

The Fitting Disc Method, a New Robust Algorithm of the Point Cloud Processing

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Abstract: This article presents a new robust LiDAR processing method. This method fits a regression plane to a point cloud in any horizontal position by fitting a disc (with R radius) on it, which contains a specified portion (q) of points under the disc plane in all three sectors of the disc. This method can be used to create digital elevation models even without any filtering process. This article also describes an analysis, which compares the results of the fitting disc method using different parameters in processing of digital elevation models.

Keywords: LiDAR; Point Cloud; Digital Elevation Model

1 Introduction

Laser scanning is a very efficient and developing technology of three-dimensional spatial data capturing. The processing of captured point clouds is a key element of these surveys. The point cloud is a typical case of big data: it contains a lot of information, and we have to assort the essential elements for our aim.

One of the most important results of the processing of airborne laser scanning (LiDAR) data is the Digital Elevation Model (DEM) of the surveyed region. The ground surface must be determined from the point cloud where the points generated by natural or artificial objects should not influence the result. There is a need for a new method that does not interfere with all points situated over the ground surface.

Usually this is achieved by methods that filter the point cloud before the surface is generated or filter the created surface. This article describes a new method (called fitting disc method) that can examine the elevation in any vertical position without filtering the point cloud. This method can be used to create digital elevation models or to recognize the terrain objects.

This article also contains an examination of the suggested method. The elevations derived from a LiDAR point cloud by the suggested method are compared to the result of surveying.

2 Solutions of the Creating Elevation Models from Point Clouds

Most of the developed methods solve this problem by using the major part of the points located over the ground surface, but the points located under the ground surface are generated only as measurement noise. The searching of the ground surface is a process of searching the lowest coherent surface.

2.1 Current Processing Methods

One group of the processing methods tries to filter the points out over the terrain surface, and uses the filtered point cloud to create a digital elevation model. The slope-based filter [14] excludes the points that result into a bigger slope than a specified angle as the maximum slope of the surface. The filtered point cloud does not contain any point pairs, where the slope between the points is more than the maximal slope. This filtering method removes the points that were created on the vegetation or buildings, if the slope resulted by these points was greater than the maximal slope. An advanced method (suggested by [12]) uses variable slope limit adapted to the terrain.

Another LiDAR data processing method filters the created surface to eliminate the effect of the non-ground points. For example, the morphological filter [3] or the methods suggested by [9].

The morphological filters are built from erosion and dilation operations, where the lower (erosion) or higher (dilation) elevation of the neighboring area (window) of the surface is assigned to an element of the elevation model. The size of this neighboring area is defined by a window. The combination of the erosion and dilation operation is called Dual Rank Filter.

The principle of the widely used Progressive Morphological Filter [17] is that the output surface of the morphological filtering is used for filtering the point cloud in the next step. The points near the surface are chosen in this filtering step. This filtered point cloud will be used in another step to create a new surface, with a simple morphological filter. These steps can be repeated several times in order to achieve the final result. The size of the window will be smaller and smaller in each step.

[2] suggests a fuzzy-based planar segmentation method for processing LiDAR data. [18] describes a method that uses cloth simulation (well-known tool in

computer graphics) for processing the ground surface: Put a cloth sheet over the upside-down point cloud, and the ground surface will be the surface of the sheet driven by the gravity and the collisions by the point of the cloud.

[13] compares different filtering methods. [10] presents details of several methods. [6] studies the possibility of the GPU-based acceleration of the LiDAR point cloud filtering methods.

2.2 Selecting the Lowest Point

The principle of our suggested method is that most of the points of a LiDAR point cloud are located higher or close to the ground surface. Or the assumption is that the theoretical surface representing the ground surface is located in such a position where only a small portion of the points remains under it. This robust method can determine the ground surface without filtering.

First, the lowest point of the selected section (for example, where the vertical distance from a position is lower than a radius) of the point cloud can be the central point of this part. The point, which is at a higher position than a certain proportion of points (for example 1 or 10 percent), can be used instead of the lowest point, which means the wrong points may be filtered this way.

