Application of Pseudo-Analysis in the Synchronization of Container Terminal Equipment Operation

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Abstract: Based on the global increase in the volume of containerized goods, there is an emerging need for analysis and improvement in all aspects of the logistic chain of container transport. Mathematical description of equipment operation in container terminal is of key importance for understanding complex container terminals. This paper presents an innovative concept in the modeling of a container reloading operation inside a container terminal and its equipment based on pseudo-analysis in the form of max-plus semiring. The presented synchronization of the equipment operation (quay container cranes and automated guided vehicles, shortly AGV) during container unloading leads to minimization of the required number of automated guided vehicles. The presented method is demonstrated in a numerical example.

Keywords: container terminal; automated guided vehicles (AGV); max-plus semiring; pseudo-analysis; synchronization of equipment

1 Introduction

Operation studies and description of container terminals, as elements of large logistic networks and therefore the key factor within their parent system of SCM (Supply Chain Management), represent a complex problem (see papers: [4], [11], [12], [33]). In addition, with the aim of increasing the accuracy of results, each part of the system should be analyzed separately, paying attention to the preservation of the general approach of the study.

The economic effects of improvements in container terminal equipment operation can be recognized by the transshipment price, which amounts to approximately 50 € per TEU. If one observes a smaller seaport or average size riverport with 100,000 TEUs, increasing the effect with the organization by only 2% brings 100,000 € more, which is not negligible. The monograph [12] presents a number of papers that deal with the subject of container terminal organization as a whole, as well as separate subsystems within these terminals. Also, several examples analyze the problem of creating the terminal concept presented on an actual example of a new terminal within ECT (Europe Container Terminal, Rotterdam). A mathematical description of the equipment operation in the container terminal is of key importance for understanding complex container terminals. However, not one paper (to the author's knowledge) has analyzed the reduction of the number of automated guided vehicles (AGVs) on the basis of the relation between the manner the container quay crane unloads containers and the distance of storage area parts where unloaded containers should be stored.

Based on the semiring structure (see [14]) developed in [21], [23], the so-called pseudo-analysis in an analogous way as classical analysis introduced $\oplus$ measure, pseudo integral, pseudo-convolution, pseudo-Laplace transform, etc. The pseudo-linear superposition principle resulted from the pseudo-analysis [13], [16], [21], where, roughly speaking, instead of the field of real numbers a semiring on the real interval is taken $[a,b] \subset [-\infty,+\infty]$, denoting the corresponding operations as $\oplus$ (pseudo-addition) and $\odot$ (pseudo-multiplication). It is successfully applied as a universal mathematical method in many fields, e.g., fuzzy systems, decision making, optimization theory, differential equations, etc.

The change in the classical linear system can often be shown in the following way:

$$x(t+1) = Ax(t), \qquad t = 0, 1, 2, \ldots,$$

where the vector x represents the state of a model, and this state evolves in time according to this equation; and $x(t)$ denotes the state in time t. The symbol A denotes the real $n \times n$ matrix (system transformation matrix).

The proceeding equation can be written in the following form:

$$x_i(t+1) = \sum_{j=1}^{n} A_{ij}\, x_j(t), \qquad i = 1,2,\ldots,n, \quad t = 0,1,\ldots$$

Depending on the technical performances of equipment in the terminal, different transformation matrices A can be determined.

Different kinds of nonlinear equations are treated in mathematics with various tools [1], [2], [6], [7], [16], [17], [20], [22], [25], [26], [28]. In the same way, max-plus algebra found its application in many areas, such as: production, traffic, communication, and image coding (see papers [1], [19]). In analogy with the approach in papers [1], [27] and [28], we shall use the following model. Now, we suppose that multiplication becomes the common addition and addition becomes the maximization. Then the corresponding equation in relation to the preceding equation is given by:

$$x_i(t+1) = \bigoplus_{j=1}^{n} A_{ij} \odot x_j(t), \qquad i=1,2,...,n, \quad t=0,1,... \,, \tag{1}$$

where: $\oplus = \max$ and $\odot = +$. The proceeding equation can be written in the following form:

$$x_i(t+1) = \max_{1\le j\le n} (A_{ij} + x_j(t)), \qquad i=1,2,....,n, \quad t=0,1,... \quad .$$

We shall use the following shorter notation for the preceding equation:

$$x(t+1) = A \odot x(t), \quad t = 0, 1, 2,... \tag{2}$$

In the case when the initial stage x_0 is known, system transformation matrix A preceding equations model deterministic systems.

