Weight Optimization of Tower Structures with Continuous Variables using Jaya Algorithm

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Abstract: A popular metaheuristic algorithm named Jaya is preferred to solve the problem of finding the minimum weight of tower structures. Joints coordinates of free nodes and the area of bar elements are constrained using the lower and upper bounds of these design variables for shape and size optimization, respectively. The constraints used in this study are the Euler buckling, allowable stress, and displacement. The presented algorithm is tested with two classic benchmark problems: the spatial truss tower with 39-bar and the transmission tower with 272-bar. The Jaya algorithm is coded in MATLAB environment and implemented into the linear finite element solver.

Keywords: weight optimization; tower structures; shape and size optimization; continuous optimization; Jaya algorithm

1 Introduction

The feasible optimal design of structures is an interesting topic to engineers in practice. The minimal cost and the optimal geometry of the construction are the basic purposes of the engineers when designing the structure by taking into account the constrained objective function. We are interested in designing structures with high reliability. Many optimal algorithms for this aim were tested, including innovative algorithms and classical methods.

Until now, many meta-heuristic methods have been presented for the structural optimization problems. Some of the most popular optimization algorithms are GA: Genetic Algorithm, SA: Simulated Annealing, PSO: Particle Swarm Optimization, HS: Harmony Search and ACO: Ant Colony Optimization.

In the recent years, JA: Jaya Algorithm was presented as a new metaheuristic technique and it has a very simple form and does not use more specific parameters. Many studies were made by using this algorithm. The optimization of cables size in cable-stayed bridge with Jaya algorithm was used by Atmaca et al. [1], optimum design of steel grillage was presented by Dede [3], braced dome structures with natural frequency constraints was studied by Grzywiński et al. [4]. Grzywiński et al. [5] studied four benchmark problems (trusses 2D ten-bar, 3D thirty-seven-bar, 3D seventy-two-bar and 2D two-hundred-bar by TLBO (Teaching-learning based optimization) algorithm. The optimization of spatial truss tower (25-bar, 39-bar, 72-bar, and 160-bar) based on Rao algorithm was analyzed by Grzywiński [6].

2 Jaya Optimization Algorithm

As a popular optimization method the Jaya was firstly presented by Rao [11, 12, 14]. The meaning of this new algorithm is the "victory" in the Sanskrit language. The basic rules of this method are to close the best solution and stay away from the worst solution in a population consist of the potential solutions for an optimization problem. The main properties of this algorithms is that it does not has a special parameter to carry out the optimization process. Like the other population-based algorithms, Jaya need a population size (Pn) and the use a generation number (Gn). The general equation for Jaya is given in Eq. (1).

$$P_{k,l,i}^{new} = P_{k,l,i} + r_{1,k,i}\left(P_{k,b,i} - \left|P_{k,l,i}\right|\right) - r_{2,k,i}\left(P_{k,w,i} - \left|P_{k,l,i}\right|\right) \tag{1}$$

Let $P_{k,l,i}$ it shows the k^th^ design variable for the l^th^ design of the population at the beginning of the i^th^ iteration. Where $P_{k,l,i}^{new}$ is the updated design variable, and $r_{1,k,it}$, $r_{2,k,it}$ are random numbers which can be change from "0" to "1".

The Jaya algorithm has of following steps:

1) first are define the size of the population (Pn) and the termination criterion,

2) next program generates the initial population randomly,

3) algorithm finds the best and worst solutions in the population,

4) after method looking for a new solution in accordance with the equation (1)

5) if the updated solution is better than the old one, the updated solution is used for the next iteration

6) if the termination criterion is satisfied, stop the optimization (else go to step 2).

3 Optimization of Tower Structures Problems

Formulation of design optimization includes the weight minimization of tower structures subjected to displacement, stress and buckling constraints. The design variables and the objective function are given as below;

obtain $A = [A_1, A_2, \cdots, A_{ng}]$ (2)

to optimize $W(A, x) = \sum_{k=1}^{ng} A_k \sum_{i=1}^{mk} \rho_i L_i x_i$ (3)

where $W(A, x)$ is the total structural weight; A_k is vector of the size optimization (cross-section area) and x_i are joint coordinates of the free nodes as shape optimization, respectively; ρ_i and L_i is the density and length of bar elements. ng and mk are the number of groups and the number of bar elements in each groups, respectively.