If this operation is done in different positions, such as at the nodes of a DEM, the digital elevation model can be created. The elevation will be the lowest elevation of the chosen area that has an unfavorable effect on a sloping terrain.

2.3 Plane (Disc) Fitting

The disadvantage of the last method (selecting the lowest point) presented above is that it derives from the terrain with a horizontal plane, and it is not the true level. Therefore, the resulted elevation does not characterize the center of the area but, rather, the elevation of the border of the area (if the small portion of the points are under the horizontal plane). To overcome this, an oblique plane can be used instead of a horizontal plane.

The horizontal plane can be defined by unique parameter (the elevation of the plane), and this one parameter can be determined by one condition: defined portion of the points will be under the plane. The oblique plane needs three conditions because it has three parameters (for example, the elevation in a position and the slope in x and y direction). The area can be divided into three parts, and by applying three conditions to these three parts we can define a portion of points under the oblique plane.

The circle-shaped area (the disc) can be divided with three half-lines started from the center and, as a result, three sectors are created (Figure 1). The three parameters of the fitted plane may be the elevations of the centers of the sectors,

and these points are called control points (other parameters of the plane can be calculated from this data-set).

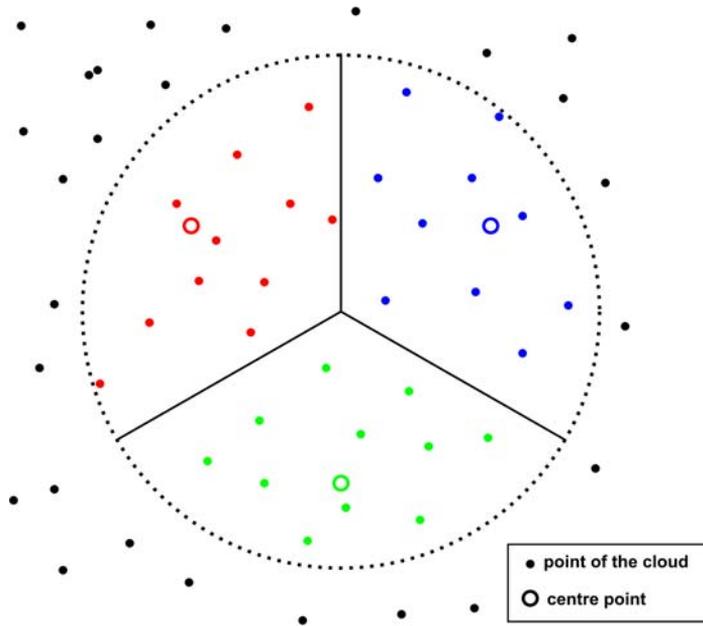


Figure 1

The points collected from the analyzed position (the area of the disc) are divided into three sectors. These points belonging to different sectors are marked by different colors. The center of each sector is represented by a small circle.

In the initial step of the plane fitting, the elevations of the sector centers will be determined as a portion (denoted by q , that means percent) of the points of this sector laying under this elevation. The q means quantile. Furthermore, the process is circulating around the sectors and it modifies the control point of the sector, according to the q portion of the points of the sector laying under the plane defined by this modified control point and the control points of the other two sectors. This step is repeated until all three sectors have q portion of the points under the plane without changing the elevation of the control points.

This method determines not only the elevation in a position (at the center of the disc), but also the slopes of the terrain are determined by the plane of the fitting disc. The result of the operation may be the elevation of the analyzed horizontal position and the slopes in x and y directions.

The principle of the method is demonstrated in the Figure 2, as a two-dimensional example.