After the introduction, the second section of the paper presents some necessary notions of pseudo-analysis. The third section provides description of the problem of the minimal number of required AGV. The fourth section gives two theorems related to the unloading of one bay. The fifth section is devoted to a synchronization model and numerical example. The sixth section covers some important related remarks. The main conclusions from this paper are given in the end.

2 Pseudo-Analysis

Pseudo-analysis is a generalization of classical analysis, where instead of the field of real numbers, a semiring is taken on a real interval $[a, b] \subset [-\infty, +\infty]$ endowed with pseudo-addition $\oplus$ and with pseudo-multiplication $\odot$, see [9], [12], [14], [16], [21], [22], [24], [29], [30].

Let $[a, b] \subset [-\infty, +\infty]$.

Pseudo-addition is a function $\oplus$: $[a, b] \times [a, b] \to [a, b]$ which is commutative, nondecreasing, associative and has a zero element, denoted by **0**.

Let $[a, b]_+ = \{x : x \in [a, b], x \ge \mathbf{0}\}$.

Pseudo-multiplication is a function $\odot$: $[a, b] \times [a, b] \to [a, b]$ which is commutative, positively nondecreasing, i.e. $x \le y$ implies

$x \odot z \le y \odot z$, $z \in [a, b]_+$; associative and for which there exists a unit element $\mathbf{1} \in [a, b]$, i.e., for each $x \in [a, b]$; $\mathbf{1} \odot x = x$. We further suppose, that $\mathbf{0} \odot x = \mathbf{0}$ and that $\odot$ is a distributive pseudo-multiplication with respect to $\oplus$, i.e.,

$$x \odot (y \oplus z) = (x \odot y) \oplus (x \odot z).$$

The structure ($[a, b]$, $\oplus$, $\odot$) is called a semiring; see [14], [16], [21]. Special cases of real semirings are investigated in papers [7], [16], [27]. In this paper, we shall use the case $\oplus = \max$ and $\odot = +$ on the interval $[-\infty, +\infty]$; see [1], [2], [7], [16], [27]. In usual real algebra we have $1 \cdot x = x$ and $0 + x = x$, but in max-plus algebra we have $0 \odot x = x$ and $-\infty \oplus x = x$, so the neutral elements for pseudo-multiplication and pseudo-addition are 0 and $-\infty$, respectively. We will denote $\mathbf{1} = 0$ and $\mathbf{0} = -\infty$. The condition $0 \odot x = 0$ ensures the convention $-\infty + \infty = -\infty$. Several other properties of pseudo-addition and pseudo-multiplication and their consequences are discussed and illustrated in paper [3].

Remark. If we compare the properties of $\oplus$ and $\odot$ with the usual operations of addition and multiplication we see that:

(i) We have lost the property: for a given a there exists only one element b such that $a + b = 0$, because for $a \neq -\infty$ we have $\max(a, b) \neq -\infty$;

(ii) We have gained the idempotency of addition;

(iii) There are no zero divisors in ($[-\infty, +\infty]$, max, +), because

$a \oplus b = -\infty \quad \Rightarrow a = -\infty$ and $b = -\infty$;

(iv) The maximum operation is no longer cancellative, because $\max(a, b) = b$ does not imply $a = -\infty$.

3 Description of the Problem

Today, over 60% of the world's deep-sea cargo is transported in containers, while some routes, especially between economically strong and stable countries, are containerized up to 100% (industrial products) [31]. Container applications, or containerization of cargo flows, has required research in the domain of organization, particularly for reloading and storing in terminals.

Container terminals constantly search for new techniques, such as automated transportation systems, and new ways of planning and control. Therefore, new planning and control concepts need to be developed for the various systems in the container terminal [34]; see also traffic models [32].There are a number of papers that analyze crane scheduling problems and overview of existing literature is presented in paper [4]. However, not one analyzes the influence of crane scheduling on the number of required AGVs necessary for the continual flow of transshipment.

