

# Adaptive Model Predictive Controller for Web Transport Systems

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*Abstract: In web transport systems (WTS), parameter variations in transported material affects the product quality and integrity of the processed material. This investigation presents an adaptive model predictive controller for WTS in process industries considering the variations of the web radius with respect to time. The proposed controller uses a radius approximation algorithm for estimating the changes in web radius. The controller then updates the WTS model with the estimated changes in web radius that enters as the disturbance in the system. Then, the controller uses the model with disturbance estimation algorithm and an optimization routine to compute the future control moves that guarantee product quality, while simultaneously satisfying the physical and operating constraints of WTS. Furthermore, it assures product quality with web radius variations. The main advantage of the proposed controller is, it combines the predictive and optimality features of MPC with adaptation provided by an adaptive controller. Simulations on WTS used in paper industries illustrates the performance and advantages derived by employing the adaptive MPC against conventional MPC. Our results show a reduction in the peak overshoot, integral absolute error, and integral time square error by using the adaptive MPC.*

*Keywords: Process industries; Adaptive model predictive controller (AMPC); Web Transport Systems (WTS); Web Transport Controllers (WTC); parametric variations*

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# 1 Introduction

Web Transport Systems (WTS) are widely used in process industries, to transport finished material in the form of sheets over long distances. To have good material integrity and for reducing downtime due to loss of production resulting from web breaks, constant tension should be maintained on the transported material and Web Tension Controllers (WTCs) are used to this extent. Therefore, WTC performance is pivotal for having good product quality and increasing production. Parameter variations of the web material affect the WTC performance adversely. Therefore, to assure product quality, the controller should maintain the constant tension in spite of the parameter variations of the web material.

The problem of web tension control has received considerable attention and many control approaches have been proposed. Proportional integral and derivative (PID) controllers are the most widely used ones (see, [6] [7] [8] [13] [14] [15] and [18]). Although PID controllers are simple and easy to tune, their performance deteriorates due to disturbances acting on the web and parameter variations. In [2] an optimal controller was studied to optimize the product quality. To compensate the effect of parameter variations on product quality, the investigations in ([1] and [11]) proposed the use of adaptive controllers. Though, the adaptive controllers designed in these investigations performed better than optimal controllers in the presence of parameter variations, they lacked optimal performance.

More recently, the investigations in ([10] and [17]) proposed the use of model predictive controllers (MPC) for WTS. The MPC uses the model of the web transport system, estimate of disturbance and an optimization routine to optimize the product quality and energy consumption in the WTS. The results showed that, combining prediction with optimization provided significant cost and quality benefits. Though, MPC showed optimal and predictive capabilities, their performance was limited by the parameter variations in the web material. Our objective in this investigation is to overcome the shortcomings with the existing approaches by designing a controller that combines adaptive, optimal and predictive features.

To reach the objectives, this investigation proposes an adaptive MPC (AMPC) that combines the benefits of adaptive and model predictive controller. The main building blocks of the AMPC are: an online parameter estimator that computes the variations in radius (web parameter), a disturbance estimator, a constrained optimization routine that computes the future optimal outputs and a receding horizon strategy. The output of the parameter estimation algorithm updates the model, during each time epoch. The AMPC uses the updated model, an optimization routine, and knowledge of constraints to compute the control moves that improves the product quality. The optimization is performed using a receding horizon approach. The use of estimation model within AMPC

guarantees that the optimization is performed considering the underlying changes in web transport dynamics and parameter variations obtained using parameter estimation technique. The presence of disturbance and parameter estimates improves the model accuracy used for designing the MPC, and handles the physical constraints and operating constraints inherently in the design. As a result, the controller is more robust to parameter variations and other disturbances acting on the plant. The AMPC combines the adaptive, optimal and predictive control features in the existing approaches and therefore, is a promising approach for web transport systems that are subjected to frequent parameter variations.

The main contributions of the investigation are: design of adaptive MPC controller for the web transport systems that incorporates parameter estimation, a simple parameter estimation algorithm, and an illustration of the proposed control methodology using parameters obtained from a prototype web transport system. The investigation highlights the advantages in employing AMPC by comparing the obtained results with a conventional MPC.

This paper is organized into six sections. Section 2 describes the mathematical model of the WTS. Section 3 provides AMPC design. Section 4 explains the AMPC algorithm for web tension control. The results and discussion are provided in Section 5. The conclusion section is presented in Section 6.

