrices. A suitable form of PDC type fuzzy control law is selected to close the feedback loops around master and slave systems. The beauty of the selected control law lies in its capability to fully utilize the method of state convergence while providing large range operation. In our earlier work, we have proved that this control law can successfully establish the state convergence behavior in a nonlinear teleoperation system [24]-[25]. Following the same lines, we show that the fuzzy transparent bilateral controller can indeed be designed for the transparency optimized state convergence architecture with a nonlinear plant model. The validity of the proposed controller is confirmed through MATLAB simulations on a one DoF nonlinear teleoperation system.

The organization of the paper is as follows: We start by reviewing the transparency optimized state convergence architecture in Section 2. The proposed fuzzy logic controller is detailed in Section 3. Results are presented in Section 4. Section 5 draws the conclusion and provides future directions.

2 Transparency Optimized State Convergence Scheme

Transparency optimized state convergence scheme is recently proposed for the bilateral teleoperation systems with time delay in the communication channel. It is a modified form of the original state convergence scheme where the objectives of reflecting the full environmental force to the operator and the desired dynamic behavior of the closed loop teleoperation could not be achieved at the same time. This restriction is resolved to some extent in the modified version on the expense of limiting the allowable time delay in the communication channel and constraining the achievable closed loop behavior. Similar to the standard state convergence scheme, transparency optimized state convergence scheme also considers the master and slave systems modeled on state space as:

$$\begin{aligned} \dot{x}_z &= A_z x_z + B_z u_z \\ y_z &= C_z x_z \end{aligned} \tag{1}$$

Where the subscript z denotes either master ($z=m$) or slave ($z=s$) systems and various matrix entries in (1) are given as:

$$A_z = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & \dots & & \\ 0 & 0 & 0 & \dots & 1 \\ -a_{z1} & -a_{z2} & -a_{z3} & \dots & -a_{zn} \end{pmatrix} \tag{2}$$

$$\begin{aligned} B_z &= \begin{pmatrix} 0 & 0 & \dots & 0 & b_{z1} \end{pmatrix}^T \\ C_z &= \begin{pmatrix} 1 & 0 & \dots & 0 & 0 \end{pmatrix} \end{aligned} \tag{3}$$

The block diagram of the transparency optimized state convergence scheme is shown in Figure 1 and various parameters forming the architecture are described below:

T: This scalar parameter represents the time delay offered by the communication channel.

F_m: This scalar parameter represents the force applied by the human operator onto the master system.

G_2: This scalar parameter measures the influence of the operator's force into the slave system.

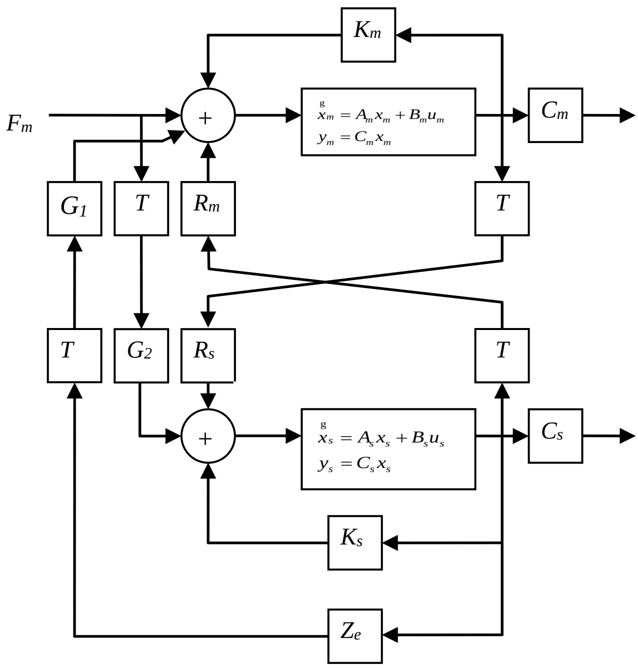


Figure 1

Transparent state convergence method

$Z_e = [z_{e1}, z_{e2}, \ldots, z_{en}]$: This vector parameter is the model of the remote slave environment. When the environment is modeled by a spring-damper system, then this vector will contain two non-zero elements and rest of the elements will be zero.

G_1: This scalar parameter represents the influence of the environmental force when reflected onto the master system.

$R_s = [r_{s1}, r_{s2}, \ldots, r_{sn}]$: This vector parameter represents the influence of the master's motion signals in the slave system.

$R_m = [r_{m1}, r_{m2}, \ldots, r_{mn}]$: This vector parameter represents the influence of the slave's motion signals in the master system.

$K_m = [k_{m1}, k_{m2}, \ldots, k_{mn}]$: This vector parameter is the state feedback controller for the master system.

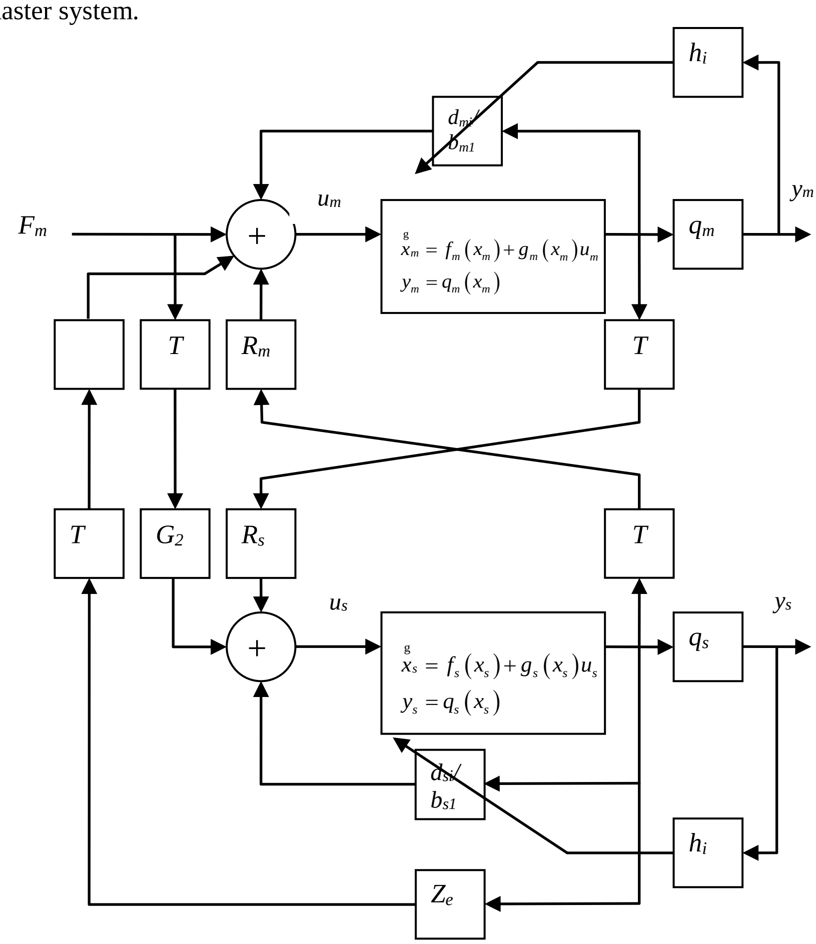


Figure 2

Prposed fuzzy transparent state convergence method

$K_s = [k_{s1}, k_{s2}, \ldots, k_{sn}]$: This vector parameter is the state feedback controller for the slave system.