Many papers analyze systems with AGVs. In paper [34], Vis discusses literature related to design and control issues of AGV systems in manufacturing, distribution, transshipment and transportation systems. In paper [36], the authors describe the development of the minimum flow algorithm to determine the number of AGVs required at semi-automated container terminals. The problem the authors analyzed in [5] was to assign each container to a yard location and dispatch vehicles to the containers so as to minimize the time required to unload all the containers from the ship. In paper [8] the goal is to determine the best allocation of resources in the yard with the objective of minimizing the costs of the terminal. The comparison of vehicle types at an automated container terminal is presented in paper [35].

In paper [10], the authors propose a heuristics search algorithm called the greedy randomized adaptive search procedure for constructing a schedule of quay cranes in a way of minimizing the makespan and considering the interference among yard cranes, but without analyzing the number of trailers. In paper [33], the authors model the seaport system with the objective of determining the storage strategy for various containers –the handling schedules. “Multi-crane oriented” is a scheduling method that yard trailers can be shared by different quay cranes. This method is presented in paper [37]. In paper [15], the objective is to minimize the weighted sum of the total delay of requests and the total travel time of yard trucks. The problem of minimizing the time taken to load the containers from the container stack yard onto the ship is presented in paper [18].

Not one paper analyzed the number of minimum required AGVs on the basis of the relation between the manner the container quay crane unloads containers and the distance of different blocks inside the container yard where unloaded containers should be stored. This paper shall deal with synchronization of equipment operation in the container terminal with the help of pseudo-analysis. Similarly, modeling on the basis of stations (in this case, the locations for transshipment of containers), as in papers [1], [27], [28], $\oplus$ (pseudo-addition) and $\odot$ (pseudo-multiplication) shall be used as operators. Containers are unloaded and loaded at locations intended for this purpose, so these locations can be regarded as stations (in analogy with railway stations in [1], [28]).

The basic division of container quay cranes is into:

- Cranes intended for operation in river terminals;
- Cranes intended for operation in sea terminals.

In this paper we analyze cases when container quay cranes do not service the container yard. Such utilization of these cranes is characteristic for all cranes that operate in sea terminals and for some that operate in river terminals. Therefore, in this case, the crane unloads the containers to the shore and further container manipulation is continued with AGVs, straddle carriers, reach stackers and forklifts. As the most expensive and complex equipment in the terminal, container quay cranes determine the operation of other equipment. The price of container quay cranes ranges between 6 and 10 million euros. This paper observes the system of container transshipment by a container quay crane, after which the containers are transferred to the container storage yard by using AGVs. Further manipulation in the container yard is done by yard cranes (This is the most common case in sea terminals). Therefore, AGVs transport containers between the shore and the container yard. One of the automated guided vehicles' characteristics is that other equipment (cranes) is necessary in order to load containers onto or unload containers from AGVs.

Container terminal is shown in Figure 1.

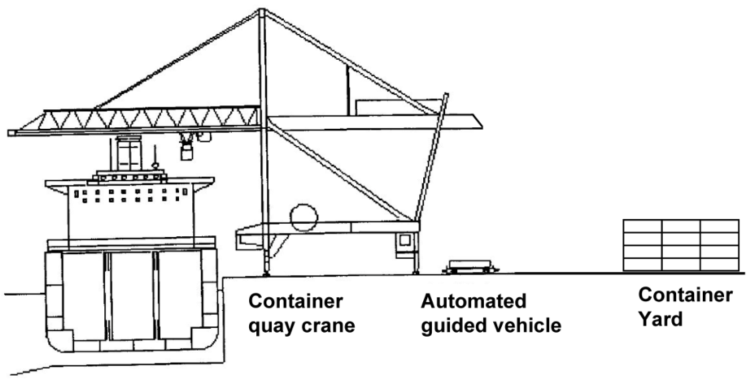

Figure 1
Container terminal

In this paper we shall analyze the relation between the container quay crane and the number of required AGVs depending on the distances of different parts of container yard. The methodology to be shown can lead to a reduction in the necessary number of AGVs for performing the same work. In this paper, it shall be proved that the manner of synchronization of the stated equipment operation shall not affect the time of transshipment and operation of the container quay crane but shall only affect the minimization of the necessary number of AGVs. In order to illustrate the methodology from the next section, a numerical example is given.

4 Unloading of One Bay

The ship has a number of container stacking compartments called bays, and only one crane can work on a bay at the same time (Figure 2). Thus, the crane most often reloads the ship by reloading one bay at a time. In this paper, it is implied that container quay crane first reloads all containers from one bay and then moves to the next bay. On the basis of the aforementioned, unloading of one bay is analyzed.

