

# Asymmetrical, Three-phase Power System Model: Design and Application

**Karel Máslo<sup>1,2</sup>, Jan Koudelka<sup>2</sup>, Branislav Bátorá<sup>2</sup>,  
Václav Vyčítal<sup>2</sup>**

<sup>1</sup>ČEPS, a.s., Transmission System Analysis Department, Elektrárenská 774/2,  
101 52 Prague, Czech Republic, maslo@ceps.cz

<sup>2</sup>Brno University of Technology, Department of Electrical Power Engineering,  
Faculty of Electrical Engineering and Communication, Technická 3082/12, 616 00  
Brno, Czech Republic, maslo@vut.cz, xkoude20@vut.cz, batora@vut.cz,  
vycital@vut.cz

---

*Abstract: The paper deals with enhancing the three-phase network model, for simulation of both symmetrical and asymmetrical networks, under both normal and faulted operation. The modelling approach, which was implemented into the MODES simulation tool, is described in a great detail. Demonstration of the application of the MODES simulation tool, newly extended by the three-phase model, in the field of simultaneous faults analysis, distribution system protection and voltage unbalance evaluation, is done, using simulations on simple test systems. Simulations are complemented by discussion of achieved results and validation is accomplished by using other simulation tools (PSCAD/EMTDC, OpenDSS). Finally, conclusions and future plans for the model extension and its possible applications are presented.*

*Keywords: power system unbalance; network asymmetry; three-phase model; dynamic simulation*

---

## 1 Introduction

For conventional studies, analyses and simulations, transmission power systems are modelled as symmetrical, which is a significant simplification. Symmetrical three-phase power system can be analyzed as a single-phase equivalent, which makes computations and analyses (e.g., load flow studies) significantly faster. The assumption of symmetry in case of transmission systems is reasonable as, generally, transmission lines are transposed and loads, i.e., transformation to distribution system, are symmetrical. In addition, symmetrical fault of three-phase short circuit is frequently analyzed (e.g., for equipment dimensioning or stability assessment). However, distribution system analysis is a different case. There are

sources of significant asymmetry, such as loads not being distributed over phases evenly and lines not being entirely transposed, sometimes even not transposed at all (at medium voltage (MV) and low voltage (LV)) levels, and, last but not least, AC traction feeders. In case of distribution system, unbalanced faults are of importance. In practice, these faults occur more often than symmetrical ones, and they are also used for dimensioning purposes, which is e.g., the case of earth fault in case of impedance-grounded systems. Simultaneous unbalanced faults, such as double earth fault or an earth fault combined with a phase interruption, are simulation challenging, as they require appropriate simulation model and simulation can be time consuming. Even though these faults are uncommon, they practically occur in the distribution system, thus, there is a demand for their analysis.

Distribution system analysis includes a vast number of problems closely related to the grid asymmetry, which must be considered in grid modelling. Usage of the single-phase equivalent in this case is often impractical, sometimes even impossible, comparing to the transmission system analysis. The endeavor to build and use a three-phase model is though evident. However, it is not a trivial issue.

## 1.1 Motivation

MODES<sup>1</sup> simulation tool is a multi-purpose tool for power system dynamics analysis using RMS values (phasors). The simulation engine is used on a daily basis by the Czech Transmission System Operator, not only for load flow calculations and analyses, but also in the dispatch training simulator [1]. Various applications of MODES have been demonstrated in the past, e.g., in the field of stability analysis or short circuit calculations [2]. The simulation engine used to work with symmetrical components only, which allowed to speed up the calculation. However, current development of power systems and planned extension of the power system model by distribution systems imply the need for extending the existing simulation engine to be able to solve problems related to network asymmetry, which means mainly building the new models of grid elements. It is planned to use MODES for simulation of wide area distribution systems, more exactly for co-simulation of Czech transmission and distribution systems.

For such a big system, other solutions allowing analysis of unbalanced power system operation and fault analysis, such as PSCAD/EMTDC or OpenDSS, which were used for validation also in this paper, are a bit inconvenient. The disadvantage is the computation time and robustness – key aspects the newly designed model in MODES is aiming to improve.

Last but not least, MODES is used in Czech universities for teaching purposes. The possibility of simulation of distribution system can enrich the classes and bring new teaching methods, leading into the students are more familiar with the issue of power system dynamics. This issue is of importance in practice.

---

<sup>1</sup> More information about MODES is available from <http://www.modesinfo.com>.

## 1.2 Aims and Paper Structure

The main aim of the paper is to describe the design of the three-phase power system model, which has been implemented to the MODES simulation tool, and its application. The issue is split into three following chapters.

The second chapter gives the description of mathematical models, used for particular grid elements. These models are necessary for analysis of the three-phase power system. Their detailed description makes them replicable for other developers aiming to build the asymmetric power system models.

The third chapter shows three case studies (simulation cases), which were done in order to validate the built model after its implementation to MODES. The first case is the analysis of simultaneous faults; the second case is the operation of the impedance-grounded distribution network, and the last case is the evaluation of voltage unbalance caused by the AC traction feeders. Each case contains test system and simulation scenario description. The chapter shows the validation of results done using other simulation tools or analytical calculations, demonstrating that results from the newly designed model, implemented to MODES, are credible.

The last section concludes the paper and outlines the possible future application of the newly designed three-phase model, which is not only limited to the cases presented in this paper.

## 2 Three-Phase Models of Grid Elements

Symmetrical three-phase power system is easy to analyze, because such a system can be treated as a single-phase equivalent, which is, unfortunately, not applicable in case of an unsymmetrical power system. In general, two basic approaches can be used for power system modelling: analyze the system using phase values (A, B, C) or use symmetrical components (0, 1, 2). Symmetrical components and transition between these approaches are well described in the literature, e.g., in [3]. For application in MODES, the existing symmetrical network model has been extended by the asymmetrical models of basic elements, such as power source, power line, transformer, loads and fault model, which are described in this chapter.