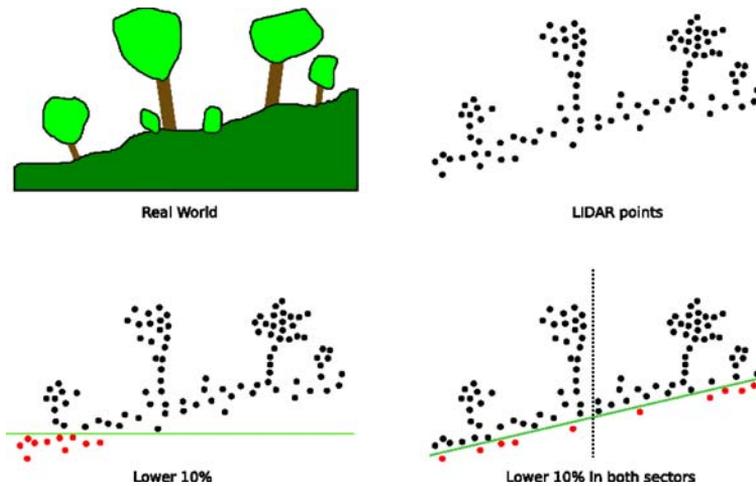


Figure 2

The principle of the method in two dimensions. The upper left picture symbolizes the real world, and the right upper picture is the LiDAR point cloud made from this terrain. In the left lower picture, a horizontal plane (here it is a line in 2D) is located over 10 percent of the points. The right lower picture shows the fitting disc method in two dimensions. The oblique plane (line) is located over 10 percent of the points in both sectors divided by a dashed line. The majority of the points are forming the trees and bushes, but the line fits to the ground surface.

2.4 Limitations

This method is based on the assumption that, as a good approximation, the terrain is flat (oblique) and close to the analyzed position. This is a better model than a terrain with a horizontal plane, but it is not too good if the terrain surface has significant curvature in the analyzed area. If the size of the area is small (the radius of the disc is small), the plane approximation will be good, but the points will form a sparse point cloud.

3 Implementation of the Proposed Method

The presented algorithm is implemented as a Python 3 application. One of the functions of the `fitdisc` module can read point cloud data from a simple text file to a point cloud object, and another function can determine the fitting disc to the point cloud at a specified horizontal position.

Inside the fitting disc function, the program uses a local coordinate system. The points are transformed because the origin of this local system is the analyzed point; the coordinates are scaled after the translation because the radius of the area

(the radius of the disc) in the local system equals 1. The equation of the transformation, where the uppercase letters (X, Y) are the original, while the lowercase letters (x, y) are the local coordinates:

$$\begin{aligned}x &= \frac{X - X_0}{R} \\y &= \frac{Y - Y_0}{R} \\z &= Z\end{aligned}\tag{1}$$

The X_0 and Y_0 are the coordinates of the analyzed horizontal position, and the R is the radius of the disc. Before applying this transformation, the program selects the points where the elevation is not changed. The program uses the original elevations for the plane fitting. Therefore, the elevation, as a result, is also calculated in the original elevation coordinate system.

The plane of the fitting disc is defined in several ways. One possibility is an equation in a local system:

$$z = z_0 + ax + by\tag{2}$$

The z_0 is the elevation of the plane in the origin (the center of the disc) of the local coordinate system, which matches the analyzed position; a and b are the slopes of the plane in the local system. The original slopes can be calculated, if these numbers are divided by R . If z_0 , a and b are known, the elevation of the plane can be calculated in an arbitrary x, y position; for example, in the horizontal positions of the points of the point cloud, these elevations may be compared to the elevation of the point to decide whether the point is under or over the plane.

Another way to define the plane is when there are three points (that do not lie on a common line) of the plane, which are defined. These points may be the centers of the sectors. The horizontal position of these points is constant, so the plane can be defined by the elevations of these points, which are denoted by z_{w0} , z_{w1} and z_{w2} . These three values are more advantageous in the plane-fitting method because one of these values influences more the plane position in its own sector than in the other two sectors. These values are denoted z_{wi} , where i is the number of the sector, an integer value between zero and two.