Of these, G_1, G_2, R_s, R_m, K_m and K_s form $4n+2$ unknown parameters. The parameter G_1 can be freely chosen and is taken as unity when perfect transparency of the teleoperation system is desirable.

3 Proposed Transparent TS Fuzzy Logic Controller

In this section, we will show the development of a transparent TS fuzzy logic controller for a class of nonlinear teleoperation systems which can be approximated by TS fuzzy models in phase variable form with common input and output matrices. Such a nonlinear teleoperation system can be given as:

$$\begin{aligned} \dot{x}_z &= f_z\left(x_z\right) + g_z u_z \\ y_z &= q_z x_z \end{aligned} \tag{4}$$

The TS fuzzy description of (1) with 'r' plant rules can be given as:

$$\begin{aligned} \dot{x}_{z1} &= x_{z2} \\ \dot{x}_{z2} &= x_{z3} \\ &\ldots \\ \dot{x}_{zn} &= -\sum_{i=1}^{r} h_i\left(x_z\right) \sum_{j=1}^{n} a_{zij} x_{zj} + b_{z1} u_z \\ y_z &= x_{z1} \end{aligned} \tag{5}$$

Where $h_i\left(x_z\right)$ is the normalized firing strength of i^{th} fuzzy plant rule which is defined in (6) and satifies the two properties as in (6):

$$h_i\left(x_z\right) = \frac{\mu_i\left(x_z\right)}{\sum_{i=1}^{r} \mu_i\left(x_z\right)}, h_i\left(x_z\right) \geq 0, \sum_{i=1}^{r} h_i\left(x_z\right) = 1 \tag{6}$$

From Figure 2, we can introduce the TS fuzzy control law for the master system as:

$$u_m = \frac{1}{b_{m1}} \sum_{i=1}^{r} h_i\left(x_m\right) \sum_{j=1}^{n} d_{mij} x_{mj} + \sum_{j=1}^{n} \left(g_1 z_{ej} + r_{mj}\right) x_{sj}\left(t-T\right) + F_m \tag{7}$$

By plugging (7) in (5), the closed loop master sysem dynamics can be obtained as:

$$\dot{x}_{mn} = \sum_{i=1}^{r} h_i\left(x_m\right) \sum_{j=1}^{n} \left(d_{mij} - a_{mij}\right) x_{mj} + b_{m1} \sum_{j=1}^{n} \left(g_1 z_{ej} + r_{mj}\right) x_{sj}\left(t-T\right) + b_{m1} F_m \tag{8}$$

Please note that we will considr n^{th} component of the system dynamics throughout the rest of the paper as in (8). Let us now introduce the time invariant coefficients for the master system as:

$$c_{mj} = d_{mij} - a_{mij} \tag{9}$$

With the coefficients in (9), the closed loop master system in (8) can be simplified as:

$$\dot{x}_{mn} = \sum_{j=1}^{n} c_{mj} x_{mj} + b_{m1} \sum_{j=1}^{n} \left(g_1 z_{ej} + r_{mj} \right) x_{sj} \left(t - T \right) + b_{m1} F_m \tag{10}$$

Now, to close the loop around slave system, the following TS fuzzy control law is introduced (see Figure 2):

$$u_s = \sum_{i=1}^{r} h_i \left(x_s \right) \sum_{j=1}^{n} \left(\frac{d_{sij}}{b_{s1}} + z_{ej} \right) x_{sj} + \sum_{j=1}^{n} r_{sj} x_{mj} \left(t - T \right) + g_2 F_m \left(t - T \right) \tag{11}$$

The closed loop slave system can now be computed using (5) and (11) as:

$$\dot{x}_{sn} = \sum_{i=1}^{r} h_i \left(x_s \right) \sum_{j=1}^{n} \left(d_{sij} - a_{sij} + b_{s1} z_{ej} \right) x_{sj} + b_{s1} \sum_{j=1}^{n} r_{sj} x_{mj} \left(t - T \right) + b_{s1} g_2 F_m \left(t - T \right) \tag{12}$$

Similar to the master system, we define the time invariant coefficients for the slave system as:

$$c_{sj} = d_{sij} - a_{sij} \tag{13}$$

With the definition in (13), the closed loop slave system in (12) can be written as:

$$\dot{x}_{sn} = \sum_{j=1}^{n} \left(c_{sj} + b_{s1} z_{ej} \right) x_{sj} + b_{s1} \sum_{j=1}^{n} r_{sj} x_{mj} \left(t - T \right) + b_{s1} g_2 F_m \left(t - T \right) \tag{14}$$

As a part of the design procedure, state convergence method assumes the time delay in the communication channel to be small and the operator's force as constant. Thus, Taylor expansion of first order can be used to approximate the time delayed terms as:

$$\begin{aligned} x_{mj} \left(t - T \right) &\approx x_{mj} - T \dot{x}_{mj} \\ x_{sj} \left(t - T \right) &\approx x_{sj} - T \dot{x}_{sj} \\ F_m \left(t - T \right) &\approx F_m - T \dot{F}_m = F_m \end{aligned} \tag{15}$$

With the approximation in (15), closed loop master dynamics of (10) can be written as:

$$\dot{x}_{mn} = \sum_{j=1}^{n} c_{mj} x_{mj} + b_{m1} \sum_{j=1}^{n} \left(g_1 z_{ej} + r_{mj} \right) x_{sj} - b_{m1} T \sum_{j=1}^{n-1} \left(g_1 z_{ej} + r_{mj} \right) \dot{x}_{sj} - \\ b_{m1} T \left(g_1 z_{en} + r_{mn} \right) \dot{x}_{sn} + b_{m1} F_m \tag{16}$$