### 2.1 Power Lines and Switches

In general, a three-phase power line with 3 phase conductors (A, B, C), neutral N, alternatively with ground wires, can be described with self-impedances  $\underline{Z}_{AA}$ ,  $\underline{Z}_{BB}$ ,  $\underline{Z}_{CC}$ , mutual impedances  $\underline{Z}_{AB}$ ,  $\underline{Z}_{AC}$ ,  $\underline{Z}_{BC}$ , self-capacitances  $C_{AA}$ ,  $C_{BB}$ ,  $C_{CC}$ , mutual capacitances  $C_{AB}$ ,  $C_{AC}$ ,  $C_{BC}$ , and shunt conductance  $G$ , representing leakage and corona losses. In case of the power line of multiple wires, e.g., low voltage (LV) line – A, B, C, N, the technique of Kron reduction [4] is applied to obtain a 3-wires

equivalent. The nodal admittance matrix (1) gives the relation between current injections to the sending (S) and receiving (R) bus  $\underline{\mathbf{I}}_S$ ,  $\underline{\mathbf{I}}_R$  and nodal voltages  $\underline{\mathbf{V}}_S$ ,  $\underline{\mathbf{V}}_R$ .

Switches are modelled also as power lines, with mutual impedance and mutual capacity equal to zero. Self-impedance has negligible value, e.g.,  $j0.1 \Omega$ .

$$\begin{bmatrix} \underline{\mathbf{I}}_S \\ \underline{\mathbf{I}}_R \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{Z}}^{-1} + \frac{1}{2}\underline{\mathbf{Y}} & -\underline{\mathbf{Z}}^{-1} \\ -\underline{\mathbf{Z}}^{-1} & \underline{\mathbf{Z}}^{-1} + \frac{1}{2}\underline{\mathbf{Y}} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{V}}_S \\ \underline{\mathbf{V}}_R \end{bmatrix} \quad \underline{\mathbf{Y}} = \mathbf{G} + j\omega\mathbf{C}$$

$$\underline{\mathbf{Z}} = \begin{bmatrix} \underline{Z}_{AA} & \underline{Z}_{AB} & \underline{Z}_{AC} \\ \underline{Z}_{AB} & \underline{Z}_{BB} & \underline{Z}_{BC} \\ \underline{Z}_{AC} & \underline{Z}_{BC} & \underline{Z}_{CC} \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} C_{AA} & -C_{AB} & -C_{AC} \\ -C_{AB} & C_{BB} & -C_{BC} \\ -C_{AC} & -C_{BC} & C_{CC} \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} G & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & G \end{bmatrix} \quad (1)$$

## 2.2 Transformers

Two conventional methods exist for modelling of a conventional, two-winding transformer: using phase values and transformer connection, or transition from symmetrical components values to phase values. Three-winding transformer can be modelled by combination of models of two-winding transformer.

### 2.2.1 Phase Values Model of Two-Winding Transformer

Phase values model of a transformer is built in three steps [5]: definition of the transformer primitive admittance matrix, calculation of the windings admittance matrix, and, finally, definition of the transformer admittance matrix using the previous two matrices and taking the vector group into consideration. This approach is described in a detail in [5] with all necessary equations.

### 2.2.2 Symmetrical Components Model of Two-Winding Transformer

The second approach is the symmetrical components model. We assume the transformer with wye-delta connection with solidly grounded neutral point. Considering the transformer admittance matrix in symmetrical components (2) to be known; P stands for primary side and S for secondary side.

$$\underline{\mathbf{Y}} \begin{bmatrix} \underline{\mathbf{I}}_{-P012} \\ \underline{\mathbf{I}}_{-S012} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{Y}}_{-PP012} & \underline{\mathbf{Y}}_{-PS012} \\ \underline{\mathbf{Y}}_{-PS012} & \underline{\mathbf{Y}}_{-SS012} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{V}}_{-P012} \\ \underline{\mathbf{V}}_{-S012} \end{bmatrix}$$

$$\underline{\mathbf{I}}_{P012} = \begin{bmatrix} \underline{\mathbf{I}}_{P0} \\ \underline{\mathbf{I}}_{P1} \\ \underline{\mathbf{I}}_{P2} \end{bmatrix} \quad \underline{\mathbf{I}}_{S012} = \begin{bmatrix} \underline{\mathbf{I}}_{S0} \\ \underline{\mathbf{I}}_{S1} \\ \underline{\mathbf{I}}_{S2} \end{bmatrix} \quad \underline{\mathbf{V}}_{P012} = \begin{bmatrix} \underline{\mathbf{V}}_{P0} \\ \underline{\mathbf{V}}_{P1} \\ \underline{\mathbf{V}}_{P2} \end{bmatrix} \quad \underline{\mathbf{V}}_{S012} = \begin{bmatrix} \underline{\mathbf{V}}_{S0} \\ \underline{\mathbf{V}}_{S1} \\ \underline{\mathbf{V}}_{S2} \end{bmatrix} \quad (2)$$

For wye-delta connection with a solidly grounded neutral point, matrices of sequence admittances have form of (3), where Y is a leakage susceptance, evaluated from the short-circuit test, and p is transformer ratio.

$$\underline{Y}_{PP012} = \frac{Y}{p^2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{Y}_{PS012} = -\frac{Y}{p} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{Y}_{SS012} = Y \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Matrices (3) reflect the fact that delta winding connection does not allow zero sequence currents to flow. After the transition to phase values, admittance matrix can be expressed as (4).

$$\underline{Y} = Y \begin{bmatrix} \underline{Y}_{PP} & \underline{Y}_{PS} \\ \underline{Y}_{PS} & \underline{Y}_{SS} \end{bmatrix}$$

$$\underline{Y}_{PP} = \frac{1}{p^2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{Y}_{PS} = -\frac{1}{3p} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & 0 & 2 \end{bmatrix} \quad \underline{Y}_{SS} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & 0 & 2 \end{bmatrix} \quad (4)$$

The matrix (4) is advantageous because it does not cause phase displacement of voltage and current phasors, caused by vector group. For instance, differential protection can be easily modelled without the need of phase displacement compensation.

### 2.2.3 Three-Winding Transformer

Typical connection of a three-winding transformer is wye-wye-delta, where primary winding (wye) neutral point is solidly grounded, secondary winding (wye) neutral point is impedance grounded through the impedance  $Z_N$ , and tertiary winding (delta) is for compensation.

Primitive admittance matrix for three-winding transformer has a size of  $10 \times 10$ . It is more convenient though to use the symmetrical components method and replace the three-winding transformer by three two-winding transformers, as shown in Figure 1.