The structural constraints used in this study are;

for tensile members, $\sigma_k \leq \sigma_k^t$ $k = 1,2, \ldots, ntm$ (4)

for compressive members, $\sigma_k \leq \sigma_k^c$ $k = 1,2, \ldots, ncm$ (5)

where σ_k is the calculated stress, σ_k^t and σ_k^c is the allowable tensile and the compressive stresses, respectively. “ntm” and the “ncm” are the number of tensile member and the number of compressive member, respectively.

$\sigma_k \leq \sigma_k^b$ $k = 1,2, \ldots, ncm$ (6)

where σ_k^b is the Euler buckling and it is given as:

$\sigma_k^b = \frac{K \cdot E \cdot A_k}{L_k^2}$ $k = 1,2, \ldots, ncm$ (7)

where E is the elasticity property of the material, and K is a constant. The constraint for the displacement is given below.

$|\mathrm{d}_i| \leq \mathrm{d}_{max}$ $i = 1,2, \ldots, nn$ (8)

where d_i is the nodal displacement, d_{max} is allowable displacement, nn and nm are number of nodes and number of bar elements, respectively.

$A_{min} \leq A_k \leq A_{max}$ $k = 1,2, \ldots, nm$ (9)

$x_{min} \leq x_i \leq x_{max}$ $i = 1,2, \ldots, nn$ (10)

To obtain best solution without the violations a penalty function is transformed another form to include the effect of the constraints. Using this function, it will be hoped that the optimization problem will find a feasible global solution. Thus, the penalized objective function (Fp) is obtained as given in Eq. 11.

$$Fp = W(A, x)[1 + Cp] \tag{11}$$

The penalized objective function including the nodal violates and member stress violates given as:

$$Cp = \sum_{i=1}^{nn} c_i + \sum_{k=1}^{ng} c_k \tag{12}$$

$$c_i = \frac{|d_i|}{d_{max}} \tag{13}$$

$$c_k = \frac{|\sigma_k|}{\sigma_{max}} \tag{14}$$

4 Testing of the Jaya Algorithm

The 272-bar transmission truss tower with 28 continuous design variables and 39-bar spatial truss tower with 11 continuous design variables were tested to show the performance of the Jaya algorithm. These structural example were taken from literature. The present the efficiency and the performance of the proposed algorithm the best feasible global solution obtained from this study were compared with the previous studies in the tables includes final design variables. In this study twenty independent runs were carried out to show the robustness of the Jaya algorithms. The Jaya algorithm, optimization tools and a standard linear elastic finite element solver were coded in the MATLAB programming by the author of this paper.

4.1 The First Numerical Example: 39-bar Truss

The first structural example is 39-bar spatial tower given in Figure 1a with the sizing and shape optimization. This tower structure was before designed by Shojaee et al. [13], Dede & Ayvaz [2], and Ho-Huu et al. [7]. Input data for this example was given in Table 1 and the elements connectivity and nodal coordinates were presented in Table 2. Among the total free nodes of the structure, the top and bottom nodes have fixed position, and the other middle nodes' coordinates are taken as design variables. At the end of the optimization process, final feasible shape obtained using proposed algorithm is given in Figure 1b. To compare the results with those given in previous studies, the optimal design variables were listed in Table 3.

The population size and number of iterations is set to 40 and 200, respectively. The Jaya algorithm found the best design after 7640 analyses and actually obtained an optimum design with the 133.51 kg. In Figure 2 is presented the converge history for the best result, and Figure 3 is shown results of 20 independent runs.

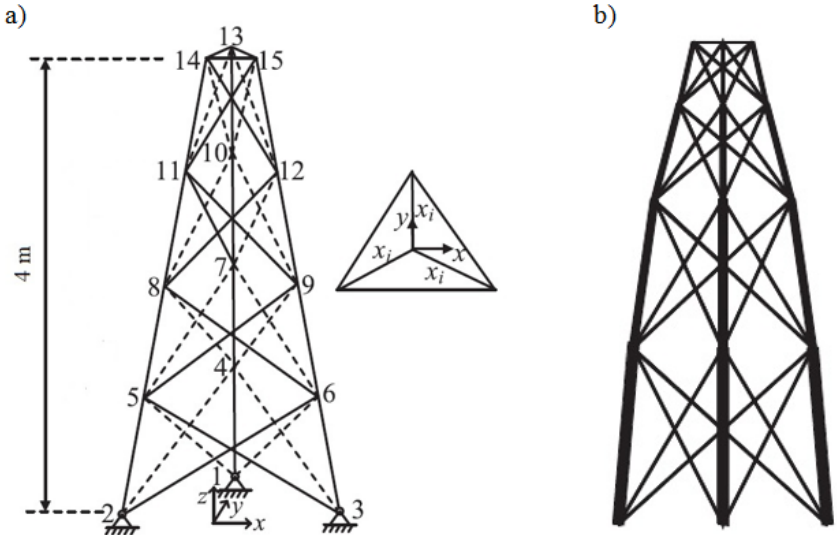

Figure 1
The first example: 39-truss spatial truss tower: a) before optimization, b) after optimization