Similarly, closed loop dynamics of the slave system in (14) can be approximated as:

$$\dot{x}_{sn} = \sum_{j=1}^{n} \left(c_{sj} + b_{s1} z_{ej} \right) x_{sj} + b_{s1} \sum_{j=1}^{n} r_{sj} x_{mj} - b_{s1} T \sum_{j=1}^{n-1} r_{sj} \dot{x}_{mj} - b_{s1} T r_{sn} \dot{x}_{mn} + b_{s1} g_2 F_m \tag{17}$$

By plugging (17) in (16) and using the phase variable representation of the slave system, closed loop master system dynamics of (16) can be written as:

$$\dot{x}_{mn} = \frac{1}{\left(1 - T^2 b_{m1} b_{s1} r_{sn} \left(g_1 z_{en} + r_{mn} \right)\right)} \begin{pmatrix} \sum_{j=1}^{n} \left(c_{mj} - T b_{m1} b_{s1} \left(g_1 z_{en} + r_{mn} \right) r_{sj} \right) x_{mj} + \\ \sum_{j=1}^{n} \left(b_{m1} \left(g_1 z_{ej} + r_{mj} \right) - T b_{m1} \left(g_1 z_{en} + r_{mn} \right) \left(c_{sj} + b_{s1} z_{ej} \right) \right) x_{sj} - \\ T b_{m1} \sum_{j=1}^{n-1} \left(g_1 z_{ej} + r_{mj} \right) x_{sj+1} + T^2 b_{m1} b_{s1} \left(g_1 z_{en} + r_{mn} \right) \sum_{j=1}^{n-1} r_{sj} x_{mj+1} + \\ \left(b_{m1} - T b_{m1} b_{s1} g_2 \left(g_1 z_{en} + r_{mn} \right) \right) F_m \end{pmatrix} \tag{18}$$

Similarly, by plugging (16) in (17) and using the phase variable representation of the master system, the closed loop slave system dynamics of (17) can be written as:

$$\dot{x}_{sn} = \frac{1}{\left(1 - T^2 b_{m1} b_{s1} r_{sn} \left(g_1 z_{en} + r_{mn} \right)\right)} \begin{pmatrix} \sum_{j=1}^{n} \left(b_{s1} r_{sj} - T b_{s1} r_{sn} c_{mj} \right) x_{mj} + \\ \sum_{j=1}^{n} \left(c_{sj} + b_{s1} z_{ej} - T b_{s1} b_{m1} r_{sn} \left(g_1 z_{ej} + r_{mj} \right) \right) x_{sj} - \\ T b_{s1} \sum_{j=1}^{n-1} r_{sj} x_{mj+1} + T^2 b_{s1} b_{m1} r_{sn} \sum_{j=1}^{n-1} \left(g_1 z_{ej} + r_{mj} \right) x_{sj+1} - \\ \left(b_{s1} g_2 - T b_{s1} b_{m1} r_{sn} \right) F_m \end{pmatrix} \tag{19}$$

Let us now define the state convergence error between master and slave systems as:

$$x_{ej} = x_{mj} - x_{sj}, \forall j = 1, 2, ..., n \tag{20}$$

We now write the closed loop master system dynamics of (18) in terms of state convergence error as:

$$\overset{g}{x}_{mn} = \frac{1}{\left(1-T^2 b_{m1} b_{s1} r_{sn}\left(g_1 z_{en}+r_{mn}\right)\right)} \left(\begin{array}{l} \sum_{j=1}^{n} \left(\begin{array}{l} c_{mj} - T b_{m1} b_{s1}\left(g_1 z_{en}+r_{mn}\right) r_{sj} + b_{m1}\left(g_1 z_{ej}+r_{mj}\right) - \\ T b_{m1}\left(g_1 z_{en}+r_{mn}\right)\left(c_{sj}+b_{s1} z_{ej}\right) \end{array} \right) x_{mj} - \\ \sum_{j=1}^{n} \left(b_{m1}\left(g_1 z_{ej}+r_{mj}\right) - T b_{m1}\left(g_1 z_{en}+r_{mn}\right)\left(c_{sj}+b_{s1} z_{ej}\right)\right) x_{ej} + \\ \sum_{j=1}^{n-1} \left(T^2 b_{m1} b_{s1}\left(g_1 z_{en}+r_{mn}\right) r_{sj} - T b_{m1}\left(g_1 z_{ej}+r_{mj}\right)\right) x_{mj+1} + \\ T b_{m1} \sum_{j=1}^{n-1}\left(g_1 z_{ej}+r_{mj}\right) x_{ej+1} + \left(b_{m1} - T b_{m1} b_{s1} g_2\left(g_1 z_{en}+r_{mn}\right)\right) F_m \end{array} \right) \tag{21}$$

Similarly, the closed loop slave system dynamics of (19) is also written in terms of state convergence error as:

$$\overset{g}{x}_{sn} = \frac{1}{\left(1-T^2 b_{m1} b_{s1} r_{sn}\left(g_1 z_{en}+r_{mn}\right)\right)} \left(\begin{array}{l} \sum_{j=1}^{n} \left(\begin{array}{l} c_{sj} + b_{s1} z_{ej} - T b_{s1} b_{m1} r_{sn}\left(g_1 z_{ej}+r_{mj}\right) + \\ b_{s1} r_{sj} - T b_{s1} r_{sn} c_{mj} \end{array} \right) x_{mj} + \\ \sum_{j=1}^{n} \left(c_{sj} + b_{s1} z_{ej} - T b_{s1} b_{m1} r_{sn}\left(g_1 z_{ej}+r_{mj}\right)\right) x_{ej} + \\ \sum_{j=1}^{n-1} \left(T^2 b_{s1} b_{m1} r_{sn}\left(g_1 z_{ej}+r_{mj}\right) - T b_{s1} r_{sj}\right) x_{mj+1} - \\ T^2 b_{s1} b_{m1} r_{sn} \sum_{j=1}^{n-1}\left(g_1 z_{ej}+r_{mj}\right) x_{ej+1} - \left(b_{s1} g_2 - T b_{s1} b_{m1} r_{sn}\right) F_m \end{array} \right) \tag{22}$$