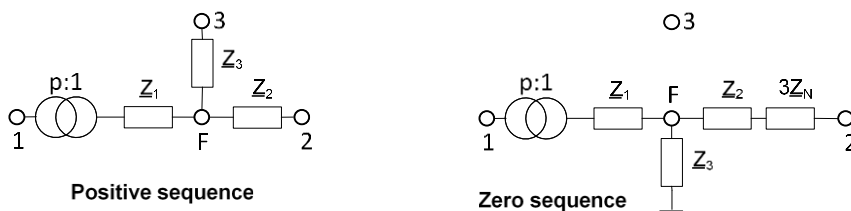


Figure 1

Equivalent diagram of three-winding transformer in symmetrical component representation

This model requires the addition of a fictive bus “F”, but previously developed models of two-winding transformers can be used. Zero sequence impedance diagram, shown in Figure 1, was taken from [6]. For simplified calculations, positive sequence impedances are equal to zero sequence ones. Tap changer is modelled on the primary side (first winding). It is assumed that transformer leakage reactance value varies with a square of transformer ratio, then tap changing does not change its value.

Values of impedances in branches  $\underline{Z}_1$ ,  $\underline{Z}_2$ ,  $\underline{Z}_3$  can be calculated using (5) from measured values from short-circuit test  $\underline{Z}_{12}$ ,  $\underline{Z}_{13}$ ,  $\underline{Z}_{23}$ , which usually have basis of the rated winding voltage and the largest apparent power of the transformer windings (usually the one with wye connection). Grounding impedance  $\underline{Z}_N$  has the basis of the winding to which neutral point the grounding impedance is connected.

$$\underline{Z}_1 = \frac{\underline{Z}_{12} + \underline{Z}_{13} - \underline{Z}_{23}}{2} \quad \underline{Z}_2 = \frac{\underline{Z}_{12} + \underline{Z}_{23} - \underline{Z}_{13}}{2} \quad \underline{Z}_3 = \frac{\underline{Z}_{13} + \underline{Z}_{23} - \underline{Z}_{12}}{2} \quad (5)$$

Following matrices  $\underline{\mathbf{Y}}_1$  and  $\underline{\mathbf{Y}}_2$  (6) can be used for the first and second winding (wye);  $\mathbf{1}$  is  $3 \times 3$  identity matrix. The third winding (delta) can be modelled by (4).

$$\underline{\mathbf{Y}}_1 = \frac{1}{\underline{Z}_1} \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \frac{1}{p} & \mathbf{1} \\ \frac{1}{p} & \mathbf{1} \end{bmatrix} \quad \underline{\mathbf{Y}}_2 = \frac{1}{\underline{Z}_2} \begin{bmatrix} \underline{\mathbf{Y}}' & -\underline{\mathbf{Y}}' \\ -\underline{\mathbf{Y}}' & \underline{\mathbf{Y}}' \end{bmatrix} \quad \underline{\mathbf{Y}}' = \begin{bmatrix} 1 - \underline{K} & -\underline{K} & -\underline{K} \\ -\underline{K} & 1 - \underline{K} & -\underline{K} \\ -\underline{K} & -\underline{K} & 1 - \underline{K} \end{bmatrix}$$

$$\underline{K} = \frac{1}{3 + \frac{\underline{Z}_2}{\underline{Z}_N}} \quad (6)$$

## 2.3 Generator

Generator can be modelled as an induced voltage  $\underline{\mathbf{E}}$  behind the impedance – reactance  $X$ . Generator is considered to be symmetrical, thus voltage phasors  $\underline{\mathbf{E}}_{ABC}$  are balanced and simply expressed as (7). For simplified calculations, induced voltage  $\underline{\mathbf{E}}$  is considered to have constant value.

$$\underline{\mathbf{E}}_{ABC} = \begin{bmatrix} 1 \\ \underline{a} \\ \underline{a}^2 \end{bmatrix} \underline{\mathbf{E}} \quad (7)$$

## 2.4 Loads

The simplest model of a complex load (given by power  $\underline{S} = P + jQ$ ) in node with voltage  $\underline{\mathbf{V}}$  has a form of constant admittances in ungrounded wye connection. After the elimination of voltage of wye connection, the following admittance matrix (8) is obtained.

$$\begin{bmatrix} \underline{Y}_A \left( 1 - \frac{\underline{Y}_A}{\underline{\Sigma Y}} \right) & -\frac{\underline{Y}_A \underline{Y}_B}{\underline{\Sigma Y}} & -\frac{\underline{Y}_A \underline{Y}_C}{\underline{\Sigma Y}} \\ -\frac{\underline{Y}_A \underline{Y}_B}{\underline{\Sigma Y}} & \underline{Y}_B \left( 1 - \frac{\underline{Y}_B}{\underline{\Sigma Y}} \right) & -\frac{\underline{Y}_B \underline{Y}_C}{\underline{\Sigma Y}} \\ -\frac{\underline{Y}_A \underline{Y}_C}{\underline{\Sigma Y}} & -\frac{\underline{Y}_B \underline{Y}_C}{\underline{\Sigma Y}} & \underline{Y}_C \left( 1 - \frac{\underline{Y}_C}{\underline{\Sigma Y}} \right) \end{bmatrix}$$

$$\underline{\Sigma Y} = \underline{Y}_A + \underline{Y}_B + \underline{Y}_C \quad \underline{Y}_i = \frac{\underline{S}_i^*}{U_i^2} \quad i = A, B, C \quad (8)$$

Also, compensation devices, such as capacitors and coils, can be modelled this way.

A special case of the load is the supply of the AC railway system from a V-connection (open delta) transformer. An example of a 25 kV system supplied from a 110 kV three-phase network is shown in Figure 2. These are actually two single-phase transformers T1 and T2, which are connected to the line-to-line voltage on the primary side (in an open delta – in V). Each of them feeds one section of the traction line from the secondary windings. For reasons of the phase changing, the individual sections are separated by a neutral section. When the train is crossing the neutral section, the power supply to the train is interrupted.

