Entropy Application for Simulation the Ballast State as a Railway Element

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Abstract: The purpose of the paper is to develop methods of entropy application for simulation of the ballast layer operation of a railway track in the tasks of predicting and controlling the service life. The author developed a method for determining the entropy of the ballast layer as an element of a railway track through calculations of the mechanical work performed by ballast as a result of the reaction to an external load. To determine the array of stresses and deformations operating in the ballast layer space, the spatial model of the stress-strained state of a railway track based was used on the elastodynamic problem. The major part of the developed method is supplemented by the technique of assessing the entropy of a system according to the deviation measuring results in its geometric position. The geometric position of a railway track was measured by a track renewal train. Files archiving to determine the randomness of data recorded in them were carried out using the LZMA algorithm. The tasks of predicting and controlling the service life of ballast have been further developed. The usage of entropy has allowed simulating the ballast degradation as a random process that depends on cyclic stresses and deformations arising in ballast from rolling stock. For the first time, the method of assessing the quality of the track surfacing through the entropy of the ballast layer is proposed. The developed mathematical tool may be used to compare the service life of ballast in various operating conditions, which allows optimizing consistency between the track design and parameters of train movements.

Keywords: railway; ballast; entropy; stress-strained state; track state

1 Introduction

The railway track, like any engineering structure, has a certain operational resource. Forecasting resource exhaustion is a key tool for planning the frequency of repairs, organizing current maintenance, and, in general, establishing reasonable operating conditions that could meet both the technical capabilities of the railway and economic feasibility.

At present, the main criterion for justifying the compliance of the railway structure with the operating conditions and forecasting inter-repair time is the passed tonnage. However, this approach does not take into consideration the structure of the train flow and excludes the possibility of assessing the impact of certain categories of trains. It is clear that there are cases when, with the same general load intensity, the impact of rolling stock on the track, and, accordingly, the timing of the disorder in its elements, would be different. First of all, it concerns main directions with the movement of passenger trains at high speeds, or freight trains with a large axle load, or tracks at industrial enterprises with special-purpose rolling stock.

To take such features into consideration, there is a need to substantiate the appropriate estimation tool.

2 Statement of the Problem

Today, there is a wealth of experience in scientific research on the issues related to diagnosing, estimating, and forecasting the state of the railway track. Thus, works [1-3] provide a fairly detailed overview of such studies with their advantages and disadvantages.

Given that the section of a railway track has uniform design characteristics and established operating conditions at a large length, the occurrence of deviations in its condition can be considered as a random process subordinated to the law of large numbers.

Methods of probability theory in the research of railway operations are increasingly used. It is shown in [4] that the accuracy of the source data has a significant impact on the results of the forecast of the geometric state of the track. Most often, specially equipped track measuring cars are used to measure the geometry of the track. They produce a large amount of information, but both the mechanics of its acquisition and subsequent mathematical processing algorithms lead to a certain uncertainty of the full-on position of the track, which necessitates the application of approaches from probability theory.

Current procedures for predicting failures of the railway track should take into consideration the likelihood of the appearance of a particular event, which makes it possible to reduce operational risks [5].

Taking into consideration various factors, including those of a random nature, makes it possible to devise system methods for optimal management of a transport enterprise. Thus, work [6] proposed an approach to determining the effective management system of a transport enterprise on the example of mining and metallurgical companies based on the theory of mass service.

The presence of changes in the state of the track directly affects its interaction with the rolling stock. This is especially true of difficult operating conditions, for example, for braking processes along sections with significant slopes [7].

Some general approaches to describing deviations in the geometry of the track using the normal law of distribution of random variables are shown in work [8]. Of the elements that make up the upper structure of the railway track, probably more difficult in terms of forecasting the state change over time is a ballast layer [9, 10]. Thus, the rail, with an adequacy enough for most problems, can be described as a homogeneous beam that works at bending. Reinforced concrete sleeper transfers the load from the rail to the ballast almost without compression or bending and has the largest established service life among other elements of the upper structure of the track. Deformation of the ballast layer as a loose material [11, 12], when working under load, is more difficult – due to the elastic movement of individual particles, which leads to their gradual destruction and mixing [13]. That causes changes in elastic properties in some places and, as a result, leads to uneven deflection and the formation of residual strains [14]. Irregularities in the ballast layer are reflected at the position of the rail-sleeper mesh and form indentation in the geometry of the track in general.