The parameters of the (2) can be calculated from the elevations of the center points:

$$\begin{aligned}z_0 &= \frac{z_{w0} + z_{w1} + z_{w2}}{3} \\a &= \frac{\sqrt{3}(z_{w2} - z_{w0})}{2} \\b &= \frac{\sqrt{3}(z_{w1} - z_{w0})}{2}\end{aligned}\tag{3}$$

The reverse calculation is simpler, because only (2) is used in the centers of the areas:

$$\begin{aligned} z_{w0} &= z_0 - \frac{\sqrt{3}a}{3} - \frac{b}{3} \\ z_{w1} &= z_0 + \frac{2b}{3} \\ z_{w2} &= z_0 - \frac{\sqrt{3}a}{3} + \frac{b}{3} \end{aligned} \quad (4)$$

In the initial step, the program determines the z_{w0} , z_{w1} and parameters. These values will be the elevations that are situated over the q portion of the points of the point cloud in the sectors around the center points.

In the next step, the z_{wi} value (i is initially zero, and in the further steps it will be $(i+1) \bmod 3$) must be modified, so that q portion of the points will be under the plane determined by z_{w0} , z_{w1} and z_{w2} in the sector number i . The $z_{w[i\pm 1 \bmod 3]}$ values are not modified, and the points of sectors number $(i\pm 1) \bmod 3$ are not analyzed in this step. The new value of z_{wi} will be an integer multiplied with t value (the default value in the program is 0.01). This step is repeated until z_{wi} is not changed in three consecutive steps. The fitting disc is calculated, finally from the z_{w0} , z_{w1} and z_{w2} parameters. The z_0 , a and b values can be calculated by (3). The z_0 is the elevation in the analyzed position, and slope can be calculated from a and b by the $\frac{a}{R}$ and $\frac{b}{R}$ formulas.

The program separates three (not two) categories when it analyzes the situation related to the plane. If a point is near the plane (closer than $1.6t$), this point will be ranked to the nearby category.

The t is the resolution of the elevation, the default $t=0.01$ means the centimeter resolution. (the z_{wi} values are integer multiple of t .)

The nearby category is necessary to avoid the infinite loop. The multiplication by 1.6 is needed for the points from the side of the disc, because the elevation of the disc's plane is changing about 1.6 times in this place rather than in the center of the sector. Other points will be ranked, obviously, to the under and to the over categories. The program counts the number of the points in under (n_{-1}), nearby (n_0) and over (n_{+1}) categories. The condition defined by a q value is true if the portion of the under category is less or equal than q and the portion of under and nearby points is greater or equal than q :

$$\frac{n_{-1}}{n_{-1}+n_0+n_{+1}} \leq q \leq \frac{n_{-1}+n_0}{n_{-1}+n_0+n_{+1}} = 1 - \frac{n_{+1}}{n_{-1}+n_0+n_{+1}} \quad (5)$$

If this condition is not true, the z_{wi} value must be increased (when portion of the under category is greater than q), or decreased (when portion of the under and nearby category is less than q). The initial value of the modification is t , and it will be doubled until the category is nearby or opposite to the initials. In the latter case,

the nearby position will be searched by the bisection method between the last two values.

The program can calculate the elevations in each node of a grid, and the created digital elevation model can be written to ArcInfo ASCII GRID format. The slopes may be calculated and exported similarly.

4 Examination of the Parameters

The fitting disc method needs two parameters. One of these is the radius of the disc (R). The other parameter is the q , the portion of the points that are under or nearby the plane in every sector.

Increasing the value of R the number of points is growing. More points provide better fitting and allow to use less q values, but the smaller details of the terrain cannot be evaluated, because the obtained value characterizes a greater area. The R value may be determined dynamically as a smaller radius around the analyzed position when every sector has at least N points from the processed point cloud.

Different q values make different surfaces. If the q is greater, the elevation is higher because more points must be located under the surface. The distances between the surfaces generated by different q values may be a feature of the point cloud. If the points of the cloud are diffused (for example in a wooden area), this value will be greater, and in other areas (fields or artificial surfaces) may be less.

5 Possible Applications of the Proposed Method

The presented method is used primarily to create digital elevation models from LiDAR data. The advantage of this method is that it can eliminate the effect of the points created around various natural or artificial objects with suitable parameters, if enough points were created in the ground surface.

The differences between the results are calculated with different parameters relating to the distribution of the points of the cloud near the analyzed position, and these values may be used similarly to multispectral data to interpret the land cover.