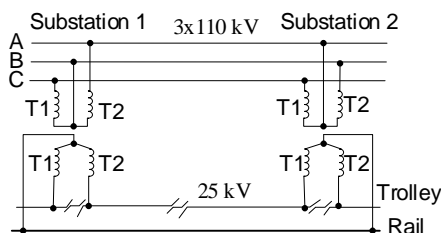


Figure 2

Diagram of power supply of the traction line from transformers connected to V

According to [6], the admittance matrix of such a power supply can be derived in the form of (9), where the left matrix is valid for Substation 1, and the right matrix is valid for Substation 2.  $N_1$  and  $N_2$  are the numbers of turns of the primary and secondary windings and  $Y_L$  is a load admittance connected to the secondary side of transformers T1 and T2.

$$\begin{bmatrix} \underline{Y}_2 & -\underline{Y}_2 & 0 \\ -\underline{Y}_2 & \underline{Y}_1 + \underline{Y}_2 & -\underline{Y}_1 \\ 0 & -\underline{Y}_1 & \underline{Y}_1 \end{bmatrix} \begin{bmatrix} \underline{Y}_1 + \underline{Y}_2 & -\underline{Y}_2 & -\underline{Y}_1 \\ -\underline{Y}_2 & \underline{Y}_2 & 0 \\ -\underline{Y}_1 & 0 & \underline{Y}_1 \end{bmatrix} \underline{Y}_i = \beta^2 \underline{Y}_{Li} \quad \beta = \frac{N_2}{N_1} \quad i = 1, 2 \quad (9)$$

If we neglect the leakage reactance of the transformers and the resistances of the supplying lines, and replace the locomotive power consumption with only the active power  $P$ , the load admittance can be simply calculated according to (10).

$$\begin{cases} \underline{Y}_{L1} = \frac{1}{R - jkX} & \underline{Y}_{L2} = 0 & \text{for } -1 \leq k < 0 & R = \frac{U^2}{P} \\ \underline{Y}_{L1} = 0 & \underline{Y}_{L2} = \frac{1}{R + jkX} & \text{for } 0 < k \leq 1 & \end{cases} \quad (10)$$

In (10),  $U$  is the voltage of the trolley (a nominal value can be considered for simplicity),  $X$  is the equivalent reactance of the entire section of the supplying lines, and  $k$  is a factor indicating the position of the train ( $k = -1$  if the train is at the end of the section fed by transformer T1 and  $k = 1$  if the train is at the end of the section fed by transformer T2). When the train outside the section is powered by transformers T1 and T2 and when the locomotive pantograph crosses the neutral section,  $k = 0$  and both admittances are zero (the power supply is interrupted).

## 2.5 Faults – Short Circuit and Phase Interruption

In phase values, both short circuit and phase interruption can be easily simulated by modification of admittance matrix (1). Short circuit or earth fault means the addition of an infinite value to the corresponding bus (assuming the fault occurred close to the bus), thus the corresponding element of admittance matrix has an infinite value. Phase interruption means the self-admittance of the faulted phase in admittance matrix is zero.

## 3 Application of the Model: Case Studies

This chapter demonstrates the use of the model for analysis of power system dynamics in three selected cases: simultaneous faults analysis, operation of arc suppression coil automatics and voltage unbalance calculations. Case studies were conducted on test networks to show the functionality of the model, but they are applicable to real power systems.

### 3.1 Analysis of Simultaneous Faults

Single unbalance fault (e.g., short circuit or phase interruption, which are the most common faults in distribution networks) can be analyzed using symmetrical components method, as described e.g., in [3]. However, situation is getting complicated when analyzing simultaneous faults. These faults can really occur in the power system – often, they happen consequently (especially due to overvoltage occurrence caused by the first fault). In case of simultaneous faults, especially if different phases are affected, it is recommended to use phase values instead of symmetrical components, which the newly implemented simulation model allows. MODES simulation tool is thus, suitable for simulation of such faults. A demonstration is completed in this section.

#### 3.1.1 Test System Description

A simple test system used for simultaneous faults analysis was taken from [8] and extended by a switch for simulation of a phase interruption. Diagram of the system is shown in Figure 3, together with parameters (reference is 116 kV, 100 MVA). The network was modelled as symmetrical; parameters are given as components values though. Matrices of phase values are created automatically by MODES (this is applicable only for symmetrical systems).



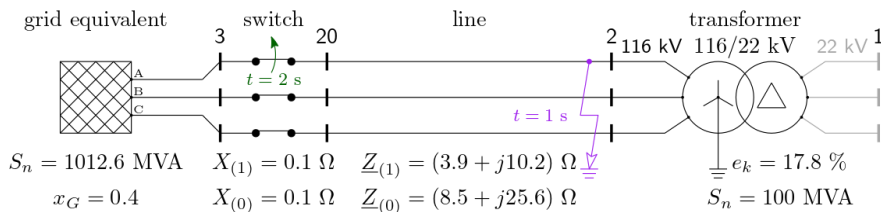


Figure 3

Diagram of the test system for analysis of simultaneous faults

### 3.1.2 Simulation of Simultaneous Earth Fault and Phase Interruption

Simulation scenario is also indicated in Figure 3: single phase A to ground fault (short circuit) near bus 2 is applied in 1 s, and interruption of phase A near bus 3 occurs in 2 s. It means that the short circuit is fed from two sides from 1 to 2 s, and from one side afterwards. Analyzed values were phase A currents on both sides of the line, which together give the fault current. Results are shown in Figure 4; solid lines are from MODES; dashed ones were obtained using PSCAD/EMTDC tool (it was used for simulation validation).

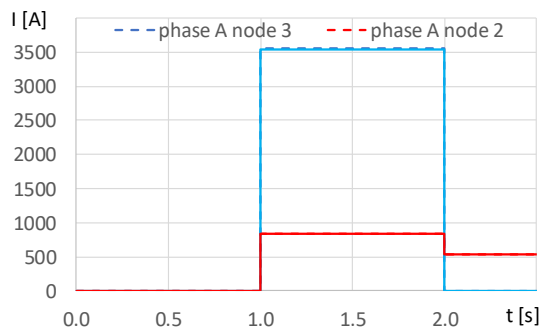


Figure 4

Phase A currents for single line to ground fault and phase A interruption; solid lines – MODES, dashed lines – PSCAD/EMTDC

The results of MODES and PSCAD/EMTDC are matching. The short-circuit current contributions from node 3 (blue waveforms) differs slightly, and the contribution from node 2 (red waveforms) matches completely. The fault current is initially given by a sum of currents from both sides, i.e., about 4400 A, and decays to approx. 500 A after the interruption. Simulation results match the fault currents computed in [8] as 3836 A and 478 A.