Deviations in the vast majority do not appear as a result of stress outside the permissible ones but are the result of fluctuations induced by relatively small but repeated loads. The measure of system order violation due to such random fluctuations can be described through entropy.

Entropy makes it possible to numerically determine the state of the system and predict its "aging". This approach is used in many areas [15-17].

The author of the paper [18] provides a justification for determining the entropy of the railway track as a system in general, by calculating mechanical work. However, the issue of the detailed description to transition to the likelihood of destruction and service life of individual elements, such as the ballast layer, and the question of determining entropy by natural geometric deviations remain unresolved.

To calculate the entropy of the full-on position of the system, it is necessary to determine the probabilities of all possible variations of its condition. With the increase in the size of the system, the complexity of the problem quickly increases, and, for most cases, solving it is impossible even for modern computing devices. In work [19], a creative solution to such a problem was proposed and substantiated. The authors of the cited work proved that the entropy of the system is proportional to the degree to which it is possible to compress a lossless byte sequence describing the predefined system configuration through discrete levels of connections of all degrees of freedom. That proves the uniformity of the approach to modeling the development (aging) of systems through entropy for various industries: from gas thermodynamics to solid-state physics, biology, chemistry, as well as information applications. The procedure proposed in [19] can be used, with a certain adaptation, as one of the elements in the entropy calculations of railway tracks.

3 Methods and Results

The condition of the track can be estimated according to different criteria. For most problems, they primarily include the geometric deviations of an actual position of the track elements relative to the values set by the project. The main ones are the position of the rail threads in the horizontal plane (plan), in the vertical plane (sagging), the width of the track, and the mutual position of rail threads in the vertical plane (distortions). These indicators refer to those that are restored during the relevant repair work and do not require direct replacement of elements of the railway track. The list of indicators reflecting the condition of the track includes another group: the wear of rails, damage to fasteners, destruction of sleepers, ballast layer diseases (clogging, abrasion of gravel, uneven sealing). Some of these deviations can be attributed to partially restorable by executing special operations (for example, grinding rails, cleaning ballast), while the elimination of others is possible only through replacement. All indicators that characterize the state of the track, either directly or indirectly are reduced to a set of linear geometric characteristics at the level of several units or several tens of millimeters. Depending on the operating conditions established in a given section, regulatory documents regulate the permissible values of such deviations. The presence of deviations to certain established limits would determine the serviceable state of the track that is, one that does not require operational restrictions. Further increase in the level of deviations would lead to the transition to a partially working state when a further operation could be possible with certain restrictions (usually the permissible speed of movement). Large values of deviations can lead to an inoperable state when further operation without their elimination is not allowed due to a threat to traffic safety. Carrying out timely scheduled repairs and proper current maintenance of the track typically makes it impossible for an inoperable state to occur, except for sudden failures such as, for example, breaking the rail.

For each of these indicators, the state of the track can be described as a set of values of its geometric position, measured in a certain step $\{\omega_i\}$, where the difference of ω_i from zero would show the level of deviation. Most deviations would be within a certain range. The presence of individual deviations beyond its boundaries is possible but their probability is insignificant $P\left(\omega_i \in [-\varepsilon;\varepsilon]\right) \to 1$, where ε is the permissible level of deviations in accordance with the established speed of movement. Then there should be an average deviation indicator

$$\exists \mu : \sum_{i=1}^{n} \left| \omega_i - \mu \right| = 0 \tag{1}$$

Taking into consideration the design and operation of the railway track $\mu \gg \varepsilon$, based on expression (1) $P\left(\omega_i \approx \mu\right) = \max$, the further process of development of deviations mainly occurs due to an increase in μ and only to some extent due to an increase in ε.

A set of values that reflects the state of the track can be represented in a discrete form

$$\Omega \in \left\{ \delta \left\lfloor \frac{\omega_i}{\delta} \right\rfloor \right\}, \tag{2}$$

where δ is the smallest difference in the geometric position of the track.

Then the indicator of its condition may be the weight of such a set

$$Z = \sum \left\lfloor \frac{\omega_i}{\delta} \right\rfloor . \tag{3}$$

The process of further degradation of the section of the railway track can be described as

$$Z_t = Z_{t-1} + \delta f\left(t, P\right), \tag{4}$$

where $f\left(t, P\right)$ is the function that shows how many discrete deviations can be added, as a result of the next external action, taking into consideration the probability that not all potential residual deformations will be implemented.

The above definitions of gradual degradation of the railway track match the principle of entropy.

