A Copula-based Approach to the Analysis of the Returns of Exchange Rates to EUR of the Visegrád Countries[1]

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Abstract: The currencies of the Visegrád countries (Poland, the Czech Republic, Hungary, and Slovakia) have been considered by the international financial community as a basket of currencies which are closely related, especially in times of their depreciations. On July 1, 2008 the official terminal exchange rate SKK/EUR was fixed. During the following 8 months, the remaining three currencies (PLN, CZK, HUF) changed their long-term behaviour to one of strong parallel depreciation. On the other hand, in the first selected long-term period (January 4, 1999 – June 30, 2008), a relatively mixed development of HUF seemed to exhibit a rather low degree of interdependence with CZK (that had been appreciating very intensively). The values of the Kendall's correlation coefficient calculated for all 3 remaining couples of returns substantially rose in the second period (indicating that similarities between the returns of these exchange rates are stronger in the times of crises). We have performed modeling and fitting of the dependencies of the above mentioned couples of returns of currencies in both the mentioned time periods by several classes of bivariate copulas, as well as by (optimized) convex combinations of their elements.

Keywords: bivariate copula; return of exchange rates; Kendall's tau; convex combinations of copulas; goodness of fit (GOF) test

1 Introduction

The aim of this paper is to further extend our earlier studies of the relations between the returns of couples of exchange rates of the Visegrád countries to EUR ([9], [10]). We have again extended the considered time span until the end of July

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2009. We have also deepened the analytical tools of our former copula approach analyses, inspired by several preceding papers dealing with exchange rates modeling ([7, 8, 12]).

The currencies of the Visegrád countries (PLN, CZK, HUF, SKK) were considered by the international financial community as a basket of currencies that were closely related especially in turbulent times. Consequently, several common features in their behavior were expected, and were often also observed.

On July 1, 2008 the official terminal exchange rate SKK/EUR was fixed. Although this country officially entered the EUR zone only 6 months after, that exchange rate was essentially frozen in the meantime.

During the following 13 months, the remaining three currencies (PLN, CZK, HUF) changed their long-term behavior to a strong parallel depreciation until March 2009, when they started to appreciate again. This change was an obvious consequence of the extremely severe crisis of the global financial system that started in the middle of 2008 and which has slightly reversed since March 2009. Let us specify that for daily values of EUR, in the considered currencies the corresponding returns are defined by $R_t = (P_t - P_{t-1})/P_{t-1}$ where P_t is the exchange rate in time t. The time series of daily values of EUR in the considered currencies and the corresponding returns are presented in the Figures 1a-1d.

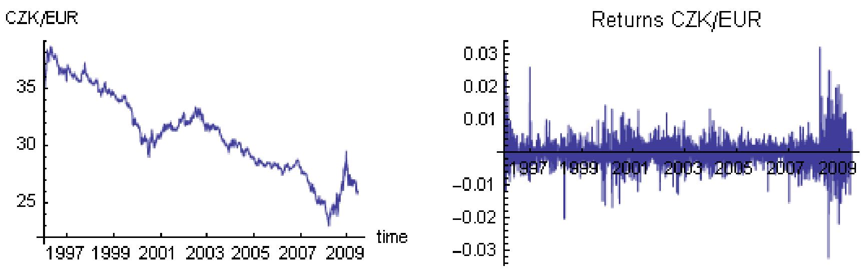

Figure 1a
Exchange rates of the Czech Crowns to EUR and the corresponding returns

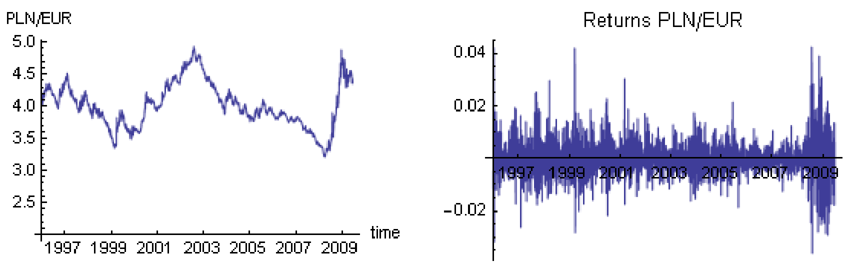

Figure 1b
Exchange rates of the Polish Zloty to EUR and the corresponding returns

