Analysis of Network Traversal and Qualification of the Testing Values of Trajectories

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Abstract: This research work is aimed directly at the study of network traversal, for the design of vehicle dynamics and driving test programs. The work can be applied more widely to the qualification of test tracks. In addition to general modelling, this method can also be used to investigate the formation of loops in order to take into account, the sub-routes, multiple times. An important area of its application is the more comprehensive analysis of complex loads, as well as, the development of learning algorithms, which can be achieved by repeating multiple traversals of certain track sections within a series of measurements, and can be used for the development of on-board vehicle systems. The mathematical modelling is presented through the application of the geometric graph and subgraphs of the track. The properties of the Markov model extracted from the connection matrix of the large-scale network model are also presented. In this way, the modelling is extended to the application of the connection matrix of the large-scale network model as well. The modelling and computational details are demonstrated by means of a computer based, algebraic program. This modelling and the results of the calculations, will allow the further development of a test program design and related evaluation methods.

Keywords: Study of network traversal; mathematical modelling; connection matrix; Markov model; computer algebra; test program design; evaluation method

1 Introduction

This research work is directly related to the study of network traversal and can be applied to the design of vehicle dynamics and driving test programs, and more broadly to the qualification of test tracks.

Within the research tasks related to the vehicle test track, the analysis of track capabilities is a key area. The work serves as an analysis of track capabilities for testing autonomous vehicles and results in a new and efficient method.

When testing autonomous vehicles [15, 18, 22, 24], both dynamic track elements and urban track elements are important to ensure that together they represent the complex effects of the real environment at the highest level.

Reducing the complexity of the calculations for this complex problem is also of great importance for our investigations.

Therefore, we propose a method that either (I) compiles an optimal test program for the existing test track under a set of criteria and selects the set of best trajectories for the given objective, or (II) determines the set of best trajectories for the given objectives in the design phase of the tracks.

The latter can provide direction for redesign and the procedure also provides algorithmic methods for designing the structure of the track models.

2 Studies on the Design of Test Tracks

The development of autonomous driving technologies is receiving considerable attention, extensive research and validation is carried out to test the autonomy and safety of vehicle systems, I. Passchier, G. v. Vugt, and M. Tideman [7]. The test track is designed and constructed to simulate the real environment along with the foreseeable risks, R. Chen, M. Arief, D. Zhao [2], so in the studies, the trajectories must adapt faithfully and flexibly to reality. These benefits have already led to the creation of several world-class CAVs, e.g. T. Stevens. [14], Harman [3], Mcity Test Facility, [4], D. Muoio. [5], ACM. Project [1]. However, the proof of the CAV evaluation theory in design is not yet widely reported in the literature. This was pointed out and a mathematical model was developed by R. Chen, M. Arief, D. Zhao [2]. Indeed, the CAV's ability to operate in the spatial, field conditions that are regularly encountered by autonomous vehicles in real-world situations is important. These include compliance with road rules, in addition to reacting to other vehicles and road users, as well as to frequently encountered hazards.

Examination of such cases leads to the capability checklist that can be used as an evaluation criterion for CAV systems, C. Nowakowski, S. E. Shladover, C.-Y. Chan, and H.-S. Tan. [6], Torc. [16].

First, R. Chen, M. Arief, D. Zhao [2] assessed a systematic approach that uses an optimal model with the aim of maximizing the testing ability of the proof of CAV evaluation.

The goal of the design approach is to map scenarios in a given space that maximize the evaluation capability of the CAV. The research is important from two perspectives: 1) It is not clear how typical scenarios are currently classified and selected from a large-scale real-world driving dataset; 2) It is currently unclear how these scenarios and proofs can be mapped.

In order to enable a particular scenario suitable for testing in their task set, they had to draw maps of a set of roads and intersections that support the accurate implementation of the concurrence of these road elements in evaluation planning. In addition, a so-called "value measure" can also be defined for it, which can be presented and used for the evaluation of road use. It is a very important finding that **the value of road assets supports versatility, multi-scenario usability, and also supports the universality of scenarios.**

Modelling considerations, constraints:

When measuring the value of road assets **S**, the pre-processed sequence **A** = $\{a_1, a_2, \ldots, a_{NA}\}$ can be used as a basis in the limited construction area.

1^0. Road elements in one asset shall not overlap roads in other assets.