By taking the time derivative of (20) and using (21)-(22), we find the closed loop error dynamics of the teleoperation system as:

$$\overset{g}{x}_{en} = \frac{1}{\left(1-T^2 b_{m1} b_{s1} r_{sn}\left(g_1 z_{en}+r_{mn}\right)\right)} \left(\begin{array}{l} \sum_{j=1}^{n} \left(\begin{array}{l} c_{mj} - T b_{m1} b_{s1}\left(g_1 z_{en}+r_{mn}\right) r_{sj} + b_{m1}\left(g_1 z_{ej}+r_{mj}\right) - \\ T b_{m1}\left(g_1 z_{en}+r_{mn}\right)\left(c_{sj}+b_{s1} z_{ej}\right) - c_{sj} - b_{s1} z_{ej} + \\ T b_{s1} b_{m1} r_{sn}\left(g_1 z_{ej}+r_{mj}\right) - b_{s1} r_{sj} + T b_{s1} r_{sn} c_{mj} \end{array} \right) x_{mj} - \\ \sum_{j=1}^{n} \left(\begin{array}{l} c_{sj} + b_{s1} z_{ej} - T b_{s1} b_{m1} r_{sn}\left(g_1 z_{ej}+r_{mj}\right) - \\ b_{m1}\left(g_1 z_{ej}+r_{mj}\right) + T b_{m1}\left(g_1 z_{en}+r_{mn}\right)\left(c_{sj}+b_{s1} z_{ej}\right) \end{array} \right) x_{ej} + \\ \sum_{j=1}^{n-1} \left(\begin{array}{l} T^2 b_{m1} b_{s1}\left(g_1 z_{en}+r_{mn}\right) r_{sj} - T b_{m1}\left(g_1 z_{ej}+r_{mj}\right) - \\ T^2 b_{s1} b_{m1} r_{sn}\left(g_1 z_{ej}+r_{mj}\right) + T b_{s1} r_{sj} \end{array} \right) x_{mj+1} + \\ \sum_{j=1}^{n-1} \left(T b_{m1}\left(g_1 z_{ej}+r_{mj}\right) + T^2 b_{s1} b_{m1} r_{sn}\left(g_1 z_{ej}+r_{mj}\right)\right) x_{ej+1} + \\ \left(b_{m1} - T b_{m1} b_{s1} g_2\left(g_1 z_{en}+r_{mn}\right) - b_{s1} g_2 + T b_{s1} b_{m1} r_{sn}\right) F_m \end{array} \right) \tag{23}$$

We now form an augmented system of the closed loop master-error systems' dynamics using (21) and (23) as:

$$\begin{pmatrix} \overset{\text{g}}{x}_{mn} \\ \overset{\text{g}}{x}_{en} \end{pmatrix} = \frac{1}{D}\sum_{j=1}^{n}\begin{pmatrix} (a_{11})_j & (a_{12})_j \\ (a_{21})_j & (a_{22})_j \end{pmatrix}\begin{pmatrix} x_{mj} \\ x_{ej} \end{pmatrix} + \frac{1}{D}\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}F_m \tag{24}$$

Where the entry $(a_{xy})_j$ implies evaluation at j^{th} state and all the entries in (24) are given as (with the zeroth index values being zero: $r_{m0}=0, r_{s0}=0, z_{e0}=0$):

$$\begin{aligned}
a_{11} &= c_{mj} - Tb_{m1}b_{s1}(g_1z_{en}+r_{mn})r_{sj} + b_{m1}(g_1z_{ej}+r_{mj}) - Tb_{m1}(g_1z_{en}+r_{mn})(c_{sj}+b_{s1}z_{ej}) + \\
&\qquad T^2b_{m1}b_{s1}(g_1z_{en}+r_{mn})r_{sj-1} - Tb_{m1}(g_1z_{ej-1}+r_{mj-1}) \\
a_{12} &= b_{m1}(g_1z_{ej}+r_{mj}) - Tb_{m1}(g_1z_{en}+r_{mn})(c_{sj}+b_{s1}z_{ej}) + Tb_{m1}(g_1z_{ej-1}+r_{mj-1}) \\
a_{21} &= c_{mj} - Tb_{m1}b_{s1}(g_1z_{en}+r_{mn})r_{sj} + b_{m1}(g_1z_{ej}+r_{mj}) - Tb_{m1}(g_1z_{en}+r_{mn})(c_{sj}+b_{s1}z_{ej}) - \\
&\quad c_{sj} - b_{s1}z_{ej} + Tb_{s1}b_{m1}r_{sn}(g_1z_{ej}+r_{mj}) - b_{s1}r_{sj} + Tb_{s1}r_{sn}c_{mj} + T^2b_{m1}b_{s1}(g_1z_{en}+r_{mn})r_{sj-1} - \\
&\quad Tb_{m1}(g_1z_{ej-1}+r_{mj-1}) - T^2b_{s1}b_{m1}r_{sn}(g_1z_{ej-1}+r_{mj-1}) + Tb_{s1}r_{sj-1} \\
a_{22} &= c_{sj} + b_{s1}z_{ej} - Tb_{s1}b_{m1}r_{sn}(g_1z_{ej}+r_{mj}) - b_{m1}(g_1z_{ej}+r_{mj}) + Tb_{m1}(g_1z_{en}+r_{mn})(c_{sj}+b_{s1}z_{ej}) + \\
&\quad Tb_{m1}(g_1z_{ej-1}+r_{mj-1}) + T^2b_{s1}b_{m1}r_{sn}(g_1z_{ej-1}+r_{mj-1})
\end{aligned} \tag{25}$$

$$\begin{aligned}
b_1 &= b_{m1} - Tb_{m1}b_{s1}g_2(g_1z_{en}+r_{mn}) \\
b_2 &= b_{m1} - Tb_{m1}b_{s1}g_2(g_1z_{en}+r_{mn}) - b_{s1}g_2 + Tb_{s1}b_{m1}r_{sn} \\
D &= 1 - T^2b_{m1}b_{s1}r_{sn}(g_1z_{en}+r_{mn})
\end{aligned} \tag{26}$$

According to the method of state convergence, error should evolve as an autonomous system. This will happen upon the satisfaction of the following conditions:

$$(a_{21})_j = 0, \forall j = 1,2,...,n \tag{27}$$

$$b_2 = 0 \tag{28}$$

Once the error will behave like an autonomous system, augmented system of (24) can be assigned the desired dynamic behavior. This leads to the following conditions:

$$\left(s-(a_{11})_j\right)\times\left(s-(a_{22})_j\right) = (s+p_j)\times(s+q_j), \forall j = 1,2,...,n \tag{29}$$

Where the coefficients p_j and q_j form the desired polynomials for master and error systems respectively:

$$
\begin{aligned}
&s^n + p_n s^{n-1} + \ldots + p_2 s + p_1 = 0 \\
&s^n + q_n s^{n-1} + \ldots + q_2 s + q_1 = 0
\end{aligned} \tag{30}
$$