In a general form, the entropy of the system is described by the following equation

$$S = k_1 \ln W, \tag{5}$$

where k_1 is the coefficient showing elementary entropy (in the classical approach –a Boltzmann constant); W is the weight of the system state, namely the number of micro conditions (variations of the interaction of its elements), corresponding to the general macrostate of the system. That is $Z \equiv W$, or, in other words, the set Ω with a metric Z is one of the possible implementations of the state of the system with entropy S, and $\Delta Z = f\left(\Delta S\right)$.

Further considerations will apply to the ballast layer as the most complex but also the most indicative element of the railway track as part of this study.

Violations of the state of ballast under the influence of external load can be considered changes in the integrity of this layer as a system that perceives pressure from sleepers and, reacting with elastic strains, transfers it to the soil bed. Such changes occur both directly due to the destruction of gravel and due to the irreversible movement of its particles. An event in which mechanical violations should be observed is an excess of permissible stresses. However, it is known that the destruction of connections between the particles of ballast and even the degradation of gravel has also happened at a much lower level of stress.

As an elementary component of external action, it is appropriate to adopt the travel of one wheel along the track. A one-time application of such a load, and at the level of stresses that do not reach the permissible values, cannot lead to the occurrence of residual deformations. However, having dozens of wheelsets in each train, dozens of trains running per day, and several years of operation, researcher shall obtain the transformation of some minor fluctuations into significant geometric irregularities.

If researcher consider a specific cross-section of the section, taking into consideration its isotropic length along the track and meeting the condition of non-exceeding permissible stresses, the occurrence of an irregularity in this very cross-section is almost zero. With an increased length of the observed section, the frequency of deviations would approach the theoretical probability predetermined by entropy. Thus, there is no exact mapping of the calculated entropy into a set that recognizes the state of the section ($S \to \Omega$), although the number of such mappings is finite and depends on W. But for the set tasks of comparing the operating conditions over a sufficiently long period, it is possible to predict changes in the state of the section (equation (4)) relative to the increase in entropy (ΔS).

A ballast layer can be described as a set of objects; their number would depend on the degree of detail of the model. Each element has several degrees of freedom of potential change in position relative to the project, related to movement, rotation, or 3D strain. Such a system works on the perception of the load transmitted to it by other elements of the railway track from the rolling stock; its entropy then would depend on a change in internal energy, which can be expressed through mechanical work. The increase in entropy (ΔS) is proportional to a change in the energy of the system exerted by external influence (δQ)

$$\Delta S = k_2 \delta Q , \tag{6}$$

where k_2 is the proportionality coefficient (for Boltzmann entropy – the value opposite to the temperature value).

The equivalent of a single energy change cycle to be adopted is the mechanical work of the ballast cross-section (A^*) when one wheel travels over it (t)

$$\delta Q = k_3 \int_t A^*(t)\,dt, \tag{7}$$

where k_3 is the proportionality coefficient; for the elements of the railway track, accepted to equal 0.066 [18].

The mechanical operation of the system is defined as the sum of mechanical works of each element, which describe the ballast layer when modeling its stressed-strained state

$$\begin{aligned} A^* &= \sum A_i; \\ A_i &= f\left(\sigma_i, \varepsilon_i\right). \end{aligned} \tag{8}$$

where σ_i, ε_i are, respectively, the stresses and elastic deformations of the *i*th element of the ballast layer.

Then the state of the system after a certain period of operation over years (T) can be determined as follows

$$\begin{aligned} W &= e^{k_3 A}; \\ A &= 365T \sum_j N_d N_c N_w \sum_i \int_t f\left(\sigma_{ij}, \varepsilon_{ij}, t_j\right) dt \end{aligned} \tag{9}$$

where N_w is the number of wheelsets in a railway car; N_c is the number of cars of the *j*-th type; N_d – the number of trains with such cars per day.

The above approach is convenient to use to compare variants of different train flows. If the base variant has a term of operation (T_0), then, for an alternative, it can be determined through the following ratio of system states

$$T_x = T_0 \frac{W_0}{W_x} \tag{10}$$

or

$$T_x = T_0 e^{k_3\left(A_0 - A_x\right)} \tag{11}$$

To determine the stressed-strained state of the ballast layer, an arbitrary mathematical model can be used, which makes it possible to obtain the necessary array of data $\left\{\sigma_i, \varepsilon_i, t\right\}$ with details sufficient for the problem being solved. For further research, within the framework of this work, a model of the stressed-strained state of the railway track was used, based on the dynamic problem of elasticity theory whose basic provisions are given in works [20, 21]. The feature of this model is the calculation of dynamic stresses and deformations using the propagation of elastic waves through the objects of the railway track. The sites of the application of external load are used to start building a set of vectors in all directions of the semi-space. The results of calculations according to such a mathematical model are the values of stresses and deformations, required for equation (9), for each time step.