2^0. The extent of road assets shall be contained within the constrained space **S**.

In Phase 1, a set representing the highest complexity asset value is selected from a subset of the feasible road sections (from all pre-generated road data sets).

In Phase 2, the pre-selected set is optimized according to **S** in the given space, taking into account the elasticities for the transitions in the scenarios.

The aim of the two-phase approach is the following:

Phase 1 is thus a model for selecting the most valuable feasible subset, taking into account constraints in a strict sense.

Phase 2: the optimization model that **takes into account the pre-selected optimal set where the objective, *based on Phase 1,* is to be maximized both in number and in the value of the elements.**

In the above, the constraint ensures that for each connected asset pair, at least one transition road connecting the boundary nodes must be selected. The constraint ensures that any designated temporary road candidate does not access another internal segment unless the case occurs that a node is constructed. Regarding the structure of the model, based on the above, it is the basis of a design framework that provides a geographic map for the implementation of a wide variety of CAV scenarios in the constrained space and the scenario provides maximum flexibility for transitions.

However, the proposed valuable method ran into a serious computational problem. Based on the prescribed model, the defined optimization procedure leads to an NP problem, mainly due to the binary tools used, which ensure the validity of the asset forms and combined patterns, and some constraint conditions are not convex either. The use of an excessive number of binary variables and non-linear constraints in this method required a significant amount of branching processes and relaxation solutions, which became the largest computational burden in the studies. Due to the resulting computational complexity, further work is needed to address the problems, according to the authors, focusing on computational solutions by breaking down the problem.

3 The New Modelling Considerations and Methods

As seen in the previous chapter, the "Xcity" study discusses a new design method for the construction of test tracks based on a complex optimization calculation. In this regard, the authors presented a design procedure using a limited number of road assets. However, the method is not applicable in practice due to its exponential computational complexity.

In our case, the "ZalaZone" test track [17], on the other hand, is already a planned and implemented test track, which is already partially operational. This test track will provide a complete test environment and multi-level testing capability for future vehicles and communication technologies, from prototype testing to series product development.

Within the framework of the first phase, the dynamic surface, the high-speed handling track, the first part of the Smart City Zone was implemented; this is followed by the completion of the motorway section, various highway sections, and the Smart City. By early 2022, the low-speed handling track, the ramps, the noise measurement surface, and the ADAS test surface will be completed.

In this case, the real track curves are composed of real sector pieces, thus real curve pieces and sectors are the building blocks of trajectories, i.e. test routes. It is very important that in this way the network approach can be perfectly applied in modelling as well [19-21] [23]. Then, taking into account all potential branch points, all possible real routes of the test track can be generated by considering each distribution point. With the algorithm we present, real test routes can be generated at high speed without any geometric and hardware-related computational constraints. The generation of all possible track trajectories, for this purpose we use the large-scale network model we have developed, in particular its connection matrix [8-13]. To determine all possible trajectories of the track, we introduce the distribution marker $\alpha_{i,j}$, which accurately indicates all routes that originate from the branch it induces. In the sample track shown in Figure 1, we consider cases A and B, which use different sector controls in each sector. Thus, different closed-curve trajectories can be constructed for testing purposes in the case of A and B and the evaluation methods can also be examined. The aim is to identify the most valuable closed track trajectories that best meet the tester's criteria.

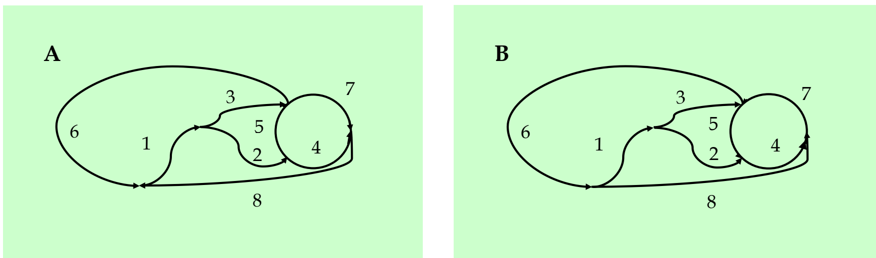

Figure 1
In the case of the track consisting of the sub-track curves Nos. 1-8, two different sets of trajectories, one of type A and one of type B, are created for the traversal due to the different sector controls

Thus, our research aims to develop applicable mathematical methods that accurately take into account the geometry of the trajectories and are also able to perform an exact qualification-evaluation procedure for the trajectories. Our goal is that the matrix transformation expansion method to be developed for generating trajectories should provide fast, real-time computational performance, and thus be efficient in this respect as well, as opposed to the Xcity method, which requires NP computational power.