Table 1
Material data and the constraints for the 39-bar spatial truss tower

Properties / constraints	Unit	Value / notes
Modulus of elasticity	E (GPa)	210
Material density	ρ (kg/m^3)	7800
Stress constraints	σ (MPa)	240 for tension -240 for compression
Displacement constraints	δ (cm)	0.4 for Y directions (nodes 13-15)
Nodal forces	F (kN)	±10 for Y directions (nodes 13-15)
Euler buckling	σ_e (MPa)	$\sigma_e \leq \frac{K_e E A_e}{L_e^2}$

Table 2
Initial data for the 39-bar spatial truss tower

Shape variables				Size variables	
joint	x (m)	y (m)	z (m)	cross-area	node-node
1	0.000	1.000	0.000	A1	(1-4), (2-5), (3-6)
2	-0.866	-0.500	0.000	A2	(4-7), (5-8), (6-9)
3	0.866	-0.500	0.000	A3	(7-10), (8-11), (9-12)
13	0.000	0.280	4.000	A4	(10-13), (11-14), (12-15)
14	-0.242	-0.140	4.000	A5	the remaining elements
15	0.242	-0.140	4.000		

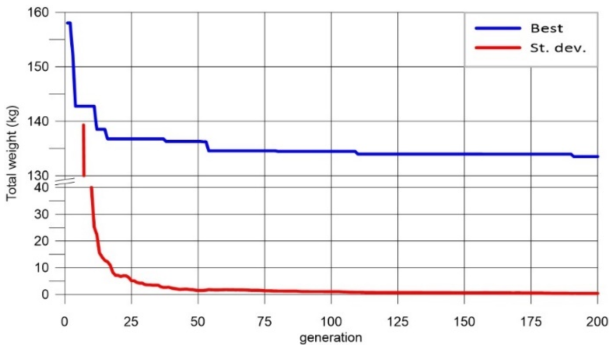

Figure 2
Convergence history of 39-bar spatial truss tower

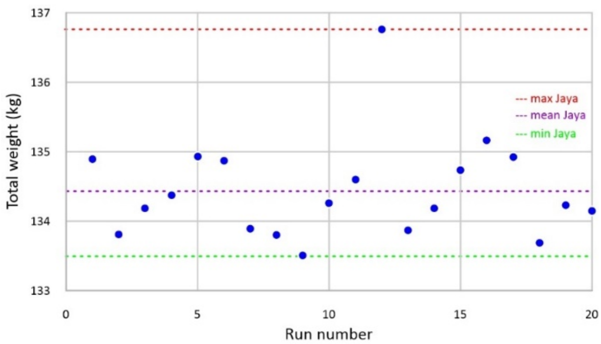

Figure 3
Results of the twenty runs for the 39-bar spatial truss tower

Table 3
Optimal results for the 39-bar spatial truss tower

Design variables	Group members	DPSO-MMA [13]	TLBO [2]	D-ICDE [7]	JA This study
1	Al (cm^2)	10.01	11.9650	13.0	11.9900
2	A2 (cm^2)	9.91	11.1457	12.9	9.7811
3	A3 (cm^2)	8.56	7.8762	9.0	6.9870
4	A4 (cm^2)	3.92	2.7013	2.7	2.0264
5	A5 (cm^2)	3.44	2.4058	1.6	1.7309
6	Y4 (m)	0.6683	0.8996	0.9232	0.8694
7	Z4 (m)	1.9000	1.3507	0.5380	1.1972
8	Y7 (m)	0.4732	0.6917	0.7958	0.6774
9	Z7 (m)	2.8734	2.3122	2.1637	2.4966
10	Y10 (m)	0.3002	0.4825	0.5105	0.4697
11	Z10 (m)	3.4415	3.3031	3.4134	3.3985

	W (kg)	176.834	154.13	140.35	133.51
	MFE	N/A	7560	1140	7640

W: weight; MFE: maximum function evaluations.

4.2 The Second Numerical Example: 272-bar Truss

The second structural example is size optimization of the 272-bar transmission tower shown in Figure 4. This tower structure was previously designed by Kaveh & Massoudi [8], Kaveh & Zaerreza [9], and Kaveh et al. [10].