Typically, a train flow moving along a given section of the railway track consists of trains that have different set speeds, weight, length, etc. This is especially inherent in mainline tracks. Moreover, this difference is not limited to simple separation into passenger and freight traffic. Both freight trains can have different purposes, different cargo, and, accordingly, different loads on the axle and other characteristics, and passenger trains can be divided into intercity, high-speed, and others. It is known that the introduction of trains in the train flow, which differ from others by the increased axle load or speed, increases the intensity of accumulation of residual deformations of the tracks, despite the fact that the stresses from their wheels in the elements of the track do not exceed the permissible values, and the total volume of transportation (cargo intensity) does not change significantly. Our method makes it possible to take into consideration such differences to some extent.

As an illustration, let us consider a problem on comparing the operational time of a ballast layer for different variants of the train flow using a numerical example.

Author accept that the train flow consists of freight cars (with an axle load of 18 t/axle, 42 cars per train) and passenger trains (15 cars per train), moving at speeds of 80 km/h and 120 km/h, respectively. Option 1: 25 freight and 8 passenger trains per day. Option 2: 16 freight and 40 passenger trains per day. The initial data are simplified and selected in such a way that both options could produce a load capacity of 30 million tons gross per km per year, which implies the same inter-repair terms.

The following structure of a railway track is adopted: rails UIC60; reinforced concrete sleepers; gravel ballast with a thickness of 0.5 m with a deformation module of 200 MPa; soil bed with a deformation module of 35 MPa. With the specified composition, the general module of deformation of the under-rail base obtained was at the level of 52 MPa.

The results of calculations according to the proposed method for both options, such as the values of stresses and mechanical work in the elements of the ballast layer, are shown in Figs. 1-4. The stress plots demonstrate the maximum values – when the wheel is passing directly above the estimated cross-section. The plots of mechanical work show the total values when a wheel travels over the estimated cross-section from the beginning of the track deflection, when the wheel is at a certain distance, to the end of the action when the wheel has already traveled further by a symmetrical distance.

The calculation results corresponding to the values of stresses in cross-section (Figs. 1 and 2) and mechanical work (Figs. 3 and 4) are given on the same scale for the possibility of comparing the effect exerted by a freight car and passenger car both in terms of values and quality. The difference is observed not only in the absolute values of stresses in the elements of the track (in this case, in the ballast layer) but also in the propagation of stresses in different zones.

According to the calculations according to equation (9), the total annual mechanical work of the ballast layer cross-section would equal 20.78 MN·m and 16.20 MN·m for the first and second options, respectively. Then, equation (11) can be represented as a ratio $T_2 = 1.35T_1$. Thus, for the considered initial data for option 2 of the train flow (dominated by passenger traffic), the accumulation of residual deformations in the ballast would occur 35% slower.

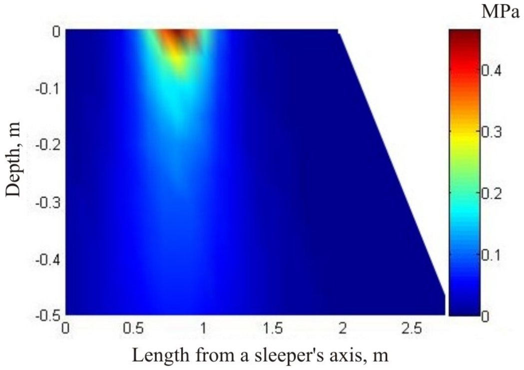

Figure 1

Propagation of maximum vertical stresses in the ballast induced by a freight car wheel in cross section (18 t/s, 80 km/h)

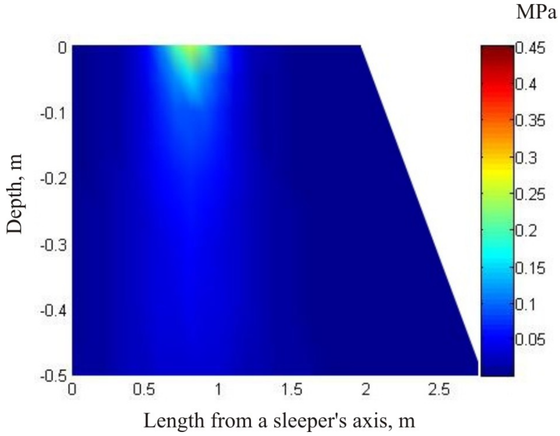

Figure 2

Propagation of maximum vertical stresses in the ballast induced by a passenger car wheel in cross section (120 km/h)

Entropy can be used not only for forecasting tasks but also to assess the current state of the system or the quality of repair work. However, the problem of calculating the entropy of a complex system based on its condition has no direct unambiguous solution.