4 Selection of the Optimal Trajectories

The possibility arises for the optimal selection of the trajectories consisting of the connected curve sections with sequentially structured dynamic programming. In this case, to perform the sequential optimization, e.g. Bellman's principle of optimality can be applied. The approach is to apply the principle of optimality to discrete, deterministic systems, taking into account that "an optimal policy can only consist of optimal sub-policies".

The reason for not doing so in our case is justified by the fact that we can define a very elegant and fast matrix transformation expansion method based on the traffic model under discussion. In terms of traffic on the test track, the connection matrix $\boldsymbol{K}_{11}$ contains the v_{ij} dynamic speed connections between the internal sector elements. For the test track consisting of *n* internal sectors ($i,j = 1,2,\ldots n$), in the case of an arbitrary sector *"j"* the connection matrix $\boldsymbol{K}_{11}$ determines to which sector or sectors *"i"* the controlled amount of material flows at an also controlled speed. To solve our problem, we form the transition probability or distribution matrix **P** from the matrix $\mathbf{K}_{11}$ by keeping only the $\alpha_{i,j}$ distribution values for the matrix elements and neutralizing the effect of the other dynamic control functions by replacing them with a value of 1. In addition, it is also necessary to replace the elements in the main diagonal with values of 0, so that the result of all considered further steps does not disappear (i.e. it remains fixed). The effects of material removals occurring in real dynamic processes are not relevant in this study, because the task is only to determine all the possibilities of further steps, at the same time it is necessary to mark and preserve all previous locations in order to reconstruct all routes. Our basic concept is that all previous splits will leave an accurate trace along the routes that resulted from the split. So **markers** determine the formation of each route. These well-identifiable markers are the $\alpha_{i,j}$ distributions. For the above, we use the Markov property derived from the matrix $\mathbf{K}_{11}$. For each *j*-th column of the matrix (j=1,2, ... , *n*), the $\alpha_{ij} \geq 0$ discrete distribution (the sum of these in column *j* is 1) depending on the condition of staying in sector *j*, determines the probability of moving from sector *j* to sector *i*: (In our case: $i \neq j$)

$$\alpha_{ij} = P(i \mid j) \tag{1}$$

Thus, the sum of the column elements of the matrix $\mathbf{P}=\mathbf{P}[\alpha_{i,j}]$ is: $\sum_{i=1}^{n} \alpha_{ij} = 1$ (j=1,2, ... , n). It can be clearly seen that the matrix **P** written in this way defines a discrete Markov chain. Note that since the $\alpha_{ij}(x(t),t)$ values can be considered constant only for short periods of time in a real traffic network and actually depend on time and also on the vehicle density state characteristic *x(t)* of the sectors, the *Markov chain defined here is inhomogeneous*. The elements of the matrix **P** cannot be said to be positive definite either, since there are a large number of 0 elements among them, so the *Markov chain under discussion is also irregular*. It is important to emphasize that when defining paths, distributions are not time- and state-dependent features. These are fixed constants because we want to consider all possible road sections in the applied procedure. The analysis is then carried out by starting from the input sector No. 1, considered as the gate of the track, and determining the distributions (scatterings) proceeding step by step. Accordingly, at the initial time, we are only in sector No. 1 with a probability of p>0. The probabilities of the initial states "1" are determined by the vector $\mathbf{p}_1$: according to the conditions, the probability of staying in sector No. 1 is p, while the probabilities of staying in the other sectors take the value of 0:

$$p_1 = \begin{bmatrix} p \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2)$$