For better validation of results, fault current of a single phase to ground fault was computed analytically using symmetrical components. Analytical computation is demonstrated in Figure 5 (reactance of the switch was neglected). Analytically calculated short circuit current is equal to 4393 A. To conclude, the newly built

asymmetric model provides credible results in case of simulation of simultaneous faults, as the results were validated against another simulation tool and also analytical calculation.

$$Z_{\text{base}} = \frac{V_{\text{base}}^2}{S_{\text{base}}} = \frac{(116 \text{ kV})^2}{100 \text{ MVA}} = 134.6 \ \Omega$$

positive sequence

$e = 1$

grid

line

$jx_{G(1)} = jx_G \cdot \frac{S_{\text{base}}}{S_n} = j0.4 \cdot \frac{100}{1012.6} = j0.0395$

$\underline{z}_{L(1)} = \underline{Z}_{(1)}/Z_{\text{base}} = \frac{3.9+j10.2}{134.6} = 0.0288 + j0.0758$

$\underline{z}_{(1)} = jx_{G(1)} + \underline{z}_{L(1)} = 0.0288 + j0.1153$

zero sequence

grid

line

transformer

$jx_{G(0)} = jx_{G(1)} = j0.0395$

$\underline{z}_{L(0)} = \underline{Z}_{(0)}/Z_{\text{base}} = \frac{8.5+j25.6}{134.6} = 0.0632 + j0.1902$

$jx_{T(0)} = j \frac{e_k}{100} = j0.178$

$\underline{z}_{(0)} = (jx_{G(0)} + \underline{z}_{L(0)}) || jx_{T(0)} = 0.0188 + j0.102$

$$I_{k1} = \frac{3e}{|2\underline{z}_{(1)} + \underline{z}_{(0)}|} \cdot \frac{S_{\text{base}}}{\sqrt{3}V_{\text{base}}} = \frac{3 \cdot 1}{|2 \cdot (0.0288 + j0.1153) + 0.0188 + j0.102|} \cdot \frac{100 \cdot 10^6}{\sqrt{3} \cdot 116 \cdot 10^3} \text{ A} = 4393 \text{ A}$$

Figure 5

Calculation of the single line to ground fault in the test system using symmetrical components method

### 3.1.3 Summary

The first case study showed the application of the created three-phase model for analysis of simultaneous earth fault and phase interruption, the most common faults in distribution networks, on a simple grid model taken from [8]. Results were validated by analytical calculation, by simulation in PSCAD/EMTDC software and also by comparison with results published in the reference [8].

## 3.2 Arc Suppression Coil Automatic Tracking System

Distribution systems, especially MV, are often operated as impedance-grounded. This way of operation increases the operational reliability, as the grid can be operated with an earth fault. On the other hand, it complicates the earth fault localization, which has to be done using advanced protection functions. Modelling of faulted distribution system operation requires advanced modelling possibilities, because asymmetry of line-to-ground capacities of power lines has to be considered, as well as shunt conductance. Otherwise, obtained results are not credible. This was also discussed in [9]. The case study demonstrates the simulation of an automation commonly installed in impedance-grounded MV distribution networks, which are arc suppression coil (ASC) tuning automation, equipped with parallel resistance connection. In addition, simulation of shunting system operation was done.

### 3.2.1 Test System Description

Application of the created three-phase power system model for distribution system analysis is demonstrated on a simple 22 kV reactance grounded network, which was taken from textbook [10]. Diagram of the test system is shown in Figure 6. The ASC with reactance  $X_{SC} = 5 \text{ k}\Omega$  is connected to the neutral point of the transformer secondary wye winding and it is equipped with a parallel resistance  $R = 708 \text{ }\Omega$  (value corresponding to  $1 \text{ }\Omega$  connected to the auxiliary 500 V winding).

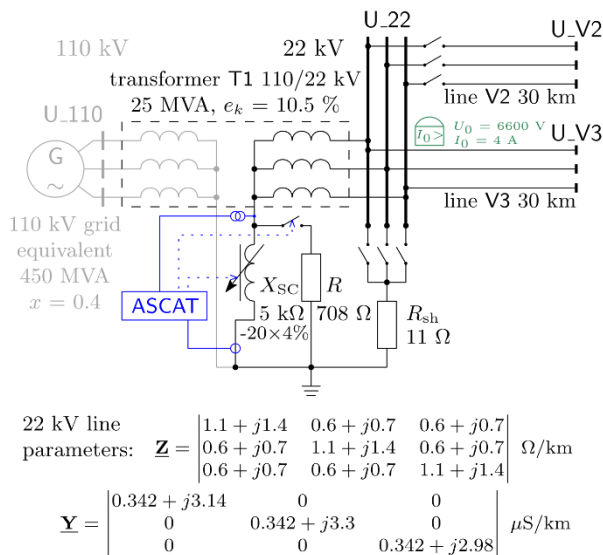


Figure 6

Diagram of the test system for analysis of ASCAT operation

The Arc Suppression Coil Automatic Tracking System (ASCAT) was newly created in MODES and provides multiple functions.

In fault-free state, ASCAT is tuning the ASC to the resonance with the line-to-ground capacitance to minimize the earth-fault current in case of fault occurrence. ASC tuning is modelled as discrete (with 20 steps of 4% each), triggered automatically when the change of voltage on ASC exceeds 20%. Achieving continuous tuning in the model is unfeasible, as each change of ASC reactance means the change of admittance matrix and finally, the whole nodal matrix needs to be recalculated making the simulation time-consuming.

In the faulted system, tuning of ASC is not allowed. But ASCAT has a different function – it connects the parallel resistor  $R$  for a short time to increase the active part of earth fault current, thus for proper operation of the directional earth fault protection (ANSI 67N; see e.g., [11] for more info).

The parallel resistor  $R$  is commonly used in impedance-grounded system, comparing to the shunting system with resistor  $R_{sh}$ , depicted also in Figure 6. The shunting method (earthing the affected phase in the transformer substation) is described in [12] and [13], and it is applied in case of an earth fault with high resistance. Its aim is to reduce the residual current in fault location by moving the earth fault to the substation. Earthing of the affected phase is done also automatically with the ASCAT system.