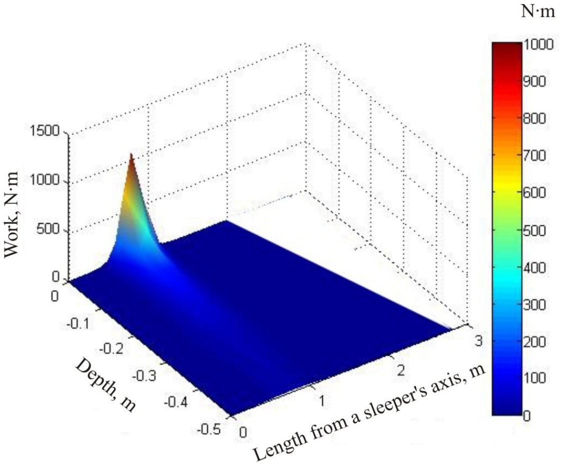

Figure 3
Distribution of mechanical work in the ballast cross-section space induced by a freight car wheel

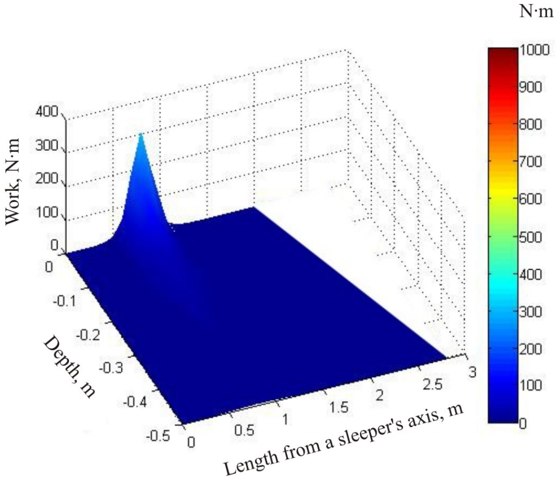

Figure 4
Distribution of mechanical work in the ballast cross-section space induced by a passenger car wheel

A hypothesis is proposed in [19] that such a problem can be solved by switching to information entropy. To this end, one determines the number of degrees of freedom of the system (D), as minimally sufficient, given the number of elements of which it consists, their possible fluctuations, and ties. Each degree of freedom is described by the required number of discrete states (n_s). A set of states of all degrees of freedom is converted to a chain of numbers (bytes). A file that is a sequence of such bytes is archived without data loss. According to the authors of work [19], the degree of archiving does reflect the entropy of the system, given that the size of the archive depends on the chaoticity of the data

$$S = \frac{CFS - ZFS}{RFS - ZFS} K_1 D \operatorname{Log} n_s , \tag{12}$$

where CFS is the size of the archive file describing the state of the system; ZFS is the archive of the file, in which all degrees of freedom are in zero position (the system is in a state of minimal entropy); RFS is the archive of the file with "white noise" – all degrees of freedom have a random value from the possible ones (the system is in a state of maximum entropy).

This approach does not contradict the understanding of ballast entropy, given that the deterioration of its condition should be characterized precisely by the unevenness of deviations of adjacent elements while uniform compaction (loosening) is not reflected in the irregularity of the track.

Irregularities in the ballast layer can be tracked for deviations in the vertical position of the rail. There are machine-based means for measuring this value along the section – by a track measuring car or a straightening-tamping machine. Paper [22] provides an example of such an entry before and after the track correction. Based on these examples, Fig. 5 shows ballast deviations in a vertical plane along a 500-m long section. The array has 800 points, the distance between points is 0.625 m.