The design conditions (27)-(29) ensure that the states' error converges to zero and the master system exhibits the desired behavior. However, the convergence of force error is not guaranteed. To achieve that the operator force matches with the environmental force in steady state, we first compute the transfer function of the closed loop augmented system of (24) under the effect of autonomous error system:

$$
\begin{aligned}
&\frac{x_{mj}(s)}{F_m(s)} = \frac{(num)_j}{(den)_j}, \forall j = 1, 2, \ldots, n \\
&(num)_j = \begin{cases} (-1)^{n+1} s^{j-1} b_1, n > 2, \forall j = 1, 2, \ldots, n \\ s^{j-1} b_1, n = 2 \end{cases} \\
&(den)_j = s^n - (a_{11})_n s^{n-1} - (a_{11})_{n-1} s^{n-2} - \ldots - (a_{11})_2 s - (a_{11})_1
\end{aligned} \tag{31}
$$

The transfer function in (31) can now be evaluated at steady state and compared against the stiffness of the environment as:

$$
\frac{x_{m1}(0)}{F_m} = \frac{b_1}{-(a_{11})_1} = -\frac{1}{z_{e1}} \tag{32}
$$

The design condition (32) ensures that the force error will converge to zero in steady state. Now, we have $4n+2$ design variables: $g_1, g_2, c_{mj}, c_{sj}, r_{mj}, r_{sj}, \forall j = 1, 2, \ldots, n$ while the number of design equations (27)-(29), (32) are $3n+2$. To create a balance, we let: $r_{mj} = -c_{mj}, \forall j = 1, 2, \ldots, n$ which will reduce the number of design variables to $3n+2$. However, to achieve that the environmental force is fully reflected to the operator, g_1 has to be unity which will again create an imbalance between the number of design variables and the design equations. To overcome this, the constant coefficient of the desired master system polynomial is constrained by the other teleoperation system's variables and now the design procedure is balanced.

4 Simulation Results

In order to validate the proposed fuzzy model based transparent controller, MATLAB simulations are carried out using a one DoF nonlinear teleoperation system which can be described in differential equation/state space form as:

$$J_z \ddot{\theta}_z + b_z \dot{\theta}_z + m_z g l_z \sin\theta_z = u_z$$
$$\begin{pmatrix} \dot{x}_{z1} \\ \dot{x}_{z2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -a_{z1}\xi_z & -a_{z2} \end{pmatrix} \begin{pmatrix} x_{z1} \\ x_{z2} \end{pmatrix} + \begin{pmatrix} 0 \\ b_{z1} \end{pmatrix} u_z \tag{33}$$

Where x_{z1} and x_{z2} are the state variables representing the position and velocity of the master/slave systems, $a_{z1} = \frac{m_z g l_z}{J_z}$, $a_{z2} = \frac{b_z}{J_z}$, $b_{z1} = \frac{1}{J_z}$ and $\xi_z(t) = \frac{\sin x_{z1}(t)}{x_{z1}(t)}$ is the corresponding scheduling variable. To construct the TS fuzzy model of the teleoperation system, we determine the extreme values of the scheduling variable over the range of its operation, which is assumed to be $[-\pi/3 \quad \pi/3]$ in this study. The extreme values are found to be $\xi_{\min} = 0.827$ and $\xi_{\max} = 1.0$ which further help in constructing the following fuzzy sets:

$$\rho_1(\xi_z) = \begin{cases} 1 & , x_{z1} = 0 \\ \frac{\xi_z - \xi_{z\min}}{\xi_{\max} - \xi_{\min}} & , x_{z1} \neq 0 \end{cases}$$
$$\rho_2(\xi_z) = 1 - \rho_1(\xi_z) \tag{34}$$

Based on (34), the two rule TS fuzzy model of (33) can now be given as:

Model Rule 1: IF ξ_z is ρ_1 THEN $\begin{pmatrix} \dot{x}_{z1} \\ \dot{x}_{z2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -a_{z11} & -a_{z12} \end{pmatrix} \begin{pmatrix} x_{z1} \\ x_{z2} \end{pmatrix} + \begin{pmatrix} 0 \\ b_{z11} \end{pmatrix} u_z$ (35)

Model Rule 2: IF ξ_z is ρ_2 THEN $\begin{pmatrix} \dot{x}_{z1} \\ \dot{x}_{z2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -a_{z21} & -a_{z22} \end{pmatrix} \begin{pmatrix} x_{z1} \\ x_{z2} \end{pmatrix} + \begin{pmatrix} 0 \\ b_{z21} \end{pmatrix} u_z$ (36)

By assuming: m_m =0.5kg, l_m=0.5m, J_m=0.0417Kg-m^2, m_s =2.0kg, l_s=1.0m, J_s=0.667Kg-m^2; the parameters in (35) and (36) for master and slave systems are computed as:

$$\begin{aligned} a_{m11} &= 58.86, a_{m12} = 12.0, b_{m11} = 24.0 \\ a_{m21} &= 48.67, a_{m22} = 12.0, b_{m21} = 24.0 \end{aligned} \tag{37}$$

$$\begin{aligned} a_{s11} &= 29.43, a_{s12} = 1.50, b_{s11} = 1.50 \\ a_{s21} &= 24.34, a_{s22} = 1.50, b_{s21} = 1.50 \end{aligned} \tag{38}$$

Besides system parameters, we need environmental model and the desired polynomials for master and error systems to obtain the control gains. The environment is assumed to behave like a spring-damper system with the following parameters:

$$Z_e = -\begin{pmatrix} 0.8 & 0.02 \end{pmatrix} \tag{39}$$

Also, the desired polynomials for master and error systems are selected as:

$$\begin{aligned} p(s) &: s^2 + 10s + p_1 = 0 \\ q(s) &: s^2 + 10s + 25 = 0 \end{aligned} \tag{40}$$

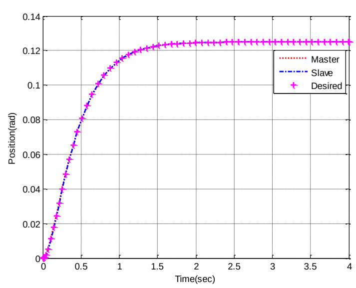
Please note that the last coefficient p_1 of the desired master polynomial cannot be chosen freely as it is constrained by other parameters and will be determined as a part of the solution. It is pertinent to mention that the selection of the desired polynomials in (40) is vital to ensure the stability and performance of the time delayed closed loop teleoperation system. Now, by considering the time delay in the communication channel to be $T = 0.01s$ and g_1 as unity, we obtain the following solution to the design equations (27)-(29),(32) through MATLAB symbolic toolbox:

$$\begin{aligned} p_1 &= 21.2416 \\ g_2 &= 19.8151 \\ C_m &= \begin{pmatrix} c_{m1} & c_{m2} \end{pmatrix} = \begin{pmatrix} 0 & 0.3805 \end{pmatrix} \\ C_s &= \begin{pmatrix} c_{s1} & c_{s2} \end{pmatrix} = \begin{pmatrix} -50.5539 & -22.8256 \end{pmatrix} \\ R_m &= \begin{pmatrix} r_{m1} & r_{m2} \end{pmatrix} = \begin{pmatrix} 0 & -0.3805 \end{pmatrix} \\ R_s &= \begin{pmatrix} r_{s1} & r_{s2} \end{pmatrix} = \begin{pmatrix} 18.6505 & 7.9608 \end{pmatrix} \end{aligned} \tag{41}$$