The Young's modulus is 200 GPa and the allowable stresses for all members is ±275 MPa. The more information about model: nodal coordinates, topology and member grouping find in Kaveh & Massoudi [8]. The transmission tower is grouped into 28 continuous design variables. The limit for the design variables of cross-sectional areas are 10 cm^2 and 160 cm^2 for the lower bounds and upper bounds, respectively. The tower has many different loading cases. The details for these cases are given in Table 4. The structural constraints in the case of displacements are limited 20 mm in z-direction and 100 mm both x- and y-direction for the joints 1, 2, 11, 20, 29.

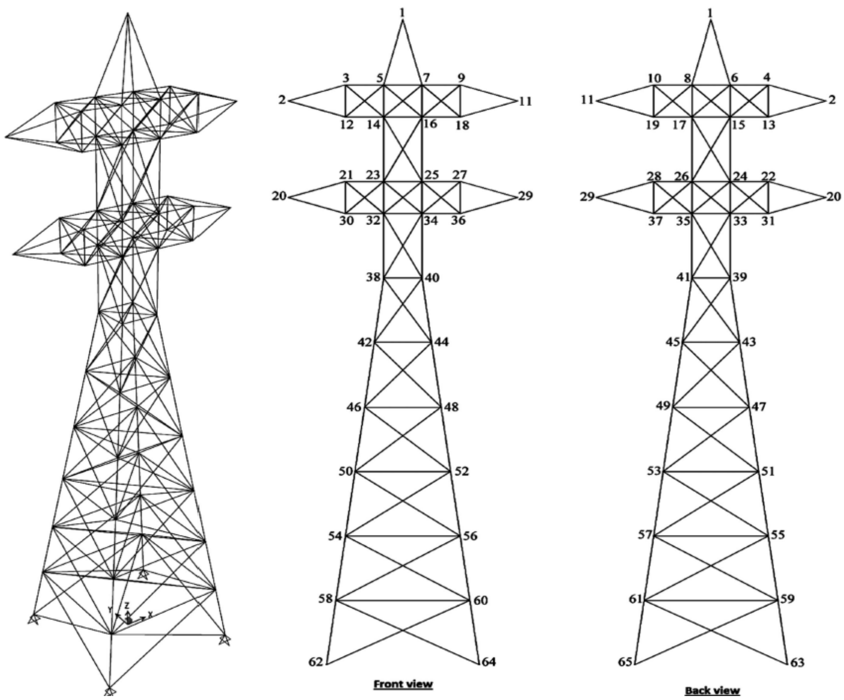

Figure 4

The second example: the 272-bar transmission tower

Table 4
Load cases for the 272-bar transmission tower

Load case	Force direction	Nodes					
		1	2	11	20	29	other free
1	F_x (kN)	20	20	20	20	20	5
	F_y (kN)	20	20	20	20	20	5
	F_z (kN)	-40	-40	-40	-40	-40	0
2	F_x (kN)	0	20	20	20	20	5
	F_y (kN)	0	3)	20	20	20	5
	F_z (kN)	0	-40	-40	-40	-40	0
3	F_x (kN)	20	0	20	20	20	5
	F_y (kN)	20	0	20	20	20	5
	F_z (kN)	-40	0	-40	-40	-40	0
4	F_x (kN)	20	20	20	0	20	5
	F_y (kN)	20	20	20	0	20	5
	F_z (kN)	-40	-40	-40	0	-40	0
5	F_x (kN)	20	0	0	0	0	5
	F_y (kN)	20	0	0	0	0	5
	F_z (kN)	-40	0	0	0	0	0
6	F_x (kN)	0	20	0	0	0	5
	F_y (kN)	0	30	0	0	0	5
	F_z (kN)	0	-40	0	0	0	0
7	F_x (kN)	0	0	0	20	0	5
	F_y (kN)	0	0	0	20	0	5
	F_z (kN)	0	0	0	-40	0	0
8	F_x (kN)	0	0	20	20	20	5
	F_y (kN)	0	0	20	20	20	5
	F_z (kN)	0	0	-40	-40	-40	0
9	F_x (kN)	0	20	20	0	20	5
	F_y (kN)	0	20	20	0	20	5
	F_z (kN)	0	-40	-40	0	-40	0
10	F_x (kN)	0	0	20	0	20	5
	F_y (kN)	0	0	20	0	20	5
	F_z (kN)	0	0	-40	0	-40	0
11	F_x (kN)	0	0	0	20	20	5
	F_y (kN)	0	0	0	20	20	5
	F_z (kN)	0	0	0	-40	-40	0
12	F_x (kN)	0	0	20	20	0	5
	F_y (kN)	0	0	20	20	0	5
	F_z (kN)	0	0	-40	-40	0	0

The optimum volume of the 272-bar transmission tower found by Jaya algorithm is shown in Table 5. The optimization result is the same as in the PGO algorithm. Jaya algorithm find optimal volume after 23100 analyses. In Figure 5 is presented the converge history for the best result 1.1681 m^3, and Figure 6 is shown results of 20 independent runs.