### 3.2.2 Tuning of ASC in Fault-Free Operation

The first simulation demonstrates the previously described operation of ASCAT in fault-free operation. In the beginning, ASCAT is tuning the ASC into the resonance with one branch (line V3; line V2 is switched off). Then another branch (V2) is switched at  $t = 1$  s, followed by tuning of ASC to resonance with the new value of grid capacitance. Finally, at  $t = 2.5$  s, second branch (V2) is switched off. ASCAT activates the tuning of ASC again. Simulation results (ASC voltage) are shown in Figure 7a.

Resonance curves for a system equipped with ASC can be calculated analytically, based on the line parameters (conductance and capacity to ground). Resonance curves for the test system from Figure 6 are shown in Figure 7b, each curve correspond to the operational stage of the network (switched branches). Tuning process of ASC is indicated by arrows in Figure 7b. Each arrow is one step change of ASC impedance. Corresponding stages of ASC tuning are marked with numbers ① – ⑤ in Figure 7.

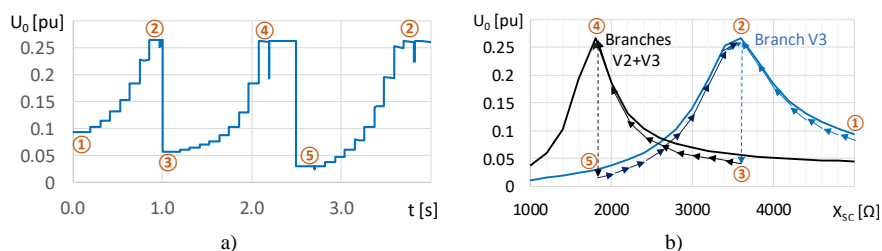


Figure 7

ASCAT test system operation (ASC tuning process) in fault-free operation: a) time course of ASC voltage  $U_0$ , b) ASC voltage  $U_0$  dependent on its impedance  $X_{SC}$  (resonance curves) for operation of one branch (V3) and two branches (V2+V3).

Theoretical value of ASC resonance reactance  $X_{SC}$  for operation of one line (V3) is given by equation (11) – it is equal to 3538  $\Omega$ . For two lines with the same length (operation of both V2 and V3), it is the half, i.e., 1769  $\Omega$ . Values of 3600  $\Omega$  and 1800  $\Omega$  respectively, used in simulation and shown in Figure 7b, are pretty matching.

$$X_{SC} = \frac{1}{B_A + B_B + B_C} = \frac{10^6}{3.14 \cdot 30 + 3.3 \cdot 30 + 2.98 \cdot 30} \Omega = 3538 \Omega \quad (11)$$

### 3.2.3 Earth Fault with Auxiliary Resistor Connection

In the following case, it is assumed that one branch (V3) is in operation, and ASC is tuned into resonance ( $X_{SC} = 3600 \Omega$ ) in the initial state. The earth fault at the end of the line V3 is applied at time  $t = 0.1$  s. ASCAT system automatically detects the earth fault (according to  $U_0$  voltage increase above 30%) and connects the auxiliary resistor for 0.1 s after the set delay of 0.1 s (processes were fastened for simulation purposes). Connecting the resistor eliminates the compensating effect of ASC, thus causes an increase of fault current to the value, which can be detected by earth fault protection.

Simulation results – line currents and ASC voltage – are plotted in Figure 8. Solid lines are the results from the MODES, dashed lines are the results obtained by the simulation validation in OpenDSS simulation tool; results are almost the same and they are overlapping in Figure 8.

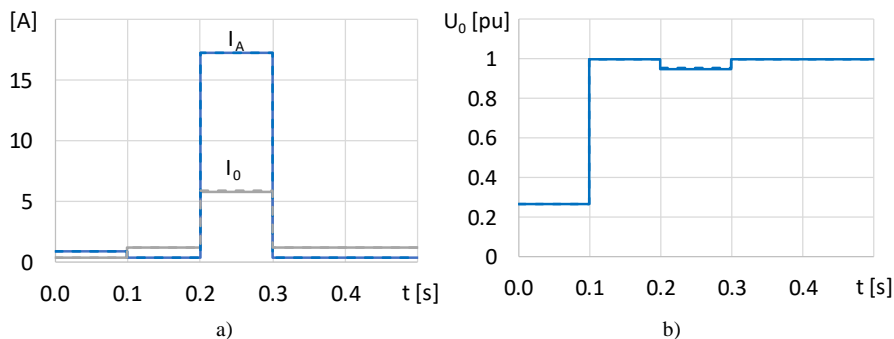


Figure 8

Simulation results of the ASCAT system operation during the earth fault: a) phase A line current  $I_A$  and zero sequence current  $I_0$ , b) ASC voltage  $U_0$ ; solid lines – MODES, dashed lines – OpenDSS

In the moment of the earth fault occurrence (0.1 s), the ASC voltage  $U_0$  increases to the nominal phase voltage (1 p.u.). Auxiliary shunt resistor connection at  $t = 0.2$  s leads to a drop in voltage – a certain ASC detuning. This is much more evident in the phase current  $I_A$  of the line, which is significantly increased as well as its zero-sequence component  $I_0$ , which exceeds the value of 5 A. The directional earth fault protection (ANSI 67N) with the threshold of 4 A detects the fault and issues a message. This message will help with fault localization (faulted branch is identified).

### 3.2.4 Shunting System

The last simulation done on this test system was the demonstration of shunting system. In this last case, both lines V2 and V3 are in operation and ASC is tuned to the resonance ( $X_{SC} = 1800 \Omega$ ). A resistance earth fault (with fault resistance of  $500 \Omega$ ) is applied at the end of the line V3 at  $t = 0.1$  s. The ASCAT actions are

noticeable from the Figure 9, showing the same variables as in previous case and also the simulation validation results from OpenDSS simulation tool; results are almost the same again and they are overlapping in Figure 9.