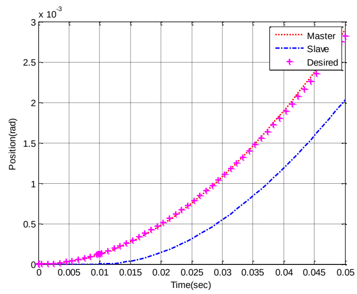
In order to implement the fuzzy logic controllers on master and slave systems, we determine the control gains based on the solution in (41), system parameters in (37)-(38) and the time invariant parameters in (9),(13) as:

$$\begin{aligned} D_{m1} &= \begin{pmatrix} d_{m11} & d_{m12} \end{pmatrix} = \begin{pmatrix} 58.86 & 12.38 \end{pmatrix} \\ D_{m2} &= \begin{pmatrix} d_{m21} & d_{m22} \end{pmatrix} = \begin{pmatrix} 48.6768 & 12.38 \end{pmatrix} \\ D_{s1} &= \begin{pmatrix} d_{s11} & d_{s12} \end{pmatrix} = \begin{pmatrix} -21.1239 & -21.3256 \end{pmatrix} \\ D_{s2} &= \begin{pmatrix} d_{s21} & d_{s22} \end{pmatrix} = \begin{pmatrix} -26.2155 & -21.3256 \end{pmatrix} \end{aligned} \tag{42}$$

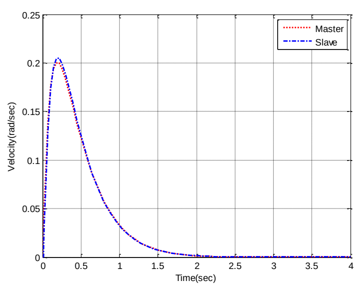
We now simulate the nonlinear teleoperation system of (33) by using the system parameters in Table 1 and the control gains in (41)-(42). The behavior of the teleoperation system as the operator applies a constant force of 0.1N is depicted in Figure 3. It can be observed that slave system is following the trajectory of the master system starting from the same initial conditions. Also, observe that the master system exhibits the desired dynamic behavior as assigned through the polynomial $p(s)$. The control inputs for both the master and slave systems in this case are also recorded and displayed in Figure 4. It should be noted that the transparency optimized state convergence scheme is found to be sensitive to actuator saturation phenomenon and can easily be driven to instability. Thus, poles of the closed loop teleoperation system should be selected carefully to avoid the actuator saturation problem. We also analyze the environmental force which is reflected onto the master system. Figure 5 shows the operator's applied force as well as the force reflected from the slave system as it interacts with the environment. It can be observed that the operator is able to fully perceive the environment during the steady state. This result coincides with the design condition (32) which only ensures the force tracking in steady state. The behavior of the closed loop teleoperation is also analyzed under the application of more realistic time varying operator's force. The position and force tracking results for this case are shown in Figure 6. It can be observed that the slave system is at different initial position than the master system and is able to catch the master system after a transient. However, a constant force reflection error is observed during the ramp period, which disappears when the force becomes constant.



(a)



(b)



(c)

Figure 3

Master and slave systems' states under an operator's force of 0.1N (a) Position signals (b) Zoomed position signals (c) Velocity signals

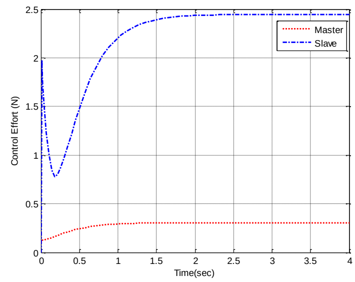


Figure 4
Control inputs for master and slave systems

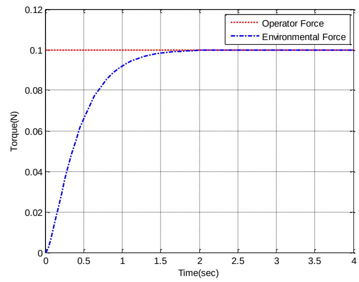
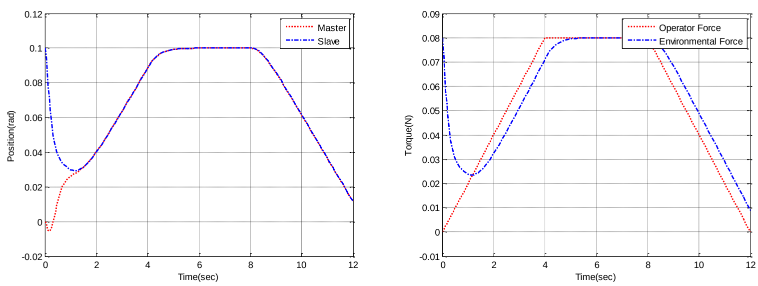


Figure 5
Operator and environmental forces

We also analyze the effect of uncertainties on the performance of the closed loop teleoperation system. To this end, we first consider 100% uncertainty in the time delay. With the same control gains as in (41)-(42) and considering $T = 0.02s$, we simulate the teleoperation system under the application of a constant applied force measuring 0.1N and the results are shown in Figure 7. It can be seen that although the state convergence between master and slave systems is achieved, a deviation from the desired dynamic behavior is evident. Further, this deviation increases as the uncertainty in the time delay increases which can be observed from figure 8 where a 200% uncertainty in the time delay ($T = 0.03s$) is considered. However, the transparency of the teleoperation system is achieved in steady state as both the slave position signal and the environmental force matches with the master position signal and the operator's applied force after the transient period.



(a) (b)

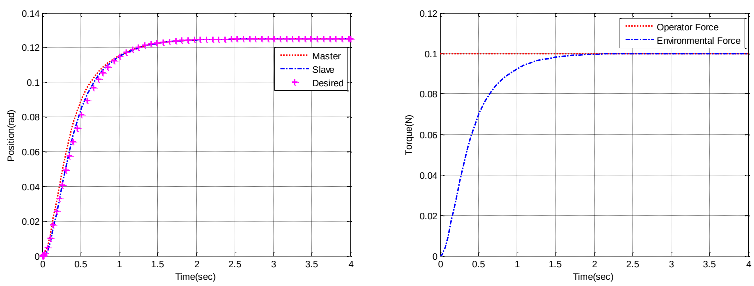
Figure 6
Performance of teleoperation system with time varying operator's force (a) Master-slave position signals (b) Operator-environment forces

Besides the uncertainty in the time delay, we also study the effect of uncertainty in coefficient of viscous friction on the performance of teleoperation system. The result of this analysis is shown in Figure 9 where a 50% uncertainty is considered in the coefficient of viscous friction of both master and slave systems. It can be seen that the teleoperation system is exhibiting a sluggish response. The effect of adding the uncertainty in the time delay and the coefficient of viscous friction at the same time is shown in Figure 10. It is evident that the uncertainty in the coefficient of viscous friction has a greater impact in degrading the system's performance.