## 2 Mathematical Model of WTS

This section describes the WTS and the dynamical equations of the system. Figure 1 shows a prototypical WTS employed widely in a process industry and it consists of a un-winder, winder, dancer, and gear transmission system. The dancer is connected to a potentiometer for measuring the displacement and a spring and damping arrangement is used for regulating its displacement. The parameters and variables that describe the dynamics of WTS are shown in Table 1. The winder and un-winder rollers are coupled to an electric motor through a geared drive. The motor torque is the input to WTS and it influences the roller velocity to maintain constant tension acting on the web. The load cell measures the web tension and provides feedback to WTC. The web tension is regulated by adjusting the torque of roller motors and dancer position.

The mathematical model of WTS describes the relation between the torque and the web tension. The dynamics can be described using the block diagram shown in Figure 2. The mathematical model of WTS consists of three sections, namely, (i) drive train, (ii) web material and (iii) dancer.

## 2.1 Drive Train

The relation between the motor torque and the angular velocity of the motor, and the relation between the angular velocity and the roller tangential velocity are given by (1) and (2), respectively by [4] as

$$\dot{\omega}_m = -\frac{B_m}{J_m} \omega_m + \frac{1}{J_m} \tau - \frac{1}{J_m} \tau_{cou} \quad (1)$$

$$V_2 = \frac{R}{N} \omega_m \quad (2)$$

Table 1  
List of parameters and variables used in WTS model

SYMBOL	DESCRIPTION
$\tau$	Input torque of the winder motor in Nm
$J_m$	Moment of Inertia of the motor in Nm <sup>2</sup>
$B_m$	Viscous friction constant of the motor in Nm.s/rad
$\omega_m$	Angular Velocity of the motor in rads/sec
$R$	Radius of the winder in m
$N$	Gear ratio
$E$	Young's modulus in Pa
$A$	Cross sectional area of the web in m <sup>2</sup>
$L$	Length of the web in m
$T$	Tension of the web in N
$V_1$	Tangential Velocity of the un-winder roller in m/sec
$V_2$	Tangential Velocity of the winder roller in m/sec
$B_d$	Viscous friction of the dancer in N.s/m
$M_d$	Mass of the dancer in Kg
$K_d$	Spring constant of damper system in N/m
$d$	Position of the dancer in m
$V_d$	Velocity of the dancer in m/sec
$\tau_{cou}$	Coupling torque in Nm
$F_x$	Disturbance forces acting on dancer in N

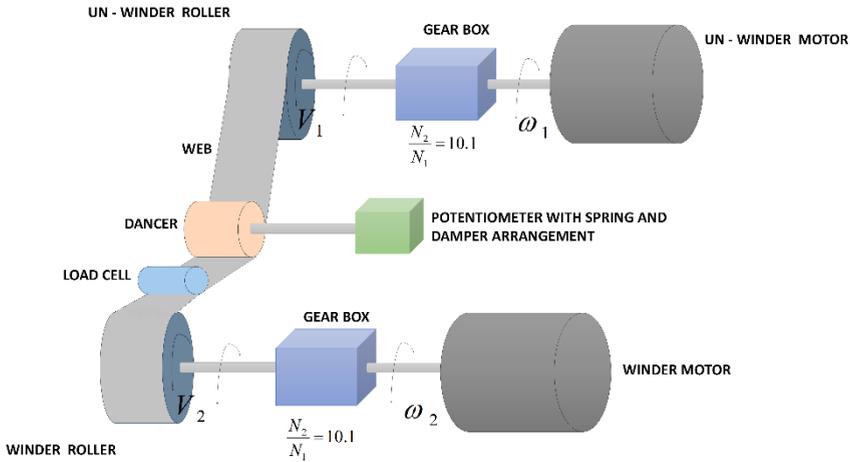


Figure 1  
Schematic of WTS with un-winder, winder and dancer arrangement

### 2.2 Web Material

Using Hook’s law and law of conservation of mass the relation between web tension ( $T$ ), angular velocity ( $\omega_m$ ) of roller and dancer velocity ( $v_d$ ) can be modelled as

$$\dot{T} = -\frac{V_1}{L}T + \frac{REA}{NL}\omega_m - \frac{EA}{L}V_1 - \frac{2EA}{L}V_d \tag{3}$$

### 2.3 Dancer

From Newton’s second law motion, the relationship between the web tension ( $T$ ) and dancer position ( $d$ ) is given by

$$\ddot{V}_d = \frac{2}{M_d}T - \frac{1}{M_d}F_x - \frac{B_d}{M_d}V_d - \frac{K_d}{M_d}d \tag{4}$$