The following state probabilities (3) are determined by the vector $\mathbf{p}_2$ calculated using the matrix **P** and the vector $\mathbf{p}_1$, where the matrix **P** contains the distributional directions and values for proceeding from sector No. 1:

$$p_2 = P \cdot p_1 \quad (3)$$

The following vector $\mathbf{p}_3$, which determines the state probabilities (4), is calculated in a similar recursive way using the matrix **P** and the vector $\mathbf{p}_2$. In this case, the matrix **P** also contains distributional directions and values for proceeding from sector No. 2:

$$p_3 = P \cdot p_2 \quad (4)$$

Finally, the vector $\mathbf{p}_n$ determining the *n*-th state probabilities is also calculated recursively using the matrix **P** and the vector $\mathbf{p}_{n-1}$:

$$p_n = P \cdot p_{n-1} \quad (5)$$

Referring to the relation for discrete Markov chains, it can be clearly seen from the above derivation that the state probability vector $\mathbf{p}_n$ can be generated by a one-step method as the product of the *n*-th power of the matrix **P** and the vector $\mathbf{p}_1$:

$$p_n = P^n \cdot p_1 \quad (6)$$

After *n* steps, if the sectors in the domains have no outflow end, i.e. there is no further transfer, and loops can be applied to the routes but not in infinite cycles, then after a finite number of steps a steady state occurs, thus the probabilities of staying in each sector are no longer modified by further application of the algorithm:

$$\mathbf{p}_n = \mathbf{p}_{n+1} = \mathbf{p}_{n+2} = \ldots = \mathbf{p}_{n+k} = \ldots \quad (7)$$

The significance of this is that the non-zero coordinates of the vector $\mathbf{p}_n$ can be used to determine how many different routes, which can be considered parallel, have led from the input sector No. 1 to any other sector. In this context, the routes that are most important are those that lead to the sectors consisting of the most elements, which can be considered as the most distant "outputs", as these can provide the most opportunities for a qualitative analysis. This will be illustrated by the examples shown in Figures 1a and 1b.

5 Presentation of the Method for Two Types of Test Tracks Consisting of 8 Sectors

The method plays an important role in the machine-based generation of all trajectories in the case of a test track after taking an arbitrary input. It can play an equally important role in the definition and evaluation of various criteria and rankings according to the test criteria. Let us consider the "test track" consisting of n=8 sectors as seen in Figures 1a and 1b, and all possible closed-curve trajectories that can be constructed from a selected sector on track A. In the case of sector No. 1 that means those, which start from and arrive at sector No. 1, while in the case of track B those starting from sector No. 1 and ending in sectors Nos. 6, 7, and 8.

Let us consider the transition probability matrices **P** of the test tracks shown in Figures 1a and 1b (8a) and (8b), which are the matrices defining the relationship between the different elements *i* and *j*.

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{3,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_{4,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_{5,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_{6,3} & 0 & \alpha_{6,5} & 0 & 0 & 0 \\ 0 & 0 & \alpha_{7,3} & 0 & \alpha_{7,5} & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_{8,4} & 0 & 0 & \alpha_{8,7} & 0 \end{bmatrix} \tag{8a}$$

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{2,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{3,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_{4,2} & 0 & 0 & \alpha_{4,5} & 0 & 0 & 0 \\ 0 & 0 & \alpha_{5,3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_{6,3} & 0 & 0 & 0 & \alpha_{6,7} & 0 \\ 0 & 0 & 0 & \alpha_{7,4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_{8,6} & 0 & 0 \end{bmatrix} \tag{8b}$$

According to the construction, the probability of staying in sector No. 1, which is considered as input, is initially p, and the probabilities of staying in the other sectors take the value of 0. This initial state is defined by the vector P1:

$$P1 = \begin{bmatrix} p \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Calculating the probabilities of staying in the sectors as we proceed, respectively: $p_2 = P \cdot p_1, \;\; p_3 = P \cdot p_2,$ ..., $p_n = P \cdot p_{n-1},$ we obtain the following sequence of vectors, which determine the state probabilities for tracks A and B:

$$P2 = \begin{bmatrix} p \\ \alpha_{2,1} p \\ \alpha_{3,1} p \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9a) \qquad P2 = \begin{bmatrix} p \\ \alpha_{2,1} p \\ \alpha_{3,1} p \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9b)$$

Where, index "a" refers to track A, while index "b" refers to track B.

During the calculation, after the 5^{th} transformation step (which has already determined all possible routes), the probability vector P_j does not change any more. This is due to the fact that the transition probability matrix used was written in such a way that in case "A", sectors Nos. 6 and 8 do not pass vehicles to any other sector any more, while in case "B", the routes end in sectors Nos. 6, 7 and 8.