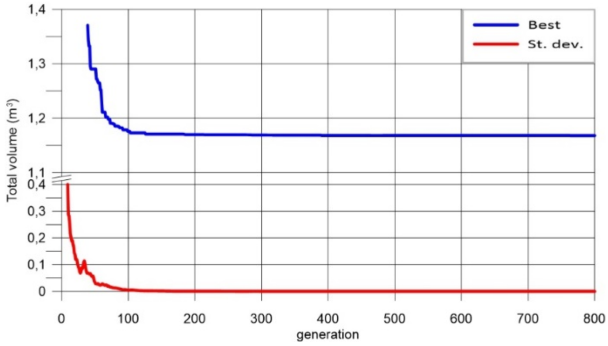

Figure 5

Convergence history for the 272-bar transmission tower

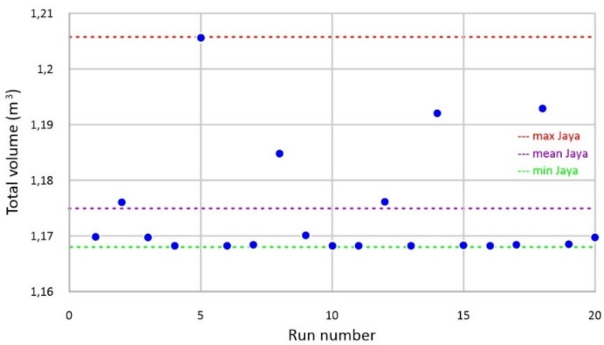

Figure 6

Results of the independent runs for the 272-bar transmission tower

Conclusions

In this article, a proposed popular optimization method named Jaya is preferred for the optimization of structural example which are the 39-bar, and 272-bar truss spatial towers. The validity of the JA is demonstrated by using these tower structures. By taking into account three different structural constraints which are the displacement, allowable stress, and Euler's buckling. The original Jaya algorithm is tested for the constrained single objective problem.

The Jaya algorithm doesn't use any special parameters to carry out the optimization process. By the help of this properties, the Jaya algorithms is a popular optimization algorithm. When compared the feasible optimal solutions obtained from this study using Jaya algorithm with those given in previous studies, it an be clearly stated that that the Jaya algorithm can be effectively used in the design of spatial tower structures.

Table 5

Best results for the 272-bar transmission tower

Group members	SSOA [9]	PGO [10]	JA This study	Group members	SSOA [9]	PGO [10]	JA This study
Al (cm^2)	10.00	10.00	10.00	A15 (cm^2)	93.20	91.19	91.77
A2 (cm^2)	12.40	12.17	12.34	A16 (cm^2)	10.00	10.00	10.00
A3 (cm^2)	24.92	24.45	24.83	A17 (cm^2)	10.00	10.00	10.00
A4 (cm^2)	10.17	10.00	10.01	A18 (cm^2)	10.02	10.00	10.00
A5 (cm^2)	96.18	95.80	95.51	A19 (cm^2)	83.90	91.19	91.77
A6 (cm^2)	10.00	10.00	10.00	A20 (cm^2)	10.01	10.00	10.00
A7 (cm^2)	120.64	122.59	123.63	A21 (cm^2)	10.00	10.00	10.00
A8 (cm^2)	10.01	10.01	10.00	A22 (cm^2)	10.03	10.00	10.00
A9 (cm^2)	10.00	10.00	10.06	A23 (cm^2)	79.82	85.41	80.29
A10(cm^2)	10.00	10.00	10.00	A24 (cm^2)	10.00	10.00	10.00
A11 (cm^2)	102.17	106.15	102.09	A25 (cm^2)	10.00	10.00	10.00
A12 (cm^2)	10.00	10.00	10.00	A26 (cm^2)	10.00	10.00	10.00
A13 (cm^2)	10.00	10.00	10.00	A27 (cm^2)	75.04	76.57	74.57
A14 (cm^2)	10.00	10.00	10.00	A28 (cm^2)	10.00	10.00	10.00
				V(m^3)	1.1682	1.1681	1.1681
				MFE	14020	23920	23100

Conflicts of Interest

The author declares no conflict of interest.

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