The state space model of WTS can be framed form equations (1), (3) and (4) as:

$$\begin{bmatrix} \dot{V}_d \\ \dot{d} \\ \dot{\omega}_m \\ \dot{T} \end{bmatrix} = \begin{bmatrix} -\frac{B_d}{M_d} & -\frac{K_d}{M_d} & 0 & \frac{2}{M_d} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{B_m}{J_m} & 0 \\ -\frac{2EA}{L} & 0 & \frac{REA}{L} & -\frac{V_1}{L} \end{bmatrix} \begin{bmatrix} V_d \\ d \\ \omega_m \\ T \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J_m} \\ 0 \end{bmatrix} \tau + \begin{bmatrix} 0 & -\frac{1}{M_d} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{J_m} \\ -\frac{EA}{L} & 0 & 0 \end{bmatrix} \begin{bmatrix} d \\ V_1 \\ F_x \\ \tau_{cou} \end{bmatrix} \tag{5}$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_d \\ d \\ \omega_m \\ T \end{bmatrix} \tag{6}$$

where,  $A$  is the system matrix;  $B$  is the input matrix;  $F$  is the disturbance matrix;  $x$  contains the states;  $d_x$  is the disturbances;  $\tau$  is the input torque applied to the WTS and  $y = T$  is the measured tension in Newton.

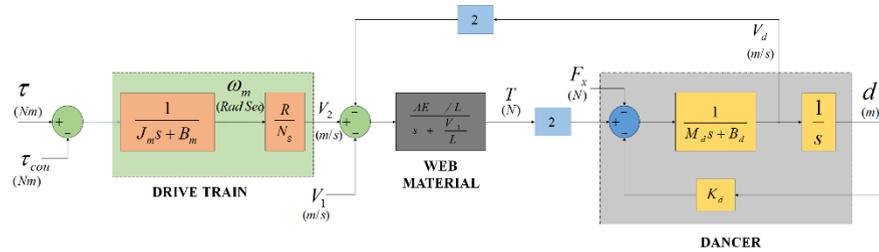


Figure 2  
Block diagram representation of WTS with rollers and dancer arrangement

### 3 Adaptive Model Predictive Controller

One major challenge in maintaining constant web tension is the continuous variation of the web radius due to winding and unwinding operations. The web tension needs to be regulated considering the current radius of the web and generally, sensors are used for this purpose. An accurate determination of web radius, using measurements is rather difficult due to acceleration and de-acceleration of the web and friction in rollers. As a result, the parameter variations in radius cause deterioration in control performance leading to the material loss, downtime, and poor material integrity. In order to overcome this performance loss, variations of the web radius need to be compensated. Furthermore, the performance needs to be optimized to guarantee product quality considering the variation in the parameters and disturbance acting on the WTS. This requires combining adaptation to parameter variations, the prediction on disturbances and optimal control for optimizing the performance in the face of uncertainties. To obtain these two features, this investigation designs an adaptive model predictive controller (AMPC) and an estimator to predict the variations in the parameter. The AMPC design has three components, namely, (i) an online model estimator, (ii) a prediction model and (iii) an online optimization routine. The schematic of AMPC design for WTS is illustrated in .

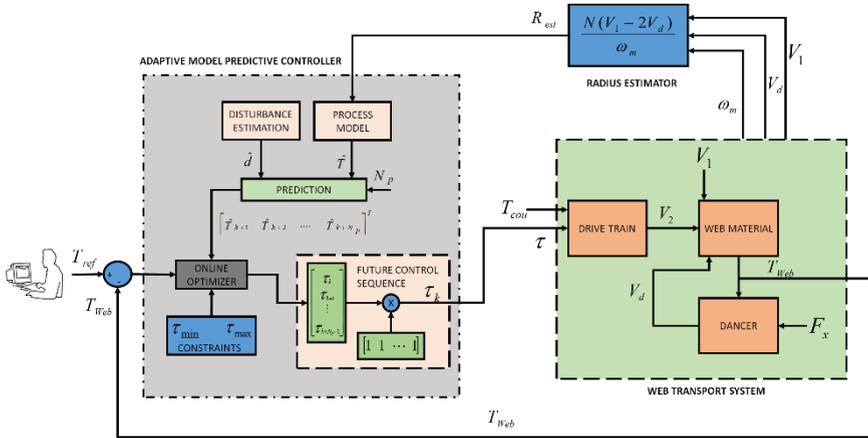


Figure 3

Schematic of Adaptive MPC design for WTS

The online estimator model estimates the web radius from the output data namely, the angular velocity of the motor, un-winder velocity, and dancer velocity, at each time epoch. The estimated radius is given by

$$R_{est} = \frac{N(V_1 - 2V_d)}{\omega_m} \quad (7)$$

The estimated web radius ( $R_{est}$ ) updates the plant model in (5) during each time period. The prediction model uses this updated model to compute the prediction matrix in (8).