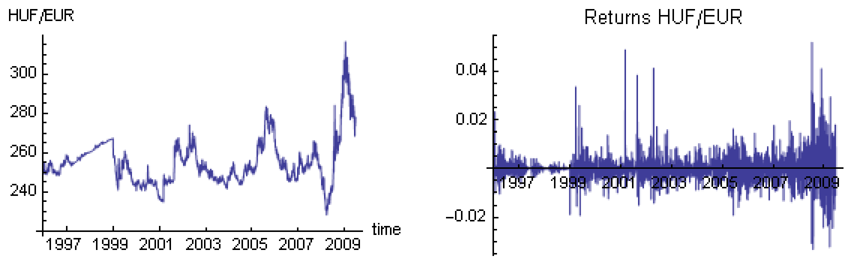

Figure 1c
Exchange rates of the Hungary Forint to EUR and the corresponding returns

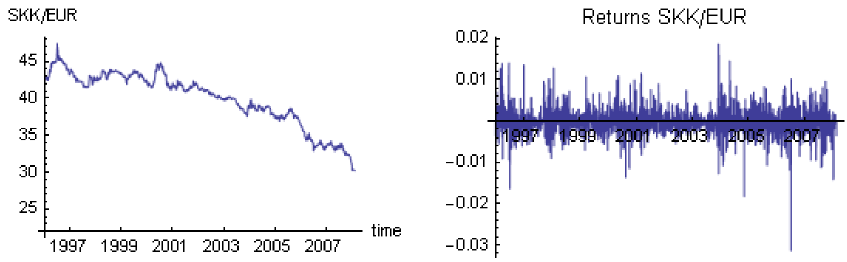

Figure 1d
Exchange rates of the Slovak Crowns to EUR and the corresponding returns

The values of the empirical versions of Kendall's correlation coefficients (cf. e.g. [1, 4, 5]) calculated for all considered couples of returns (presented in Table 1 for the first period January 4, 1999 – June 30, 2008, the second crisis period July 1, 2008 – July 31, 2009 and for the merged period January 4, 1999 – July 31, 2009) are in accordance with the previous qualitative reasoning. Their respective values for all three remaining couples of returns of exchange rates rose substantially in the second period (indicating that similarities between returns of exchange rates of the couples of these currencies are stronger in times of crises).

Table 1
The values of the empirical Kendall's coefficients τ for returns

Couple	**4.1.1999 – 30.6.2008**	**4.1.1999 – 31.7.2009**	**1.7.2008 - 31.7.2009**
(SKK/EUR, CZK/EUR)	0,231	x	x
(SKK/EUR, PLN/EUR)	0,214	x	x
(SKK/EUR, HUF/EUR)	0,240	x	x
(CZK/EUR, HUF/EUR)	0,167	0,209	0,443
(CZK/EUR, PLN/EUR)	0,217	0,246	0,423
(PLN/EUR, HUF/EUR)	0,319	0,345	0,509

We subsequently performed modeling and fitting of the dependencies of the above mentioned couples of returns of currencies separately for 2 periods, before and after July 1, 2008, as well as for the whole considered time period (January 4, 1999 – July 31, 2009) by several classes of bivariate copulas as well as by convex combinations of their elements. Based on our previous modeling experiments we utilized 3 well known 1–parametric classes of Archimedean copulas (Gumbel, Clayton, Frank) and the 2–parametric Joe BB1 copula.

2 Theoretical Basis

Recall that the most important applications of 2–dimensional copulas are related to a well known and very convenient alternative for expressing the joint distribution function F of a vector of continuous random variables (X, Y) in the form

$$F(x, y) = C\,(F_X(x), F_Y(y)), \tag{1}$$

where F_X, F_Y are the marginal distribution functions. Note that the copula $C:[0,1]^2 \to [0,1]$ is unique whenever X and Y are continuous random variables (see e.g. [11]).