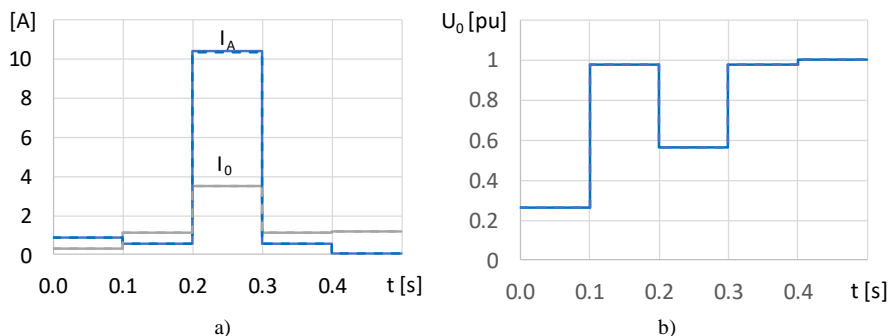


Figure 9

Simulation results of the ASCAT system operation, equipped with shunting system, during the earth fault: a) phase A line current  $I_A$  and zero sequence current  $I_0$ , b) ASC voltage  $U_0$ ; solid lines – MODES, dashed lines – OpenDSS

Earth fault occurrence at  $t = 0.1$  s leads to the increase of voltage  $U_0$  close to the nominal value (1 p.u.), connection of auxiliary shunt resistor at  $t = 0.2$  s for a limited period of time (0.1 s) means an increase of the phase current  $I_A$  of the line, which is slightly smaller than in the previous case. The zero-sequence component of the current is lower than the set threshold of 4 A, so the directional earth fault relay does not detect the fault. At time  $t = 0.4$  s, the affected phase is earthed in the substation through a resistance of 11  $\Omega$ . The fault is thus transmitted to the substation from its actual location, and the fault current  $I_A$  drops to zero. The power system should be still treated as faulted, but moving the fault to the substation minimizes the risks in the fault location (e.g., the arc should extinguish).

### 3.2.5 Summary

The three previous cases demonstrated the use of the newly built asymmetric model for MV distribution system analysis. The possibility to model the asymmetry of power lines and also the shunt conductance, which significantly affects the results (it is impossible to tune the ASC if all lines are entirely symmetrical), is of importance. A complex model of ASCAT system as well as a protection model were implemented in MODES. Presented cases are really simple and deals with the most common fault in the distribution system, which is the earth fault, thus, they are suitable also for education purposes. Results were validated using analytical calculations and OpenDSS simulation tool.

### 3.3 Voltage Unbalance Calculation

One of the causes of asymmetry in power system is the load unbalance caused by uneven distribution of loads over phases. This phenomenon is known especially on lower voltage levels (LV and MV), where some consumers and prosumers have only single-phase connection. However, it is also a problem of AC single-phase railway traction feeders, connected to the HV distribution system [14]. In this case, the use of created three-phase system model for voltage unbalance calculation caused by the railway traction feeders (V-connected) is demonstrated.

#### 3.3.1 Test System Description

For voltage unbalance calculation, modified test system depicted in Figure 10 was used. It is a part of the eight-node network published in [8].

The network was assumed to be balanced, except for loads in buses 7 and 8. These loads are the traction feeders 25 kV, fed from the 220 kV grid through the V-connection (open delta) transformers, according to Figure 2.

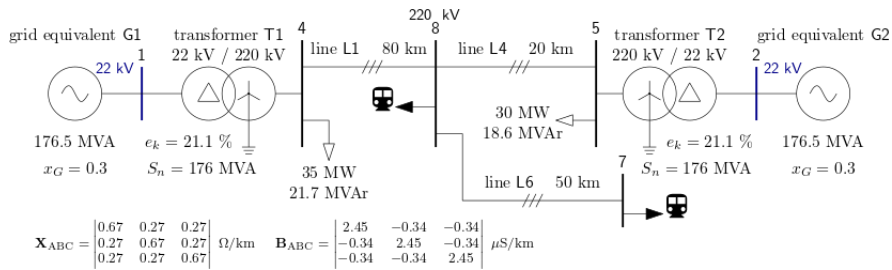


Figure 10

Single-line diagram of the test system for voltage unbalance calculation

#### 3.3.2 Voltage Unbalance Caused by the Train Operation

For voltage unbalance calculation, two scenarios were considered. The first one was the load consumption of  $P = 6$  MW in bus 8, caused by the train operation in the railway segment fed from the transformer T2 in bus 8. The second scenario was the consumption of  $P = 6$  MW in both buses 7 and 8, caused by the train operation in the railway segment fed from transformers T2 connected to these two buses.

It is assumed that the transformer connection at node 8 corresponds to substation 1 in the Figure 2, and the transformer connection at node 7 corresponds to substation 2. The values of power consumption of traction transformers supplied from the 220 kV network can be derived using the relations (9) and (10), the simplifying assumption of symmetrical phase voltages and neglecting reactance  $X$  in the relation (10). Phase values of these powers are  $\underline{S}_A = (3 + j1.73)$  MVA,  $\underline{S}_B = (3 - j1.73)$  MVA,  $\underline{S}_C = 0$  for both nodes. It is worth noting that although the locomotive takes only active power  $P$ , reactive power also flows in the network, causing voltage drops and losses.

When we know the power consumption in the individual phases, we can calculate the voltage asymmetry these consumptions will cause in the 220 kV network. A measure of asymmetry is the negative sequence voltage component. This value for the test system and defined two cases was calculated using the newly implemented network model in MODES; the results in per unit are given in Table 1. In both cases, positive sequence voltage value was almost 1 p.u. Validation calculation was done in PSCAD/EMTDC, results of validation are given also in Table 1 and they are almost the same.

Table 1  
Calculated values of negative sequence voltage

Simulation case	Negative sequence voltage in p.u.			
	Bus 7		Bus 8	
	MODES	PSCAD	MODES	PSCAD
1. – load of 6 MW in bus 8	0.01	0.01	0.01	0.01
2. – load of 6 MW in buses 7 and 8	0.023	0.023	0.0204	0.0201

Results in Table 1 are of interest. The standard EN 50160 [15] gives the admissible value of negative sequence voltage to be 0–2% of positive sequence voltage as the criterion for voltage unbalance. The requirement of EN 50160 is thus not fulfilled in the second case (negative sequence voltage reaches 2.3% and 2.04% respectively).

### 3.3.3 Summary

The chapter shows the use of the three-phase model for the evaluation of voltage unbalance caused by power consumption of traction feeders. The model makes possible to check whether the voltage unbalance does not exceed the limit defined in the standard EN 50160. Results were validated using PSCAD/EMTDC tool.

## 4 General Summary and Future Work

Two general concepts for simulation of three phase power system respecting the asymmetry are applicable, either the symmetrical components (positive, negative and zero sequence) or phase values (phase A, phase B, phase C). The choice of the concept is dependent on the problem being analyzed and to what extent the grid asymmetry needs to be respected.