$$P = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix} \text{ and } H = \begin{bmatrix} CB & 0 & 0 & \dots \\ CAB & CB & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \dots \end{bmatrix} \quad (8)$$

To obtain optimal control input, an optimization routine ( $J$ ) is formulated with the variables of interest, the reference tension ( $T_{ref}$ ), prediction model ( $P$  and  $H$ ), control input ( $\tau$ ) and the disturbance matrix ( $F$ ). The optimization problem in (9) models the adaptive MPC controller for the prediction horizon and is solved using quadratic programming (by invoking conic solvers) considering the constraints on the WTS.

$$J = \|T_{ref} - H\Delta\tau - P\hat{x}_k - Fd_x\|_2^2 + \lambda\|\Delta\tau\|_2^2 \quad (9)$$

Sub.to.

$$\tau_{min} \leq 0 \leq \tau_{max}$$

Solution to the constrained optimization problem provides the control moves for the prediction horizon ( $N_p$ ). The first among the control input is applied and the procedure is repeated during each step.

## 4 AMPC Algorithm for Web Tension Control

This section presents the AMPC algorithm for the web tension controller. The execution sequence of AMPC has six steps as shown in Figure 4.

### Step1: Update the parameter and measurement

In the first step of the algorithm, the measurements on web tension measured using load cells are updated. Then, the web radius ( $R_{est}$ ) information obtained from the online parameter estimator block updates the plant model during the current time epoch.

### Step 2: Model update and construction of prediction matrices

Using the estimated web radius ( $R_{est}$ ), the prediction matrices in (8) are updated.

### Step 3: Compute control inputs

The reference web tension ( $T_{ref}$ ), prediction models ( $P$  and  $H$ ), disturbance model ( $F$ ) and the operating constraints ( $\tau_{max}, \tau_{min}$ ) are used to solve the constrained optimization problem in (9) to obtain control inputs ( $\tau$ ).

### Step 4: Receding horizon input

The online optimizer provides control inputs ( $\tau$ ) for  $N_p$  time steps. Out of  $N_p$  control inputs, the first control input is applied and the rest are discarded to implement a receding horizon control.

### Step 5: Estimate the parameter

The control input ( $\tau$ ) is applied to WTS, and the web tension ( $T_{web}$ ), un-winder velocity ( $V_1$ ), dancer velocity ( $V_d$ ) and angular velocity of the winder ( $\omega_m$ ) are measured using sensors. These measurements are used to estimate the web radius given by (7).

### Step 6: Write the output

The web tension ( $T_{web}$ ) and the estimated radius ( $R_{est}$ ) are given as the feedback to the controller to update the process model and to calculate the prediction matrices for the next iteration  $t + 1$ .

This execution sequence is repeated for the entire run time. The effects of web radius variations on the web tension, the significance of online estimation model and the performance of AMPC for WTS are discussed in the following section.