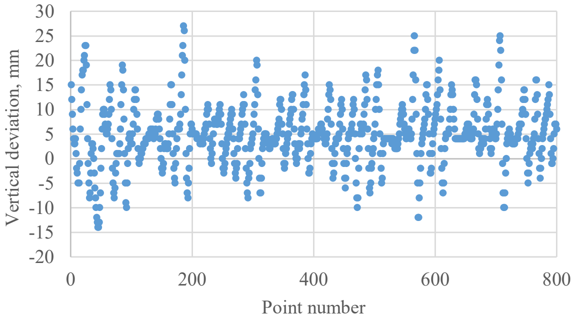

Figure 5
Registering track deviations in a vertical plane

The data given in Fig. 5, consisting of a sequence of deviations in the position of the track, indirectly reflect the position of the rail in the vertical plane. For the correct perception of the picture, it should be noted that the horizontal and vertical scales vary greatly – variations in the range from -15 mm to +30 mm (vertical axis) occur along a 500-m long section (horizontal axis). That is, the considered section has certain deviations but, in general, its profile is quite even.

To create files according to equation (12), the position of each point was written in one byte with a sampling level of 1 mm through the conversion $[-127...128] \rightarrow [0...255]$. The LZMA algorithm was used for archiving. Given that ballast is considered as a system that obeys the law of large numbers and has approximately the same state in its length, the sample size should ensure that the ratio of data archiving is approximated to a constant value, Fig. 6.

If researcher talk about the correction and not about repairs with a complete replacement of the upper structure of the track, then its execution does not imply the return of the outline of the section to the design position. This is due to the inability to significantly lower the level of the track, which requires cutting ballast, and other technical limitations of track machines. Thus, the correction work requires preliminary calculations, which are the solution to the optimization problem of finding a compromise between the quality of the track outline and the magnitude of its displacement, with restrictions on ballast volumes, machine capabilities, and others. Such an optimization problem does not have an unambiguous solution, and, therefore, requires the existence of a method for assessing the position of the track both at the stage of calculations and immediately after work.

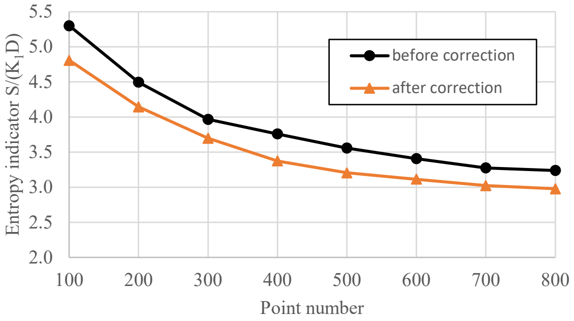

Figure 6

Entropy indicator dependence on a sample size

Figure 6 shows, in addition to entropy calculations for the data reflecting the actual state of the section before the correction (Fig. 5), the corresponding results for the state of the section after correction.

The method of correction and the numerical data on its application are given in [22]. Regarding the example under consideration (Fig. 6), it can be concluded that the entropy of the ballast is reduced after correction. According to equation (11), its lifetime has been restored by $e^{(3.238-2.977)}T_0 = 1.298T_0$, that is, by 30% of the base operating life.

Conclusions

This paper reports the devised method for determining the entropy of the ballast layer as an element of the railway track using the calculations of mechanical work performed by the ballast as a result of a reaction to an external load. The main part of the devised method is supplemented with the procedure for estimating the entropy of the system according to the results of measuring deviations in its geometric line.

Thus, the tasks of forecasting and managing the lifetime of the ballast have been advanced. The use of entropy has made it possible to simulate ballast degradation as a random process, depending on cyclic stresses and deformations that arise in the ballast due to rolling stock. A procedure for evaluating the quality of track correction through the entropy of the ballast layer has been proposed for the first time.

The built mathematical toolset can be used to compare the resource of ballast operation under different operating conditions, which makes it possible to optimize the correspondence between the track design and train traffic indicators. This is of particular importance under difficult operating conditions: high-speed passenger traffic, excessive weight of freight trains, specialized trains of industrial enterprises. This paper gives a numerical example of comparing the resource of ballast work for two variants of the train flow, which are identical in cargo intensity but significantly differ in the ratio of passenger and freight traffic.

As part of the initial data of the considered example, it was concluded that when passenger traffic dominates, the accumulation of residual deformations occurs 35% slower.

A numerical example of the use of entropy to assess the quality of track correction is also provided. Along the considered section, the correction operation was carried out by the VPR-02 machine according to the procedure proposed in [22]. The assessment showed that after the correction, the entropy of the ballast decreased, which increased the resource of its operation by 30%.

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