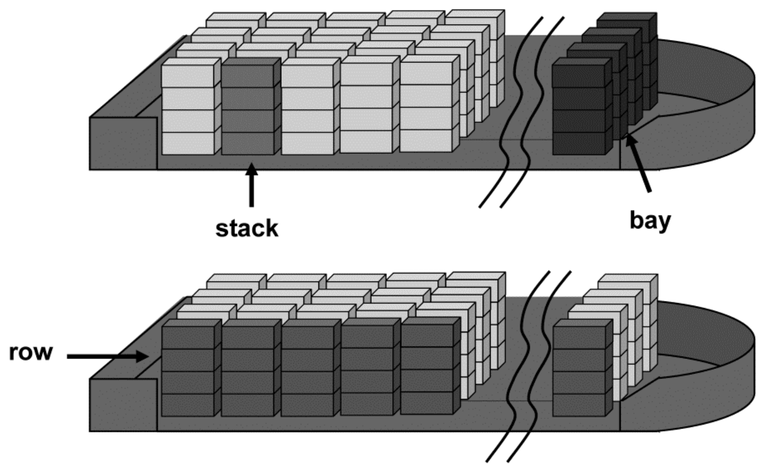

Figure 2
Container ship, bay

We shall prove that there is no difference if a container quay crane unloads containers from a bay "one after the other" or "in any other way".

Theorem 1: *If a container quay crane unloads containers from a bay "one after the other" and if container quay crane unloads containers from a bay "in any other way", time of unloading is the same.*

Proof: We stress that "one after the other" implies that all containers from the first stack are unloaded first and then all containers from the second stack, and so on, until the last container from the last stack is unloaded.

We stress that "in any other way" implies ways that do not lead to unnecessary increase of the number of transshipped containers (e.g., second container from the first stack cannot be transshipped if the first container from the first stack is not transshipped. This is presented in Figure 4 in the following manner: u_2 is unloaded, and then u_1 is unloaded – if such thing occurs, it would lead to an unnecessary increase in the number of transshipped containers, i.e., it would lead to an unnecessary increase in rehandled containers because u_1 then behaves as a rehandled container).

If $l_i \quad , i = 1,2,3....,n$ denotes the distance of each container (which is to be unloaded) from location S_1 (the location where the container quay crane loads containers onto the AGVs), the sum of all distances remains the same

$$2(l_1 + l_2 + l_3 + ... + l_n) = l_{uk},$$

until unloading which shall increase the number of traveled distances (i.e., until the unnecessary number of transshipped containers) occurs. This means that the time of unloading shall remain the same until unloading, which will unnecessarily increase the number of transshipped containers occurs. In other words: the time of unloading the ship does not depend on the order of unloading containers.

In the next theorem we stress the connection of the order of unloading containers with the number of AGVs.

Theorem 2: *Order of unloading containers affects currently required number of AGVs.*

Proof: Let us assume that only the order of unloading of containers which shall not unnecessarily increase the number of transshipped containers is analyzed. If $r_i, \quad i = 1,2,3..., n$ denotes the distance from location S_1 to the container yard location $S_j \quad j = 2,3,4..., m$ the total sum of traveled distances from S_1 to S_j is given by

$$2(r_1 + r_2 + r_3 + ... + r_n) = r_{uk}.$$

The total amount of traveled distances of containers from their location on a ship to the container yard location is given by

$$2(l_1 + r_1 + l_2 + r_2 + l_3 + r_3 + ... + l_n + r_n) = l_{uk} + r_{uk} .$$

The way of forming the sum plays a key role. If we specify that

$$2(l_1 + r_1) = c_1; \quad 2(l_2 + r_2) = c_2; \quad 2(l_3 + r_3) = c_3; \quad ... 2(l_n + r_n) = c_n,$$

the total sum of traveled distances is

$$c_1 + c_2 + c_3 + ... + c_n = c_{uk} .$$

Since $c_i + c_{i+1}$ does not have to be equal to $c_i + c_{i+2}$, the distances AGVs must travel during these transshipments do not have to be equal. It happens that the AGVs return to station S_1 before schedule, which again makes them ready for transshipment; i.e., the order of unloading containers affects the currently required number of AGVs.