All routes starting from input sector No. 1 can be specified by determining how many routes lead to each of the end sectors separately. (In case "A" from No. 1 to outputs Nos. 6 and 8; in case "B" from No. 1 to outputs Nos. 6, 7 and 8.)

For case "A": For the routes leading from sector No. 1 to output sector No. 6, the 6^{th} coordinate of the vector must be examined. In this regard, we see two sums resulting from the fact that we reached No. 6 on two parallel routes.

$$\mathrm{P3} = \begin{bmatrix} p \\ \alpha_{2,1}\, p \\ \alpha_{3,1}\, p \\ \alpha_{4,2}\, \alpha_{2,1}\, p \\ \alpha_{5,2}\, \alpha_{2,1}\, p \\ \alpha_{6,3}\, \alpha_{3,1}\, p \\ \alpha_{7,3}\, \alpha_{3,1}\, p \\ 0 \end{bmatrix} \quad (10a)$$

$$\mathrm{P3} = \begin{bmatrix} p \\ \alpha_{2,1}\, p \\ \alpha_{3,1}\, p \\ \alpha_{4,2}\, \alpha_{2,1}\, p \\ \alpha_{5,3}\, \alpha_{3,1}\, p \\ \alpha_{6,3}\, \alpha_{3,1}\, p \\ 0 \\ 0 \end{bmatrix} \quad (10b)$$

$$\mathrm{P4} = \begin{bmatrix} p \\ \alpha_{2,1}\, p \\ \alpha_{3,1}\, p \\ \alpha_{4,2}\, \alpha_{2,1}\, p \\ \alpha_{5,2}\, \alpha_{2,1}\, p \\ \alpha_{6,3}\, \alpha_{3,1}\, p + \alpha_{6,5}\, \alpha_{5,2}\, \alpha_{2,1}\, p \\ \alpha_{7,3}\, \alpha_{3,1}\, p + \alpha_{7,5}\, \alpha_{5,2}\, \alpha_{2,1}\, p \\ \alpha_{8,4}\, \alpha_{4,2}\, \alpha_{2,1}\, p + \alpha_{8,7}\, \alpha_{7,3}\, \alpha_{3,1}\, p \end{bmatrix} \quad (11a)$$

$$P4 = \begin{bmatrix} p \\ \alpha_{2,1}\, p \\ \alpha_{3,1}\, p \\ \alpha_{4,2}\,\alpha_{2,1}\, p + \alpha_{4,5}\,\alpha_{5,3}\,\alpha_{3,1}\, p \\ \alpha_{5,3}\,\alpha_{3,1}\, p \\ \alpha_{6,3}\,\alpha_{3,1}\, p \\ \alpha_{7,4}\,\alpha_{4,2}\,\alpha_{2,1}\, p \\ \alpha_{8,6}\,\alpha_{6,3}\,\alpha_{3,1}\, p \end{bmatrix} \tag{11b}$$

$$P5 = \begin{bmatrix} p \\ \alpha_{2,1}\, p \\ \alpha_{3,1}\, p \\ \alpha_{4,2}\,\alpha_{2,1}\, p \\ \alpha_{5,2}\,\alpha_{2,1}\, p \\ \alpha_{6,3}\,\alpha_{3,1}\, p + \alpha_{6,5}\,\alpha_{5,2}\,\alpha_{2,1}\, p \\ \alpha_{7,3}\,\alpha_{3,1}\, p + \alpha_{7,5}\,\alpha_{5,2}\,\alpha_{2,1}\, p \\ \alpha_{8,4}\,\alpha_{4,2}\,\alpha_{2,1}\, p + \alpha_{8,7}\left(\alpha_{7,3}\,\alpha_{3,1}\, p + \alpha_{7,5}\,\alpha_{5,2}\,\alpha_{2,1}\, p\right) \end{bmatrix} \tag{12a}$$

$$P5= \begin{vmatrix} p \\ \alpha_{2,1}\,p \\ \alpha_{3,1}\,p \\ \alpha_{4,2}\,\alpha_{2,1}\,p + \alpha_{4,5}\,\alpha_{5,3}\,\alpha_{3,1}\,p \\ \alpha_{5,3}\,\alpha_{3,1}\,p \\ \alpha_{6,3}\,\alpha_{3,1}\,p \\ \alpha_{7,4}\left(\alpha_{4,2}\,\alpha_{2,1}\,p + \alpha_{4,5}\,\alpha_{5,3}\,\alpha_{3,1}\,p\right) + \alpha_{7,8}\,\alpha_{8,6}\,\alpha_{6,3}\,\alpha_{3,1}\,p \\ \alpha_{8,6}\,\alpha_{6,3}\,\alpha_{3,1}\,p \end{vmatrix} \qquad (12b)$$