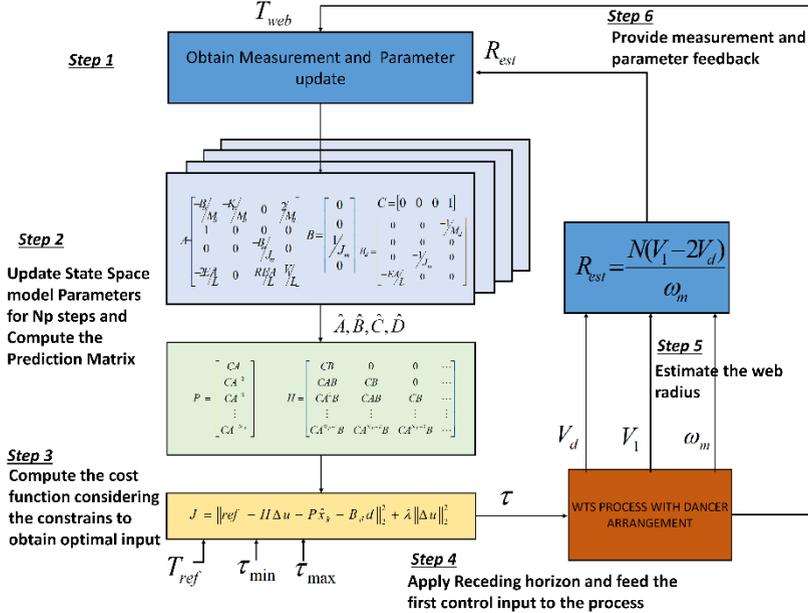


Figure 4  
Adaptive MPC algorithm for WTS with parametric variations

## 5 Results and Discussion

The proposed AMPC is illustrated using simulations with parameters of a web transport system obtained from a paper industry. This section, studies the benefits of adapting the AMPC to control the WTS. To illustrate the benefits of the AMPC, its performance is compared with the conventional MPC used for controlling the WTS presented in Section 2.

During the simulations, the radius of the winder roller is varied from 0.57 to 0.16 m. As the radius changes, the dancer roll moves up or down to compensate the parameter variations. The effect of the radius variations on dancer position is illustrated in Figure 5. It can be observed that as the radius of the winder increases, the dancer roll moves down in the un-winder section to compensate for parameter changes. The estimated web radius obtained using equation (7) and the actual web radius is shown in Figure 6 for the operating range considered in our simulations, and an error of 0.4% was observed in the estimated radius.

The parameters used in our simulations for studying WTS performance are listed in Table 2. The control parameters and the operating constraints for MPC and AMPC controllers are shown in Table 3. These parameters were selected based on the fast dynamics exhibited by WTS and the parameter variations. The prediction horizon should capture the relevant transient information in it to foresee the effect of parameter variations and disturbances. Therefore, based on open loop response characteristics the prediction horizon was selected to be 30 time epochs. Though a control horizon of 10-15 is enough for implementing AMPC, to account for the fast changes in disturbances and parameter variations, a control horizon of 20 epochs was selected. The physical and operating limits used in the simulations are obtained from the process dynamics, these constraints are used in the optimization problem to reflect the actual conditions. The set-points are selected based on the paper variety being processed in the paper industry.

Table 2  
Simulation Parameters of WTS model

PARAMETER	VALUE
$A$	$4.35 \times 10^{-6}$
$E$	$4 \times 10^9$
$L$	0.61
$J_m$	0.0324
$B_m$	$0.55 \times 10^{-3}$
$R$	$57.3 \times 10^{-3}$
$N$	10.1
$B_d$	500
$M_d$	6.762
$K_d$	1131
$V_1$	0.051
$\tau_{cou}$	$0.0081 \times 10^{-5}$
$F_x$	66.3

The changes in web tension for a tension set-point of 70 N using AMPC and MPC controllers are shown in Figure 7. From the result, it is observed that the percentage overshoot is 10.43% with the MPC, as against 5.12% in AMPC. The percentage overshoot reflects the quality loss due to winding operations. Therefore, a reduction of about 50.5% in overshoot that also reflects in the quality of transported material is achieved during the transient stage of the web processing due to AMPC. The improvement is mainly due to the accurate compensation of the web radius in AMPC that in general causes degradation of material quality with the MPC. Second, the settling time of the AMPC is

reduced by 6% leading to faster settling times to ensure required material integrity, thereby reducing material loss significantly. These results illustrate the material savings and quality enhancements achieved using AMPC in process industries.

The improvements achieved using proposed AMPC are illustrated by analyzing performance measures such as, integral square error (IAE), and integral time square error (ITSE). The IAE indicates the improvements in the transient part of the response, whereas the ITSE represents the steady-state improvements. Our results indicate that the proposed AMPC has significant benefits in improving the performance of the WTS by up to 4.2% that reflects the improvements achieved using the proposed AMPC. The improvement is mainly due to the incorporation of parameter estimation model within the AMPC controller.

Table 3  
Controller parameters of MPC and Adaptive MPC

PARAMETERS	ADAPTIVE MPC	MPC
Prediction Horizon ( $N_p$ )	30	30
Control Horizon ( $N_c$ )	20	20
$\tau_{\min}$	-0.01	-0.01
$\tau_{\max}$	0.09	0.09
$\Delta\tau_{\min}$	-0.1	-0.1
$\Delta\tau_{\max}$	0.3	0.3
$\lambda$	0.4	0.4