Newly designed simulation model, implemented to MODES simulation tool and described in this article, can deal with both concepts – it allows data entering in phase values or symmetrical components. The computation method can be also selected as either symmetrical components or phase values. Criteria of selection are summarized in Table 2.



Comparing to other simulation tools, the newly designed three phase model implemented to MODES allows to simulate different fault-free and faulted operation of power system in RMS values, making the computation fast. It is using the MODES simulation engine, which has already implemented advanced protection and automation models. Existing models can be used also in newly build projects, dealing with unbalanced power systems. The asymmetric model is supposed to have various applications in the future, mainly in power system analysis, and additional models of network elements are being prepared. Currently, it is a source model representing single-phase connection of power sources, which is typical for LV networks. Such a model will allow to analyze problems related to prosumers or charging of electric vehicles.

Table 2  
Calculated values of negative sequence voltage

Computation method	Data entering	
	Symmetrical components	Phase values
Symmetrical components	Symmetrical both grid and load. Simulation of simple faults: single line to earth fault, line to line fault, double line to earth fault, single and double phase interruption.	Internal computation of parameters in symmetrical. Both grid and load are symmetrical. Simulation of simple faults: single line to earth fault, line to line fault, double line to earth fault, single and double phase interruption.
Phase values	Symmetrical both grid and load. Internal computation of parameters in phase values. Model allows to simulate also simultaneous faults including phase interruptions and single line to ground faults.	Neither grid nor load need to be symmetrical. It is e.g., possible to simulate the tuning of ASC. Model allows to simulate also simultaneous faults including phase interruptions and single line to ground faults.

## Conclusions

This work shows the process for the creation of a computational tool, allowing calculation of load flow problems, short circuit currents and transient stability assessment using phase values. The MODES simulation tool, newly extended by the asymmetric model, can deal with the grid unbalance, in both initial (fault-free) stage and also those occurring in the grid, due to unbalanced faults, comprising simultaneous faults.

Applications of the model were demonstrated using 3 selected problems from power systems operations – simultaneous faults, operation of arc suppression coil automatic tracking with earth fault protection and assessment of voltage unbalance caused by traction feeders. Results were validated by both analytical calculations and simulations using other tools. Simulation model implemented to MODES is

robust, allowing the analysis of power systems – especially distribution systems – under both fault-free and faulted operation. MODES is supposed to be used for analysis of different phenomena and also topical issues, such as, spread of distributed sources in the power system, in the future. Last but not least, it can be used for educational purposes, to demonstrate selected phenomena, in the grids, for the training of university students. The MODES tool usage is convenient and intuitive, thanks to the graphical user interface.

### **Acknowledgement**

This research work has been carried out in the Centre for Research and Utilization of Renewable Energy (CVVOZE). Authors gratefully acknowledge financial support from the Ministry of Education, Youth and Sports of the Czech Republic under BUT specific research program (grant number FEKT-S-23-8403).

### **References**

- [1] Máslo, K., Kolcun, M.: Simulation engine for dispatcher training and engineering network simulators, Proc IFAC CIGRE/CIREN Workshop on Control of Transmission and Distribution Smart Grids (CTDSG'16), Oct. 2016, pp. 395-399
- [2] Chladová, M., Máslo, K., Křištof, V.: Short circuit calculations and Dynamic stability assessment within dispatch control environment, operational requirements and SW reality, Proc. 10<sup>th</sup> International Scientific Symposium on Electric Power Engineering, Sept. 2019
- [3] Anderson, P. M.: Analysis of faulted power systems, Wiley-IEEE Press, 1995
- [4] Kersting, W. H.: Distribution System Modelling and Analysis, Third Edition, ISBN 978-1-4398-5622-2, CRC Press, 2012
- [5] Coppo, M., Bignucolo, F., Turri, R.: Generalised transformer modelling for power flow calculation in multi-phase unbalanced networks. IET Gener. Transm. Distrib. 2017, 11: 3843-3852. doi: 10.1049/iet-gtd.2016.2080
- [6] IEC TR 60909-4:2000 – Short-circuit currents in three-phase a.c. systems – Part 4: Examples for the calculation of short-circuit currents, 2000
- [7] Chen, B. K., Guo, B. S.: Three phase models of specially connected transformers, IEEE Trans. Power Delivery, Vol. 11, pp. 323-330, Jan. 1996
- [8] Roy L., Rao N. D.: Exact calculation of simultaneous faults involving open-conductors and line-to-ground short circuits on inherently unbalanced power system, IEEE Trans. Power App. Syst., Vol. 101, No.8, pp. 2738-2746, Aug. 1982
- [9] Koudelka, J., Topolanek, D., Toman, P., Fabian, M.: Analysis of Fault Condition Caused by Phase Interruption of HV Overhead Line, CIGRE 2021

- The 26<sup>th</sup> International Conference and Exhibition on Electricity Distribution, 2021, pp. 1572-1576, doi: 10.1049/icp.2021.1720
- [10] Trojánek, Z., Chladová, M.: Transients in electrical power systems (Přechodné jevy v ES; In Czech), CTU publishing house, 1988, p. 145
- [11] Topolanek, D., Lehtonen, M., Toman, P., Orsagova, J., Drapela, J.: An earth fault location method based on negative sequence voltage changes at low voltage side of distribution transformers, *Int J Electr Power Energy Syst* 2020;118: 105768, doi: 10.1016/j.ijepes.2019.105768
- [12] Kouba, D., Procházka, K.: The Analysis of Efficiency of Shunt Resistor During a Single-phase Earth Fault Using the Two-port Network Theory, *Proc.of Conference CIRED*, 2013, pp. 1-4, doi: 10.1049/CP.2013.1066
- [13] Topolánek, D., Toman, P., Orságová, J., Dvořák, J.: The Method of the Additional Earthing of the Affected Phase During an Earth Fault and Its Influence on MV Network Safety, *Proc. of the IEEE PowerTech 2011*, pp. 1-8
- [14] Bayliss, C. R., Hardy, B. J.: *Transmission and Distribution Electrical Engineering*, Elsevier, 2012, doi: 10.1016/C2009-0-64342-7
- [15] EN 50160:2010 – Voltage characteristics of electricity supplied by public electricity networks, CENELEC, 2010