2.1 Tail Dependencies between Random Variables

For a given copula $C(x, y)$, the upper and lower tail dependencies can be defined with reference to how much probability is in regions near (1, 1) (upper-right-quadrant tail) and (0, 0) (lower-left-quadrant tail). Let (X, Y) be a vector of continuous random variables with marginal distribution functions F_X, F_Y. The coefficient λ_U of *upper tail dependence* of (X, Y) is (see e.g. [2])

$$\lambda_U = \lim_{u \uparrow 1} P\left\{Y > F_Y^{-1}(u) \mid X > F_X^{-1}(u)\right\} = \lim_{u \uparrow 1} \frac{1 - 2u + C(u,u)}{1-u} \tag{2}$$

provided that the limit $\lambda_U \in [0, 1]$ exists. If $\lambda_U > 0$, X and Y are said to be asymptotically dependent in the upper tail; X and Y are said to be asymptotically independent in the upper tail if $\lambda_U = 0$.

The coefficient λ_L of *lower tail dependence* of (X, Y) is

$$\lambda_L = \lim_{u \downarrow 0} P\left\{Y < F_Y^{-1}(u) \mid X < F_X^{-1}(u)\right\} = \lim_{u \downarrow 0} \frac{C(u,u)}{u} \tag{3}$$

provided that the limit $\lambda_L \in [0, 1]$ exists. If $\lambda_L > 0$, X and Y are said to be asymptotically dependent in the lower tail; X and Y are said to be asymptotically independent in the lower tail if $\lambda_L = 0$.

2.2 Some Classes of Bivariate Copulas

Table 2 presents a summary of basic facts (presented e.g. in [2, 11, 14]) that are related to the families of classes copulas that we utilize in our analyses.

It is well known ([14]) that the Gumbel class is a limiting case of the Joe BB1 class for $a \to 0$, while its special case for $b = 1$ is the Clayton class.

Table 2

Characteristics for some Archimedean copulas

Family of copulas	**Parameters**	**Bivariate copula C(u,v)**	λ_L	λ_U
Gumbel	$b \geq 1$	$\exp\left\{-\left[(-\ln(u))^b + (-\ln(v))^b\right]^{1/b}\right\}$	0	$2 - 2^{1/b}$
Clayton	$a > 0$	$\left(u^{-a} + v^{-a} - 1\right)^{-1/a}$	$2^{-1/a}$	0
Frank	$\theta \in \Re$	$-\frac{1}{\theta}\ln\left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{(e^{-\theta} - 1)}\right)$	0	0
Joe BB1	$b \geq 1$, $a > 0$	$\left\{1 + \left[\left(u^{-a} - 1\right)^b + \left(v^{-a} - 1\right)^b\right]^{1/b}\right\}^{-1/a}$	$2^{-1/ab}$	$2 - 2^{1/b}$

We can observe that the coefficients λ_L and λ_U can attain values in the whole interval (0, 1) for Joe BB1 copulas, while the same holds for λ_L for strict Clayton copulas and for λ_U in case of Gumbel copulas. Both λ_U and λ_L are equal to 0 for Frank copulas, while $\lambda_L = 0$ for Gumbel copulas and $\lambda_U = 0$ for Clayton copulas. It is also well known (see [2]) that $\lambda_L = \lambda_U = 0$ for so-called normal copulas. More detailed analyses (accompanied by graphical illustrations)) related to the tail dependence coefficients can be found in [14]. These coefficients (called also parameters) are there treated as the limit values of the left and right tail concentration functions

$$L(u) = P(V < u \mid U < u) = P(U < u \mid V < u)$$

and

$$R(u) = P(V > u \mid U > u) = P(U > u \mid V > u)$$

with $U = F_X(x)$, $V = F_Y(y)$, (that yields $P(U < u) = P(V < u) = u$).

For the Joe BB1 class, it is shown in [14] that the values of the theoretical Kendall correlation coefficient $\tau = 1 - \frac{2}{b(a+2)}$ determine a growing system (in τ) of decreasing dependencies between λ_L and λ_U (which can attain maximum values of λ_U slightly greater then τ). Consequently, Gumbel copulas have λ_U greater than any Joe BB1 copulas with the same value of τ. Similarly, Clayton copulas have λ_L greater than any Joe BB1 copulas with the same value of τ.