Finally, we compare the performance of the proposed fuzzy logic controller with the existing linear controller in achieving the transparency during large range operation. The linear controller is derived from the proposed controller using (43) and is given in (44).

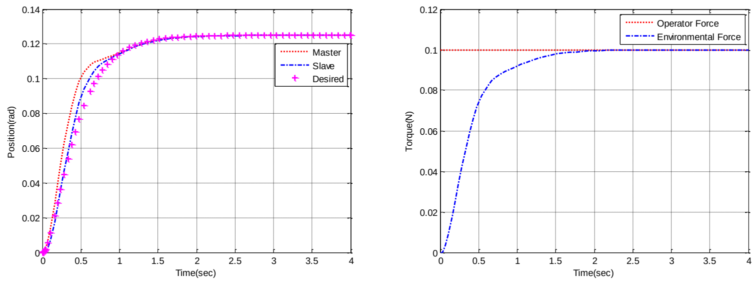
$$\begin{aligned} k_{mj} &= \frac{1}{b_{m1}}\left(a_{mj}+c_{mj}\right), \forall j = 1,2,\ldots,n \\ k_{sj} &= \frac{1}{b_{s1}}\left(a_{sj}+c_{sj}\right), \forall j = 1,2,\ldots,n \end{aligned} \tag{43}$$

$$\begin{aligned} K_m &= \begin{pmatrix} k_{m1} & k_{m2} \end{pmatrix} = \begin{pmatrix} 2.4525 & 0.5159 \end{pmatrix} \\ K_s &= \begin{pmatrix} k_{s1} & k_{s2} \end{pmatrix} = \begin{pmatrix} -14.0826 & -14.2171 \end{pmatrix} \end{aligned} \tag{44}$$



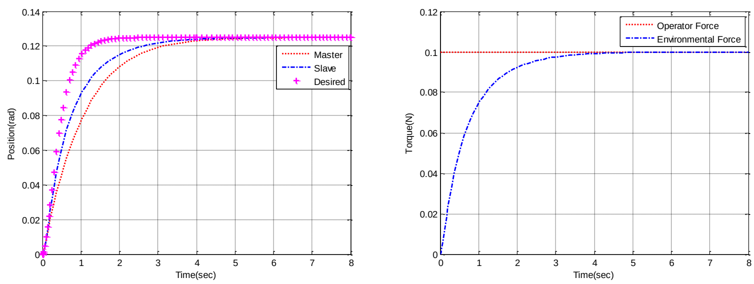
(a) (b)

Figure 7
Effect of 100% uncertainty in time delay on the performance of teleoperation system (a) Master-slave position signals (b) Operator-environment forces



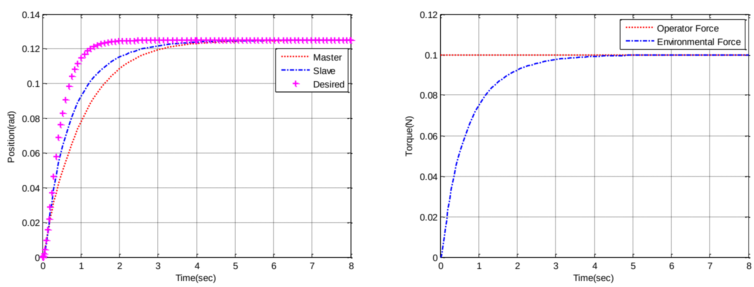
(a) (b)

Figure 8
Effect of 200% uncertainty in time delay on the performance of teleoperation system (a) Master-slave position signals (b) Operator-environment forces



(a) (b)

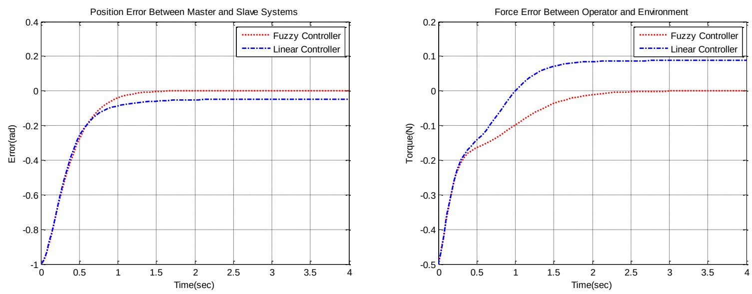
Figure 9
Effect of 50% uncertainty in the coefficient of viscous friction on the performance of teleoperation system (a) Master-slave position signals (b) Operator-environment forces



(a) (b)

Figure 10
Effect of 50% uncertainty in the coefficient of viscous friction and time delay on the performance of teleoperation system (a) Master-slave position signals (b) Operator-environment forces

By considering the final position to be reached as 1 rad, the nonlinear teleoperation system is simulated and the performance of the two controllers is recorded. The errors in the position and the force signals for the two cases are then computed and are displayed in Figure 11. It can be observed that the proposed fuzzy logic controller has shown superior performance as the error in position and force signals has converged to zero while a constant position and force error is seen in case of the linear controller. Thus, perfect transparency is achieved by the proposed fuzzy logic controller.



(a) (b)

Figure 11
Comparison of proposed fuzzy logic controller and existing linear controller (a) Master-slave position errors (b) Operator-environment force errors

Conclusions

This paper has presented the design of a fuzzy model based transparent controller for a nonlinear teleoperation system based on its TS fuzzy description. The proposed TS fuzzy logic control laws for the master and slave systems allow using the method of state convergence in its true sense. The feasibility of the presented approach is evaluated through simulations in MATLAB environment on a one DoF tele-manipulator. It is concluded that the proposed approach can control a nonlinear teleoperation system with a small time delay in the communication channel. Future work involves enabling the scheme to work in the presence of time varying delays. The robustness of the scheme to parameter uncertainties need to be improved as well.