$$P6 = \begin{bmatrix} p \\ \alpha_{2,1}\,p \\ \alpha_{3,1}\,p \\ \alpha_{4,2}\,\alpha_{2,1}\,p \\ \alpha_{5,2}\,\alpha_{2,1}\,p \\ \alpha_{6,3}\,\alpha_{3,1}\,p + \alpha_{6,5}\,\alpha_{5,2}\,\alpha_{2,1}\,p \\ \alpha_{7,3}\,\alpha_{3,1}\,p + \alpha_{7,5}\,\alpha_{5,2}\,\alpha_{2,1}\,p \\ \alpha_{8,4}\,\alpha_{4,2}\,\alpha_{2,1}\,p + \alpha_{8,7}\left(\alpha_{7,3}\,\alpha_{3,1}\,p + \alpha_{7,5}\,\alpha_{5,2}\,\alpha_{2,1}\,p\right) \end{bmatrix} \qquad (13a)$$

$$P6=\left|\begin{array}{c} p \\ \alpha_{2,1}\,p \\ \alpha_{3,1}\,p \\ \alpha_{4,2}\,\alpha_{2,1}\,p+\alpha_{4,5}\,\alpha_{5,3}\,\alpha_{3,1}\,p \\ \alpha_{5,3}\,\alpha_{3,1}\,p \\ \alpha_{6,3}\,\alpha_{3,1}\,p \\ \alpha_{7,4}\left(\alpha_{4,2}\,\alpha_{2,1}\,p+\alpha_{4,5}\,\alpha_{5,3}\,\alpha_{3,1}\,p\right)+\alpha_{7,8}\,\alpha_{8,6}\,\alpha_{6,3}\,\alpha_{3,1}\,p \\ \alpha_{8,6}\,\alpha_{6,3}\,\alpha_{3,1}\,p \end{array}\right| \tag{13b}$$

Test routes are shown by the right-to-left readings of the second indexes of the $\alpha_{i,j}$ distributions.

$$\mathbf{P8}[6]=\alpha_{6,3}\,\alpha_{3,1}\,p+\alpha_{6,5}\,\alpha_{5,2}\,\alpha_{2,1}\,p \tag{14}$$

The first route, denoted by I, leads from No. 1 to No. 3 and from No. 3 to No. 6;

The second route, denoted by II, leads from No. 1 to No. 2, then from No. 2 to No. 5 and from No. 5 to No. 6.

The routes leading from No. 1 to No. 8 are similarly shown by the 8th coordinate of the vector. Here we see the sum of 3 members as follows, so we can get from No. 3 to No. 8 on 3 different, parallel routes.

$$\mathbf{P8}[8]=\alpha_{8,4}\,\alpha_{4,2}\,\alpha_{2,1}\,p+\alpha_{8,7}\left(\alpha_{7,3}\,\alpha_{3,1}\,p+\alpha_{7,5}\,\alpha_{5,2}\,\alpha_{2,1}\,p\right) \tag{15}$$

The first route, denoted by III, leads from No. 1 to No. 2, then from No. 2 to No. 4, and finally, from No. 4 to No. 8.

The second route, denoted by IV, leads from No. 1 to No. 3, then from No. 3 to No. 7, and finally, from No. 7 to No. 8.

The third route, denoted by V, leads from No. 1 to No. 2, then from No. 2 to No. 5, from No. 5 to No. 7, and finally, from No. 7 to No. 8.

It can be clearly seen that formulas (14) and (15) obtained by the computer-algebraic method are parametric mathematical formulas. Their evaluation for determining the route codes is done by processing strings and characters. These formulas consist of sums of products and each product is separated from the next by a "+" sign. These products form the strings in which, when analyzing the characters from right to left, only the indices of the alpha character are collected and two identical indices following each other are considered only once.

The sequence of numbers thus determined consists of the serial numbers of the consecutive and interconnected sector elements that form the route. (See routes I, II, ... , V.)