The preceding two theorems indicate that order of unloading containers from the ship directly affects the required number of AGVs without an influence on container quay crane, i.e., on time required for transshipment.

5 A Model of Synchronization

The model in Figure 3 represents the system that consists of the container quay crane, the AGVs and the yard cranes. In this paper, the model of the synchronization of the equipment operation in an example of unloading a ship is presented . S_i , $i = 1,2,3,...$ are reloading locations, such as: S_1 - location on the shore where the container quay crane loads the containers onto the automated guided vehicles; S_2 , S_3 , S_4 , S_5 -different blocks inside the container terminal.

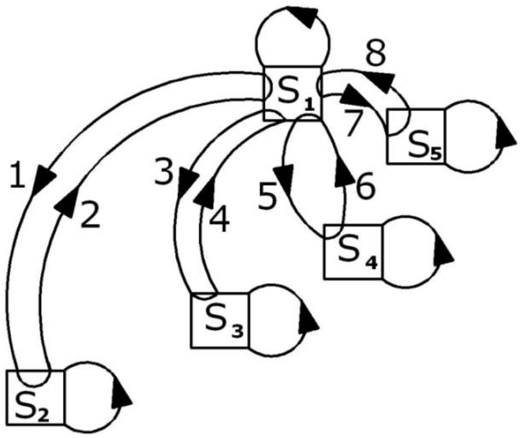

Figure 3

Model with five locations for loading and unloading of containers

Adopted assumptions:

- As aforementioned, container quay cranes are the most expensive and the largest machines in a container terminal and what is to be adopted here is that for each container the crane locations onto the shore there must exist a free AGV onto which the crane loads the container; i.e., the crane must not at any moment wait for the AGV.
- Since it is adopted in the example that only one quay crane transships the containers (in such way that the flow of containers is not large), the yard cranes are always prepared to unload the container from the AGV when it arrives and to transfer the container to container storage yard location.
- The AGV always travel the same distance within the same time.
- Each container on a ship has its pre-specified storage yard location.

A mathematical description of the presented system is given in the following form:

$$x_i(t+1) = \bigoplus_{j=1}^{n} A_{ij}(t) \odot x_j(t), \qquad i = 1,2,...,n, \quad t = 0,1,... \tag{3}$$

We shall use the following shorter notation for the preceding equation:

$$x(t+1) = A(t) \odot x(t)\,, \quad t = 0, 1, 2,... \tag{4}$$

Unlike expressions (1) and (2), expressions (3) and (4) have the transformation matrix $A(t)$ which depends on the time interval. If instead of time $t = 0, 1, 2,...,$ we observe events $k = 0, 1, 2,...$ that changed the system, and equation (4) can be written in the following form:

$$x(k+1) = A(k) \odot x(k)\,, \quad k = 0, 1, 2,... \tag{5}$$

$x(k)$ - marks the k - th departure time of the AGVs in direction i ,

$A(k)$ - system transformation matrix.

If we adopt that time intervals between locations (for container transshipment) are: $s_{12} = s_{21} = 60$; $s_{13} = s_{31} = 50$; $s_{14} = s_{41} = 20$; $s_{15} = s_{51} = 15$.

Then we can assume that container quay crane should unload the bay as in Fig. 4:

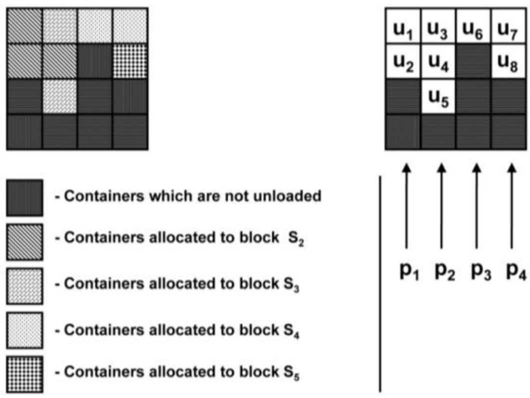

Figure 4
An example of a bay in a ship which is to be unloaded (side view)

Figure 4 displays a bay which is to be unloaded. Let the time units for unloading of containers in the bay be adopted:

$u_1 = 2$; $u_2 = 3$; $u_3 = 4$; $u_4 = 5$;
$u_5 = 6$; $u_6 = 7$; $u_7 = 8$; $u_8 = 9$;

It is stressed that all adopted values are given as an example only to show the methodology of synchronization.