References

[1] M. Ferre, M. Buss, R. Aracil, C. Melchiorri, C. Balaguer: Advances in Telerobotics, Springer, 2007

[2] P. F. Hokayem, M. W. Spong: Bilateral Teleoperation: an Historical Survey, Automatica, Vol. 42, 2006, pp. 2035-2057

[3] R. Muradore, P. Fiorini: A Review of Bilateral Teleoperation Algorithms, Acta Polytechnica Hungarica, Vol. 13, No. 1, 2016, pp. 191-208

[4] J. Artigas, G. Hirzinger: A Brief History of DLR's Space Telerobotics and Force Feedback Teleoperation, Acta Polytechnica Hungarica, Vol. 13, No. 1, 2016, pp. 239-249

[5] R. Anderson, M. W. Spong: Bilateral Control of Teleoperators with Time Delay: IEEE Transactions on Automatic Control, Vol. 34, No. 5, 1989, pp. 494-501

[6] G. Niemeyer, J. J. E. Slotine: Stable Adaptive Teleoperation, IEEE Journal of Oceanic Engineering, Vol. 16, No. 1, 1991, pp. 152-162

[7] J. Ryu, D. Kwon, B. Hannaford: Stable Teleoperation with Time-domain Passivity Control, IEEE Transactions on Robotics and Automation, Vol. 20, No. 2, 2004, pp. 365-373

[8] A. Ziminejad, M. Tavakoli, R. V. Patel, M. Moallem: Transparent Time-delayed Bilateral Teleoperation Using Wave Variables, IEEE Transactions on Control Systems Technology, Vol. 16, No. 3, 2008, pp. 548-555

[9] L. Chan, F. Naghdy, D. Stirling: Application of Aadaptive Controllers in Teleoperation Systems, IEEE Transactions on Human-Machine Systems, Vol. 44, No. 3, 2014, pp. 337-352

[10] A. Hace, M. Franc: FPGA Implementation of Sliding Mode Control Algorithm for Scaled Bilateral Teleoperation, IEEE Transactions on Industrial Electronics, Vol. 9, No. 3, 2013, pp. 1291-1300

[11] M. Boukhnifer, A. Ferreira: H∞ Loop Shaping Bilateral Controller for a Two-fingered Tele-micromanipulation System, IEEE Transations on Control Systems Technology, Vol. 15, No. 5, 2007, pp. 891-905

[12] V. T. Minh, F. M. Hashim: Adaptive Teleoperation System with Neural Network Based Multiple Model Control, Mathematical Problems in Engineering, Vol. 2010, pp. 1-16

[13] J. Zhu, W. Gueaieb: Adaptive Fuzzy Logic Control for Time-delayed Bilateral Teleoperation, Autonomous and Intelligent Systems, Vol. 7326, 2012, pp. 106-115

[14] Y. Yang, C. Hua, X. Guan: Adaptive Fuzzy Finite-time Coordination Control for Networked Nonlinear Bilateral Teleoperation System, IEEE Transactions on Fuzzy Systems, Vol. 22, No. 3, 2014, pp. 631-641

[15] H. O. Wang, K. Tanaka, M. F. Griffin: An Approach to Fuzzy Control of Nonlinear Systems: Stability and Design Issues, IEEE Transactions on Fuzzy Systems, Vol. 4, No. 1, pp.14-23, 1996

[16] L. K. Wong, F. H. F. Leung, P. K. S. Tam: Design of Fuzzy Logic Controllers for Takagi-Sugeno Fuzzy Model Based System with Guaranteed Performance, International Journal of Approximate Reasoning, Vol. 30, pp. 41-55, 2002

[17] R-E. Precup, M-L. Tomescu, S. Preitl: Fuzzy Logic Control System Stability Analysis Based on Lyapunov's Direct Method, International Journal of Computers, Communications and Control, Vol. 4, 2009, pp. 415-426

[18] Y. Wei, J. Qiu, H. R. Karimi: Reliable Output Feedback Control of Discrete Time Fuzzy Affine Systems with Actuator Faults, IEEE Transactions on Circuits and Systems, Vol. 64, No. 1, 2017, pp. 170-181

[19] Y. Wei, J. Qiu, P. Shi, M. Chadli: Fixed-Order Piecewise-Affine Output Feedback Controller for Fuzzy-Affine-Model-Based Nonlinear Systems with Time Varying Delay, IEEE Transactions on Circuits and Systems, Vol. 64, No. 4, 2017, pp. 945-958

[20] Z. Lendek, A. Sala, P. Garcia, R. Sanchis: Experimental Application of Takagi-Sugeno Observers and Controllers in a Nonlinear Electromechanical System, Journal of Control Engineering and Applied Informatics, Vol. 15, No. 4, 2013, pp. 3-14

[21] T-C. Lin, T-Y. Lee, V. E. Balas: Adaptive Fuzzy Sliding Mode Control for Synchronization of Uncertain Fractional Order Chaotic Systems, Chaos, Solitons & Fractals, Vol. 44, 2011, pp. 791-801

[22] U. Farooq, J. Gu, M. E. El-Hawary, V. E. Balas, M. U. Asad: Experimental Study of Optimal Takagi Sugeno Fuzzy Controller for Rotary Inverted Pendulum, Proc. IEEE FUZZ, 2015, pp. 1-7

[23] X. Yang, C. C. Hua, J. Yan, X. P. Guan: A New Master-Slave Torque Design for Teleoperation System by T-S Fuzzy Approach, IEEE Transactions on Control Systems Technology, Vol. 23, No. 4, 2015, pp. 1611-1619

[24] U. Farooq, J. Gu, M. El-Hawary, M. U. Asad, G. Abbas: Fuzzy Model Based Bilateral Control Design of Nonlinear Tele-operation System Using Method of State Convergence, IEEE Access, Vol. 4, 2016, pp. 4119-4135

[25] U. Farooq, J. Gu, M. El-Haway, M. U. Asad, Faiqa Rafiq: Transparent Fuzzy Bilateral Control of a Nonlinear Teleoperation System Through State Convergence, Proc. IEEE International Conference on Emerging Technologies, 2016, pp. 1-7.

[26] J. M. Azorin, O. Reinoso, R. Aracil, M. Ferre: Generalized Control Method by State Convergence of Teleoperation Systems with Time Delay, Automatica, Vol. 40, No. 9, 2004, pp. 1575-1582

[27] M. A. Artigas, R. Aracil, M. Ferre, C. Garcia: Bilateral Adaptive Control by State of Convergence in Teleoperation Systems, Proc. IEEE International Conference on Control Applications, 2009, pp. 548-553

[28] R. Aracil, J. M. Azorin, M. Ferre, C. Pena: Bilateral Control by State Convergence Based on Transparency for Systems with Time Delay, Robotics and Autonomous Systems, Vol. 61, No. 2, 2013, pp. 86-94