2.3 Convex Combinations of Copulas

A very useful tool for fitting the investigated couplas of time series has been obtained in the classes of convex combinations of copulas $C_{\theta_1}(u,v)$ and $C_{\theta_2}(u,v)$ with the weight coefficients α and $(1-\alpha)$ that have the form

$$C_{\theta_1,\theta_2,\alpha}(u,v) = \alpha C_{\theta_1}(u,v) + (1-\alpha) C_{\theta_2}(u,v)$$

for $\alpha \in [0, 1]$. It is obvious that the relations

$$\lambda_L = \alpha \lambda_{1,L} + (1-\alpha)\lambda_{2,L}, \quad \lambda_U = \alpha \lambda_{1,U} + (1-\alpha)\lambda_{2,U}$$

hold for the coefficients of lower and upper tail dependencies of the considered original and resulting copulas.

2.4 Fitting of Copulas

In practical fitting of the data we utilized the *maximum pseudolikelihood method* (MPLE) of parameter estimation (with initial parameters estimate received by the minimalization of the mean square distance to the empirical copula C_n presented e.g. in [5]). It requires that the copula C_θ (u, v) is absolutely continuous with density $c_\theta(u,v) = \frac{\partial^2}{\partial u\, \partial v} C_\theta(u,v)$. This method (described e.g. in [5]) involves maximizing a rank-based log-likelihood of the form

$$L(\theta) = \sum_{i=1}^{n} \ln\left(c_\theta\left(\frac{R_i}{n+1}; \frac{S_i}{n+1} \right)\right) \tag{4}$$

where n is the sample size and θ is vector of parameters in the model. Note that arguments $\frac{R_i}{n+1}, \frac{S_i}{n+1}$ equal to corresponding values of empirical marginal distributional functions of random variables X and Y.

2.5 Goodness of Fit (GOF) Test

We followed the approach of [13] and [15] for goodness of fit test measuring the size of misspecification in the form of the statistics with assymptotical distribution of the type $\chi^2_{p(p+1)/2}$ where p is the number of the estimated parameters. We use a simplified version of this statistics suggested for practical purposes in [15]. As a compensation for this simplification we only reject the tested models if the corresponding P–value < 0, 01.

To compare goodness of fit of the models from several classes of copulas, we apply the Takeuchi criterion TIC ([6]) that is a robustified version of the famous Akaike criterion.

3 Review of Results

For each of considered periods (4.1.1999 – 31.7.2009, 4.1.1999 – 30.6.2008, 1.7.2008 – 31.7.2009), each couple of considered returns of exchange rates and each class of the considered Archimedean copulas (as well as for all convex combinations of their couples) we perform the following sequence of procedures:

1 least squares initial estimates of the model parameters θ (by minimizing the L_2 distance $d(C_\theta, C_n)$ from the empirical copula),

2 calculation of MPLE estimates of the model parameters θ and *TIC*,

3 goodness of fit tests (rejecting the models with P–value < 0, 01).

4 Finally, we choose among the considered classes of copulas with non-rejected models according to the minimalization of the *TIC* criterion. Subsequently, we calculate lower and upper tail dependencies λ_L and λ_U (using their relations to the model parameters, where we enter the MPLE estimates of those parameters).

3.1 Models for the First Period (4. 1. 1999 – 30. 6. 2008)

a) Archimedean Copulas

Among 4 considered Archimedean copulas only the Gumbel class provided models for all 6 considered couples that had not been subsequently rejected by the GOF test described above. The Clayton class provided such models for the first, fourth, fifth and sixth couples, while the Frank class did it for the last three couples.

The values of the *TIC* criterion were minimized for the Gumbel class models for the first four couples and for the Frank class models for the remaining two couples.

Note that no models in the 2-parametric Joe BB1 class passed the GOF tests.

Tables 3(a) and 4(a) present the MPLE estimates $\hat{\theta}$ of parameters for optimal copulas for all 6 couples of currencies, P-values corresponding to goodness of fit test statistics χ^2 as well as the minimizing values of *TIC* and the respective values of the L_2 distances to empirical copulas (which may be reduced in comparison with local minima found in the original least squares error approximation). Finally we also present the values of the coefficients of tail dependencies λ_L and λ_U. Note that the values of the coefficients $\hat{\theta}$ are close to each other and also the distances $d(C_\theta, C_n)$ from the corresponding empirical copulas are not dramatically different.

b) Convex Combinations of Copulas

The optimal models for all couples of currencies with the corresponding results of model parameters, P-values, *TIC*s, L_2-distances, λ_L and λ_U for optimal models are presented in Tables 3(b) and 4(b).