For a crane to unload the first container (which is on top) from the first stack, $2+2$ time units are required (i.e., 2 time units from location S_1 to first container in the first stack and 2 time units from the first container in the first stack to location S_1). For a crane to unload the second container from the first stack, $3+3$ time units are required (i.e., 3 time units from location S_1 to the second container in the first stack and 3 time units from the second container in the first stack to location S_1).

Case 1 Analyse the unloading of container with the stacks order p1,p2,p3,p4 (Figure 4). This makes it clear that the departure times for containers from station S_1 are: 4,10,18,28,40,54,70,88. **Case 2** (order p4,p1,p2,p3) shows other departure times for containers from station S_1 (see Table 2). In order to obtain integer numbers in matrices, time units are used in this paper.

Let us assume that the bay presented in Figure 4 should be unloaded, with the existence of a system with AGVs and yard cranes (as in Figure 3). The description of the given system can be presented by max-plus equation:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ x_5(k+1) \\ x_6(k+1) \\ x_7(k+1) \\ x_8(k+1) \end{bmatrix} = \begin{bmatrix} s_{11}(k) & s_{21}(k) & \mathbf{0} & s_{31}(k) & \mathbf{0} & s_{41}(k) & \mathbf{0} & s_{51}(k) \\ s_{12}(k) & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & s_{21}(k) & s_{33}(k) & s_{31}(k) & \mathbf{0} & s_{41}(k) & \mathbf{0} & s_{51}(k) \\ \mathbf{0} & \mathbf{0} & s_{13}(k) & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & s_{21}(k) & \mathbf{0} & s_{31}(k) & s_{55}(k) & s_{41}(k) & \mathbf{0} & s_{51}(k) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & s_{14}(k) & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & s_{21}(k) & \mathbf{0} & s_{31}(k) & \mathbf{0} & s_{41}(k) & s_{77}(k) & s_{51}(k) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & s_{15}(k) & \mathbf{1} \end{bmatrix} \odot \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \\ x_7(k) \\ x_8(k) \end{bmatrix} \quad (6)$$

In the presented equation, s_{ij} represents the time required for the AGVs to travel from location S_i to location S_j, and in the case when $i = j$, s_{ij} represents activation time of location S_1 (this is the connection with transshipment done by container quay crane). The transformation matrix depends directly on the manner of transshipment and the way the locations S_i are connected.

If we observe the first AGV (order (p4, p1, p2, p3)), it can be concluded that the system will change four times.

$x(1) = A(0) \odot x(0)$; $x(2) = A(1) \odot x(1)$; $x(3) = A(2) \odot x(2)$;

$x(4) = A(3) \odot x(3)$; i.e.,

$$\begin{bmatrix}1\\1\\1\\1\\16\\1\\1\\1\end{bmatrix}=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&16&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{bmatrix}\odot\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix};\quad \begin{bmatrix}1\\1\\1\\1\\16\\36\\1\\1\end{bmatrix}=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&20&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{bmatrix}\odot\begin{bmatrix}1\\1\\1\\1\\16\\1\\1\\1\end{bmatrix};$$

$$\begin{bmatrix}1\\1\\1\\1\\56\\36\\1\\1\end{bmatrix}=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&20&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{bmatrix}\odot\begin{bmatrix}1\\1\\1\\1\\16\\36\\1\\1\end{bmatrix};\quad \begin{bmatrix}62\\1\\1\\1\\56\\36\\1\\1\end{bmatrix}=\begin{bmatrix}62&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{bmatrix}\odot\begin{bmatrix}1\\1\\1\\1\\56\\36\\1\\1\end{bmatrix}$$

By the application of the given mathematical model (equation (6)), the following results can be presented:

Due to easier track of changes of system state, the changes are given in tables.