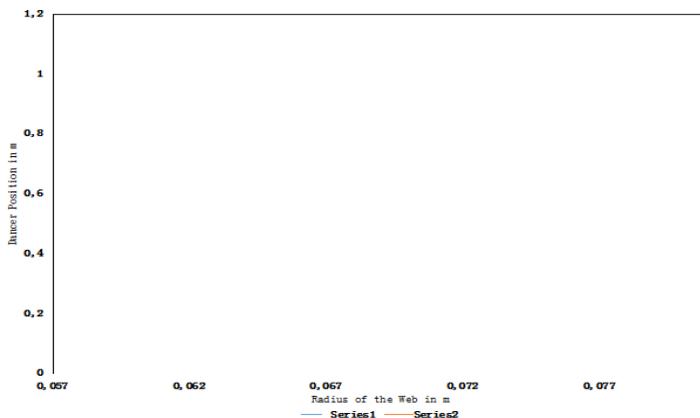


Figure 5  
Position of the dancer with respect to the variation in the winder radius

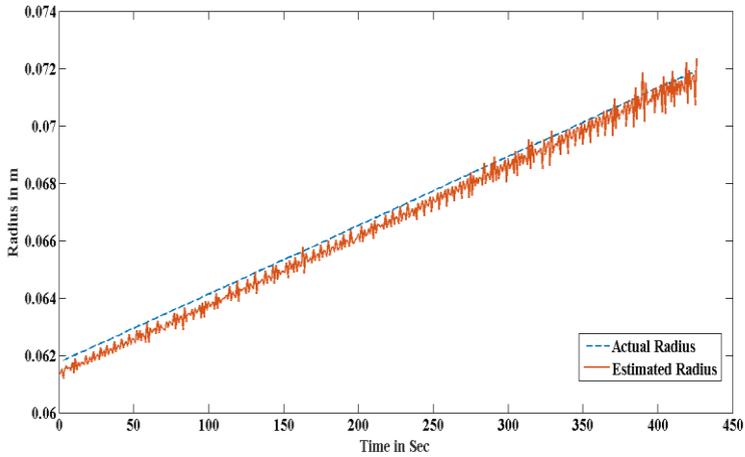


Figure 6

Comparison between actual winder radius and the online estimated radius

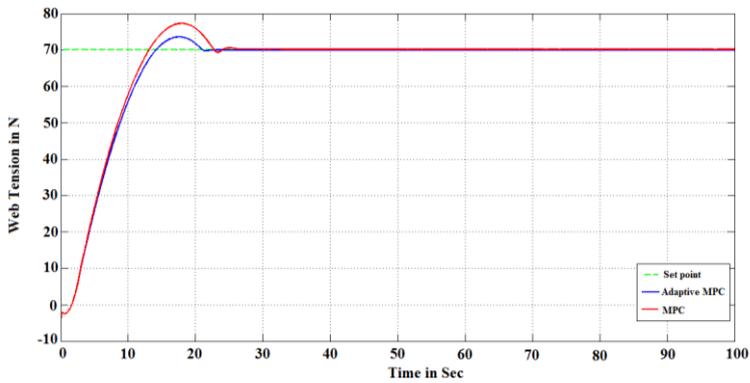


Figure 7

Responses of Adaptive MPC and MPC to a reference web tension of 70 N

Table 4

Transient performance of Adaptive MPC and MPC controller

CONTROLLER	RISE TIME (Sec)	SETTLING TIME (Sec)	PEAK OVERSHOOT (%)
ADAPTIVE MPC	7.75	20.9	5.16
MPC	7.53	22.3	10.43

Table 5

Performance indices of Adaptive MPC and MPC controller

CONTROLLER	IAE	ISE	ITSE
ADAPTIVE MPC	5.19	259.54	723.29
MPC	5.43	256.6	725.95

## Conclusion

This investigation proposed an adaptive model predictive controller for web transport systems that combines adaptive, optimal and predictive features. The confluence of these highly desirable features in web transport systems led to improvements in performance, quality and reduced downtime. The main building blocks of the controller are a parameter estimation block, prediction model and an online optimization routine. The parameter estimation block uses the variables from WTS process to estimate the changes in web radius. The web radius estimation obtained is close to the actual radius with an estimation error of 0.4%. Using the estimated web radius, the prediction model is updated at each time epoch to handle the parametric variations. An online optimization routine is used to provide an optimal control input considering the physical and operating constraints. To verify the performance improvement obtained by AMPC, its performance is compared to conventional MPC. Our result shows that AMPC handles the parameter variations effectively with 6% increase in material integrity and 50.5% increase in material quality. Further, the AMPC controller performance in regulating web tension shows improvement up to 4.2% than conventional MPC.

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