5.1 Creating DEM

The presented method can calculate an elevation from the point cloud in any horizontal position. These positions may be the points of a grid, and a digital elevation model can be created. Raster files can be created from other values of

this method, such as slope and aspect data. Figure 3 shows some DEM generated by the described algorithm from LiDAR data with different R and q parameters.

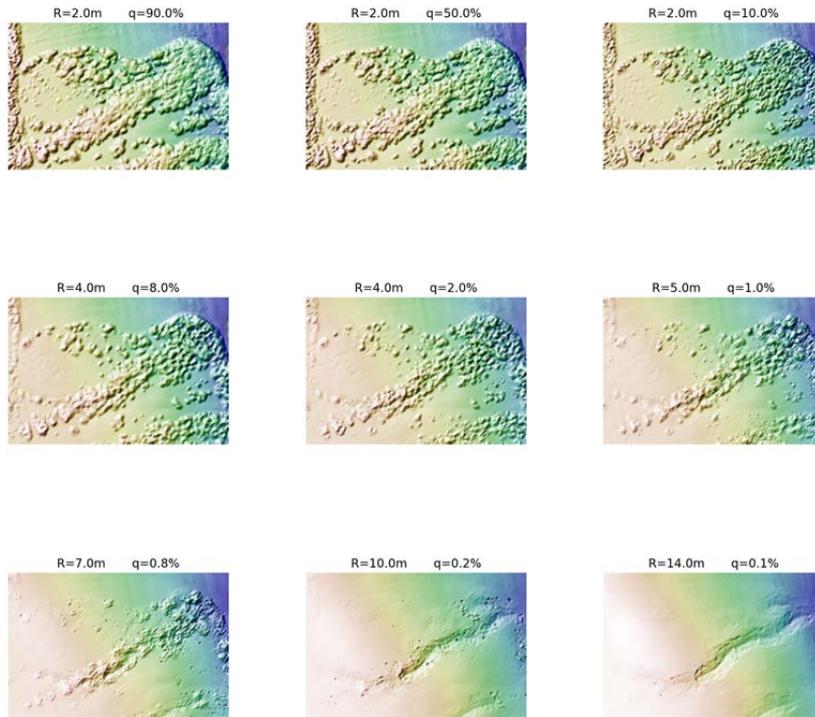


Figure 3.

Different elevation models created by different q parameters. The $R=10m$. The impact of the trees and bushes is less with lower q value.

TIN (Triangulates Irregular Network) models may be created by a method similar to method described in [5]. Another TIN generation method was described in [1].

5.2 Examination of the Ground Surface in Woodland Area

[11] and [8] analyzed the problem of the LiDAR data processing in woodland areas. [5] called this function as Virtual DeForestration (VDF). [16] described an additional method for processing LiDAR data in a woodland area.

The method presented in this article is suitable for this task with appropriate parameters: sufficiently low q and enough large R to the sectors has enough points (the points count is proportional to R^2). Of course, this method is not universal. If the vegetation is too dense, and points were not created in the ground surface, the

method cannot determine the elevation of the terrain (more correctly, a wrong elevation is calculated).

The presented method may be used for separation of the woodland areas. The different surfaces created by different q values may be useful for this work.

5.3 Recognizing Artificial Objects

In the urban areas, many points are created on the roofs of the buildings. The roof usually contains continuous and plane parts. The roofs are similar to the terrain surface, as points under these surfaces are not created because the laser beam cannot pass through them.

The presented method may be useful to recognize and evaluate roofs because it can provide a spatial position of a fitting disc, not only an elevation. The closer points may be grouped to a roof element, if the fitting discs match together. Another useful feature of this method is that this fitting disc process can be calculated in any vertical position. The edges of the roof elements may be interpreted smoother, if these advantages will be utilized.

The road surfaces can be recognized by a fitting disc that has a bit lesser R than the half of the width of the road. [7] and [15] demonstrated some their algorithm for road surface processing from LiDAR data.

6 Study of the Results

One LiDAR dataset was processed by different software applications to study the results. We made a surveying in the area of the LiDAR survey.