Table 1
Results of numerical example (order p1,p2,p3,p4)

	Order of unloading of stacks (p1, p2, p3, p4)							
AGV	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	4	64	-	-	-	-	-	-
2	10	70	-	-	-	-	-	-
3	-	-	18	68	-	-	-	-
4	28	88	-	-	-	-	-	-
5	-	-	40	-	-	-	-	-
6	-	-	-	-	54	74	-	-
7	-	-	-	-	70	-	-	-
8	-	-	-	-	-	-	88	-

Table 2
Results of numerical example (order p4,p1,p2,p3)

AGV	Order of unloading of stacks (p4, p1, p2, p3)							
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	62	-	-	-	16	36	-	-
2	-	-	74	-	-	-	34	49
3	38	-	-	-	-	-	-	-
4	44	-	-	-	-	-	-	-
5	-	-	52	-	-	-	-	-
6	-	-	-	-	88	-	-	-

From Tables 1 and 2 it can be seen that the transfer of the last unloaded container, in both cases, began at station S_1 in the 88th time unit. This means that, as Theorem 1 showed, the time of the ending the unloading, from the aspect of container quay crane, is the same in both cases. As Theorem 2 showed, the number of required AGVs is not the same in both cases (In the first case AGV=8, while in the second case AGV= 6). From the given calculation, it can be seen that in the 36th time unit, one AGV departs from location S_4 towards location S_1, while it can also be seen that in the 49th time unit, another AGV departs from location S_5 towards S_1. These two AGVs return to station S_1 in the 56th and 64th time unit. This means that these two AGVs are again used for unloading of containers, and therefore fewer AGVs were required to perform the same work.

As in the numerical example, during transshipment of containers by the container quay crane, it is possible to simultaneously unload several containers that are being transferred (by AGVs) to the most remote storage yard locations; thus AGVs are then busy for a longer period of time.

6 Important Notes

This paper presents the model made on the basis of pseudo-operator ($\oplus$ (pseudo-addition), $\odot$ (pseudo-multiplication)) as a result of already presented models [1], [27], [28]. The presented models can be used for various types of system optimization. However, this paper presents a manner of the synchronization of equipment in a container terminal, where it is possible to optimize the number of AGVs.

<u>We should aim at a constant number of AGVs</u>

During reloading of containers by the quay crane, it has been proved that the order of unloading containers does not affect the time necessary for unloading the ship.

However, it is quite clear, on the basis of the aforementioned, that the number of AGVs during unloading varies at times. By using the given method, the best optimum way of using the AGVs can be achieved by minimizing the oscillations of the number of AGVs. In other words, this would be aimed at achieving a constant number of vehicles.

<u>The effect of rehandled containers</u>

During reloading, there are containers that need to be rehandled. Such containers cause the time of reloading the ship to extend. When a crane takes the container which later needs to be put back onto the ship, it places it on the shore or some other place on the ship. Nevertheless, AGVs are not required during transshipment of such containers.

This implies that while a crane is unloading such containers, AGVs are not needed in the station S_1.

<u>Even if there are priorities, savings are possible</u>

When unloading large ships, there are containers with priority that need to be unloaded first. Nevertheless, even in such systems, there are containers with the same priority. In such cases, it is possible to apply the stated synchronization and in such manner reduce the required number of AGVs in the process of transshipment.

<u>A larger number of cranes</u>

Unlike the above presented system, it is possible to observe the state in which a large number of cranes unload containers from one ship. Then, the transformation matrix would change the system faster, and the optimization which is possible would be greater. In sea terminals, usually more cranes are used for big ships (2 to 4), which would be appropriate for this system. The matrix would then contain a higher number of changes in the same time moment, i.e., event. It implies that the effects of yard cranes would have to be modeled in the system (i.e., AGVs would at some location have to wait until the yard crane is free to handle them).

Conclusions

This paper presents a possible synchronization of the container quay crane and AGVs' operation on the basis of modeling by using the pseudo-analysis. It was shown that if containers are unloaded in specified order, the number of AGVs can be reduced without any effect on the time necessary to unload the ship. It is possible to track the system and acknowledge whether a required reduction of the number of AGVs is possible for each container terminal (i.e., terminal scheduling) and the frequency of different ships being handled in the terminal. The size of the ship, the number of cranes, the size of the storage yard location and the technical performances of the equipment will affect the possible minimization of the required number of AGVs. It is stressed that this way of synchronization can lead only to a reduction in the required number of AGVs. To the author's knowledge, no similar research for this kind of equipment synchronization has been conducted so far. Finally, as was stressed at the beginning: resulting from the global increase in the volume of containerized goods, there is an emerging need for analysis and improvement in all aspects of the logistic chain of container transport.

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