Table 3

Results for the pairs of returns of exchange rates including SKK/EUR

a) Archimedean copulas class

Couple	**(SKK/EUR, CZK/EUR)**	**(SKK/EUR, PLN/EUR)**	**(SKK/EUR, HUF/EUR)**
Copula's type	**Gumbel**	**Gumbel**	**Gumbel**
θ	1,279	1,248	1,294
P-value	0,470	0,166	0,155
TIC	-500,47	-271,09	-265,26
$d(C_\theta, C_n)$	0,420	0,403	0,347
λ_L	0,000	0,000	0,000
λ_U	0,281	0,258	0,291

b) The optimal convex combinations of Archimedean copulas

Couple	**(SKK/EUR, CZK/EUR)**	**(SKK/EUR, PLN/EUR)**	**(SKK/EUR, HUF/EUR)**
Copula's type	**Frank+Joe BB1**	**Clayton+Gumbel**	**Gumbel+Joe BB1**
α	0,140	0,124	0,864
θ_1	6,369	1,595	1,208
$\theta_2 = (b_2; a_2)$	(1,189; 0,066)	(1,215; x)	(2,210; 0,296)
P-value	0,118	0,123	0,344
TIC	-529,82	-279,76	-288,89
$d(C_\theta, C_n)$	0,262	0,305	0,315
λ_L	0,0004	0,080	0,047
λ_U	0,180	0,202	0,280

Interestingly, models including the Joe BB1 copulas also passed the GOF tests. This enables us to model simultaneously non-zero lower and upper tail dependencies (which is also possible for convex combinations of Gumbel and Clayton classes). Furthermore, we can observe that for most couples the values of λ_U of optimal models are substantially larger than those of λ_L (this first period was dominated by the appreciation of the considered currencies, mainly SKK and CZK). The only exception is the couple (CZK/EUR, HUF/EUR) where the model in the combination of classes Clayton – Frank had a lower value of *TIC* than one in the Gumbel – Joe BB combination (with $\lambda_U > \lambda_L > 0$), which also passed the GOF test.

Table 4

Results for the returns of exchange rates for the remaining couples of exchange rates for the first period 4.1.1999 - 30.6.2008

a) Archimedean copulas class

Couple	**(CZK/EUR, PLN/EUR)**	**(CZK/EUR, HUF/EUR)**	**(HUF/EUR, PLN/EUR)**
Copula's type	**Gumbel**	**Frank**	**Frank**
θ	1,245	1,554	3,117
P-value	0,067	0,033	0,111
TIC	-262,98	-152,03	-500,06
$d(C_\theta, C_n)$	0,545	0,309	0,418
λ_L	0,000	0,000	0,000
λ_U	0,255	0,000	0,502

b) The optimal convex combinations of Archimedean copulas

Couple	**(CZK/EUR, PLN/EUR)**	**(CZK/EUR, HUF/EUR)**	**(HUF/EUR, PLN/EUR)**
Copula's type	**Frank+Gumbel**	**Clayton+Frank**	**Frank+Joe BB1**
α	0,666	0,503	0,658
θ_1	1,568	0,001	3,101
$\theta_2 = (b_2; a_2)$	(1,459; x)	(3,407; x)	(1,363; 0,138)
P-value	0,151	0,100	0,202
TIC	-279,44	-154,88	-587,87
$d(C_\theta, C_n)$	0,269	0,284	0,427
λ_L	0,000	0,000	0,009
λ_U	0,131	0,000	0,115

3.2 Models for the Second Period (1. 7. 2008 - 31. 7. 2009)

The results for the second period are presented in Table 5. For all 3 considered pairs of exchange rates, optimal models in all three 1-parametric Archimedean copulas classes passed the GOF tests. The optimal models in the Joe BB1 class again did not pass the GOF tests for either of the 3 couples of exchange rates. The minimal values for the TIC criterion were attained for the optimal model in the Gumbel class for the first couple and in the Frank class for remaining 2 pairs.

This time we have $\lambda_L > \lambda_U$ for the last 2 pairs for exchange rates and the dominance of λ_U over λ_L is also dramatically reduced for the first pair. This dramatic change (in comparison to the corresponding models for the first period) can be related to the fact that all 3 considered currencies strongly depreciated in the second period.

Note that the value of $d(C_\theta, C_n)$ increased dramatically in comparison to the corresponding value for the first period.