6.1 The LiDAR Survey

The LiDAR survey was made in May 2008, in the area of Iszkaszentgyörgy, close to Székesfehérvár, by the TELECOPTER Company. This survey project resulted in a true orthophoto from this area, which is used for samples for some figures of this article. The area covers six by four kilometers in E18.2280-18.3085 and N47.2130-47.2495. The flight altitude was 1400 meters (4593 ft).

The presented method, and the other processing software, use the last echo points. The maker guaranteed 0.15 meter as the maximum height error.

6.2 Surveying

One part of the LiDAR survey was surveyed by geodetic technology. This is about 25 hectares around the N47.2370 E18.2560 point. We surveyed 195 points in all types of vegetation (field, bushes and woods).

We used Leica TC 407 total station and Sokkia Stratus GNSS station in the survey. All of the points were surveyed by the total station, the GNSS observations were used to measure the positions of the station. We carried out the survey in August 2015.

The result of the surveying will be the reference in later analyzes. The processed LiDAR data will be compared to this surveying.

6.3 LiDAR Data Processing by Other Methods

The last echo LiDAR data was processed by GRASS [4]. As a first step, the point cloud was filtered by the `v.outlier` command. In the next step, we used the `v.lidar.edgedetection` command to search edges in the point cloud and, after that, we applied the `v.lidar.growing` and `v.lidar.correction` commands. In the final step, a surface was created by the `v.surf.bspline` command. We created another surface by the `v.surf.bspline` command from the original, unfiltered last echo point cloud.

The LiDAR data had been processed by the TopoSys software previously. This data was provided by the maker of the LiDAR survey. One of this data sets is the Digital Terrain Model (DTM) file, which was calculated from the last echo points. This file contains empty values where the ground surface was not determined reliably. In the FDTM (Filled DTM) file, these holes are filled in starting from the neighboring known values.

6.4 The Result of the Studies

The elevations were calculated in the horizontal positions of the surveyed points by bilinear interpolation from the data of the analyzed elevation models (denoted H_{DEM}), and these values were compared to the elevation of the surveying (denoted H_{GEOD}). We calculate the mean, the median and the

deviation of the difference ($H_{DEM}-H_{GEOD}$), and we also calculated the mean of the absolute value of the differences ($\frac{\sum |H_{DEM}-H_{GEOD}|}{n}$), and square mean of the differences ($\sqrt{\frac{\sum (H_{DEM}-H_{GEOD})^2}{n}}$).

A part of points (37 points) are in the holes of the TopoSys DTM data, the analysis can be done at the other 158 points. In the other models, the values can be calculated from all of the 195 surveyed points.

In the analysis of the fitting disc method, we calculated the elevations in the horizontal positions of the surveyed points (this method can calculate an elevation in any horizontal position), and these elevations are compared to the surveyed elevations. The analysis was done with more R and q parameter combinations.

In the higher q values, the mean of the differences is positive, and in the lower q values it changes into negative value. This ensures the principle of the method: when only few points must be under the plane, the plane is located lower. The trend of the medians of the differences is the same as it is with the mean values.

The standard deviations of the differences are lower with lower q parameters. The minimal value depends on the R parameter.

The trends of the mean of the absolute values of the differences and the square mean of the differences are similar to the standard deviations of the differences.

After trying different parameter combinations, the best value of the mean of the absolute values of the differences is 0.174 where $R=3.67$ and $q=0.015$. It is better than the surface created by GRASS (0.904), and it is only less unfavorable than the FDTM surface of the TopoSys software (0.166) but the fitting disc method did not use filtering. The values of TopoSys FDTM and the fitting disc method are near to the error of the LiDAR survey (0.15 meter).

The differences are depending on the vegetation of the neighboring area with analyzed points. This is expressed by a value, which is calculated by the difference of elevations with $q=0.95$ and $q=0.053$ where $R=4m$, and this value may be called as the thickness of the point cloud.

The values of the Table 1. were calculated separately from the points where the thickness of the point cloud is thinner than 40 centimeters. The part thinner than 40 cm has 98 points, and the part thicker than 40 cm has 97 points. All of the 37 points, which are in the holes of the DTM data are located in the thicker area.

Table 1

The comparison of the elevation from fitting disc method with different R and q values and other software. The values were calculated in different areas depending on the thickness of the point cloud.

thickness	method	GRASS		TopoSys		Fitting disc method				
		original	filtered	DTM	FDTM	$R=2m$ $q=0.09$	$R=4m$ $q=0.016$	$R=5.19m$ $q=0.014$	$R=10.37$ $q=0.0107$	
< 40 cm	mean	0.058	0.060	0.042	0.055	0.005	-0.006	-0.036	-0.054	-0.117
	median	0.062	0.061	0.048	0.048	0.002	-0.007	-0.037	-0.047	-0.094
	standard deviation	0.079	0.070	0.158	0.070	0.055	0.056	0.061	0.080	0.145
	mean of abs. diff.	0.080	0.076	0.085	0.072	0.044	0.045	0.055	0.071	0.131
	sq.mean of diff	0.097	0.092	0.162	0.162	0.055	0.056	0.070	0.097	0.186
> 40 cm	mean	1.467	0.683	0.064	0.091	0.471	0.281	-0.007	-0.039	-0.117
	median	0.858	0.371	0.090	0.094	0.071	0.054	-0.003	-0.012	-0.062
	standard deviation	1.710	1.086	0.170	0.199	1.154	0.966	0.422	0.229	0.253
	mean of abs. diff.	1.507	0.718	0.126	0.138	0.517	0.366	0.164	0.116	0.151
	sq.mean of diff	2.247	1.278	0.181	0.218	1.240	1.002	0.420	0.231	0.277

The fitting disc method can provide better or, in some cases, only a bit worse result than other methods; it depends the values of the parameters (q and R). The method is especially good in the mean values.

The presented method can provide elevation data from the LiDAR point clouds of similar quality to the other analyzed methods with ideal parameters. The key issue is the correct adjustment of the R and q parameters, which are dependent on the characteristic of the terrain surface. The dense vegetation needs low q value and the low q value needs big R value for enough points, but the big R value is very good for minor details of the ground surface.

Conclusions and Future Work

The fitting disc method is an effective and robust solution for processing LiDAR point clouds. An elevation model can be made by this algorithm in any optional resolution because the fitting disc method can calculate an elevation in any horizontal position (not only the nodes of a raster grid). The method can work even without any filtering process because some wrong points do not influence the result.

The method can be adapted to the characteristic of the terrain by varying the parameters. The choice of the optimal parameters needs additional research. The choice is not a constant pair of R and q values; rather, it will be a method that can determine the parameters from the point cloud data.

The differences of the results, which were calculated from different parameters, may provide useful information of the surveyed ground surface (for example, the thickness, in the analysis, in the Table 1). These values can help to search for the optimal parameters to create a reliable elevation model.

The practical application needs more research for the parameters of the method, and combine the method with filter algorithms. The study of the method in other areas and other measurements (different instruments, flight altitude, etc.) is also required.

The principle of the fitting disc method may be used with polynomial surfaces. The area must be divided into many parts (similar to the sectors in the fitting disc method) as many parameters determine the surface. The parameters of the polynomial surface can be calculated from the elevations of the center points of the parts. The elevations of the center points may be modified in sequence to q portion of the points of the part lying under the surface, until this, criterion does not need modification in all of the parts. The fitting disc method is a special case of this general principle.

The fitting disc method may be regarded as a two-dimensional interpolation method, where the interpolated value is the elevation. The principle is applicable in higher dimension spaces as well. The two-dimensional space must be divided

into three sectors, the N -dimensional space must be divided into $N+1$ parts because a linear function has $N+1$ parameters in an N -dimensional space.

The test programs were made in Python 3 language because the ideas invented during the research were implemented easier this way. The final algorithms may be implemented in a low-level programming language (for example C or C++), and this program would be more effective. One important advantage of this method is an excellent parallel calculation because many fitting disc processes can be run at the same time.

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