Table 5
Results for the returns of exchange rates for the remaining couples of exchange rates for the second period 1.7.2008 - 31.7.2009

a) Archimedean copulas class

Couple	**(CZK/EUR, PLN/EUR)**	**(CZK/EUR, HUF/EUR)**	**(HUF/EUR, PLN/EUR)**
Copula's type	**Gumbel**	**Frank**	**Frank**
θ	1,716	4,831	5,952
P-value	0,078	0,454	0,159
TIC	-126,07	-130,47	-177,49
$d(C_\theta, C_n)$	2,688	2,177	1,770
λ_L	0,000	0,000	0,000
λ_U	0,502	0,000	0,000

b) The optimal convex combinations of Archimedean copulas

Couple	**(CZK/EUR, PLN/EUR)**	**(CZK/EUR, HUF/EUR)**	**(HUF/EUR, PLN/EUR)**
Copula's type	**Clayton+Gumbel**	**Frank+Joe BB1**	**Frank+Joe BB1**
α	0,450	0,196	0,221
θ_1	0,554	9,052	3,639
$\theta_2 = (b_2; a_2)$	(2,632; x)	(1,367; 0,454)	(1,674; 0,665)
P-value	0,016	0,247	0,043
TIC	-139,34	-140,96	-188,53
$d(C_\theta, C_n)$	1,758	1,551	1,286
λ_L	0,129	0,273	0,417
λ_U	0,384	0,263	0,379

3.3 Models for Whole (Merged) Period

Despite the dramatic differences between respective models for the first and the second period, we also calculated models for 3 pairs of currencies that can be analyzed through the whole merged period (4. 1. 1999 – 31. 7. 2009).

The results of computations for the whole period are presented in the Table 6. We can observe that the resulting optimal models have distances to the empirical copulas that are comparable to those of the corresponding models for the dominating first period.

On the other hand, despite the fact that the second period represents less than 10% of the data, the parameters of tail dependencies for the resulting optimal models among convex combinations of Archimedean copulas for the whole time period moved disproportionally closer to those for the corresponding models for the second period.

Table 6

Results for the returns of exchange rates for the remaining couples of exchange rates for the whole (merged) period

a) Archimedean copulas class

Couple	**(CZK/EUR, PLN/EUR)**	**(CZK/EUR, HUF/EUR)**	**(HUF/EUR, PLN/EUR)**
Copula's type	**Gumbel**	**Gumbel**	**Gumbel**
θ	1,319	1,266	1,483
P-value	0,01	0,06	0,17
TIC	-462,33	-356,65	-807,75
$d(C_\theta, C_n)$	0,374	0,349	0,677
λ_L	0,000	0,000	0,000
λ_U	0,309	0,271	0,402

b) The optimal convex combinations of Archimedean copulas

Couple	**(CZK/EUR, PLN/EUR)**	**(CZK/EUR, HUF/EUR)**	**(HUF/EUR, PLN/EUR)**
Copula's type	**Gumbel+Joe BB1**	**Frank+Joe BB1**	**Frank+Joe BB1**
α	0,295	0,115	0,285
θ_1	1,007	-3,089	2,013
$\theta_2 = (b_2; a_2)$	(1,390;0,236)	(1,251; 0,229)	(1,420; 0,335)
P-value	0,011	0,338	0,207
TIC	-498,06	-399,79	-868,51
$d(C_\theta, C_n)$	0,306	0,280	0,469
λ_L	0,088	0,079	0,166
λ_U	0,251	0,229	0,264

Conclusions

- The character of dependencies between the first and the second period changed dramatically (resembling situations described by the regime switching methodology in the time series theory (which has been presented in detail e.g. in [3]).
- Utilizing models in the form of convex combination of Archimedean copulas helped to improve substantially the quality of fitting of empirical copulas. Also tail dependencies for the second period became more pronounced for these types of models. This is in accordance with the occurrence of frequent simultaneous highly extremal returns (of both orientations) in this period.
- Although the couple (PLN/EUR, HUF/EUR) has the largest values of the Kendalls correlation coefficient, its optimal models reach closest fit only for the second (crisis) period.
- Although the Kendalls correlation coefficients were larger for the second period (for all 3 considered couples), the quality of fit for this period (measured by $d(C_\theta,C_n)$) was much worse than for the first period.

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