In case "B", similarly to the above, all routes starting from sector No. 1 and ending in either sector No. 6, No. 7 or No. 8 are defined:

$$\mathbf{P8}[6] = \alpha_{6,3}\,\alpha_{3,1}\,p \tag{16}$$

$$\mathbf{P8}[7] = \alpha_{7,4}\left(\alpha_{4,2}\,\alpha_{2,1}\,p + \alpha_{4,5}\,\alpha_{5,3}\,\alpha_{3,1}\,p\right) + \alpha_{7,8}\,\alpha_{8,6}\,\alpha_{6,3}\,\alpha_{3,1}\,p \tag{17}$$

$$\mathbf{P8}[8] = \alpha_{8,6}\,\alpha_{6,3}\,\alpha_{3,1}\,p \tag{18}$$

I: leads from No. 1 to No. 3 and from No. 3 to No. 6; (route defined by P8 [6])

II: leads from No. 1 to No. 3, then from No. 3 to No. 6, from No. 6 to No. 8, and finally, from No. 8 to No. 7; (route defined by P8 [7])

III: leads from No. 1 to No. 3, then from No. 3 to No. 5, from No. 5 to No. 4, and finally, from No. 4 to No. 7; (route defined by P8 [7])

IV: leads from No. 1 to No. 2, then from No. 2 to No. 4, and finally, from No. 4 to No. 7; (route defined by P8 [7])

V: leads from No. 1 to No. 3, then from No. 3 to No. 6, and finally, from No. 6 to No. 8; (route defined by P8 [8])

6 Presentation of Matrix Operations for the Calculations

In summary, in both cases a total of 5 different routes led from input sector No. 1 so that each of the sectors used in a route was only considered once. Based on the above matrix transformation algorithm, all trajectories were determined for the test tracks shown in *Figs. 1a and 1b*. These trajectories, i=I., II., … , V, are encoded in the rows of the matrix **Tr** (19a and 19b) in such a way that where 1 is in the i-th row, that sector j is part of the i-th trajectory:

$$\mathbf{Tr} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad (19a) \qquad \mathbf{Tr} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad (19b)$$

The elements [i,j] of the matrix **Ck** (20a and 20b) contain the values of the individual trajectory elements (there are 8 sector elements) for the given test program.

$$\mathbf{Ck} = \begin{bmatrix} 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \\ 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \\ 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \\ 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \\ 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \end{bmatrix} \quad (20a)$$

$$\mathbf{Ck} = \begin{bmatrix} 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \\ 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \\ 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \\ 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \\ 1.5 & 2.5 & 3.2 & 5.7 & 4.1 & 7.6 & 3.8 & 9.5 \end{bmatrix} \quad (20b)$$

The elements [i,j] of the matrix **Cv** (21a and 21b) give the values of the sector change for each trajectory element for the given test program. The value is assigned to the transferring sector.

$$\mathbf{Cv} = \begin{bmatrix} 0 & 0 & 0.5 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 1.6 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 1.2 & 0 & 0 & 0 & 1.3 \\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0.2 & 1.1 \\ 0 & 0.5 & 0 & 0 & 1.6 & 0 & 1.2 & 1.3 \end{bmatrix} \quad (21a)$$

$$\mathbf{Cv} = \begin{bmatrix} 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0.2 \\ 0 & 0 & 0.9 & 0 & 0.1 & 0.1 & 0 & 0 \\ 0 & 0.5 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (21b)$$

The elements [i,j] of the matrix **C** (22a and 22b) are the summed values for each trajectory element for the given test program:

$$\mathbf{C = Ck + Cv} = \begin{bmatrix} 1.5 & 2.5 & 3.7 & 5.7 & 4.1 & 7.7 & 3.8 & 9.5 \\ 1.5 & 3.0 & 3.2 & 5.7 & 5.7 & 8.1 & 3.8 & 9.5 \\ 1.5 & 3.0 & 3.2 & 6.9 & 4.1 & 7.6 & 3.8 & 10.8 \\ 1.5 & 2.5 & 3.7 & 5.7 & 4.1 & 7.6 & 4.0 & 10.6 \\ 1.5 & 3.0 & 3.2 & 5.7 & 5.7 & 7.6 & 5.0 & 10.8 \end{bmatrix} \tag{22a}$$

$$\mathbf{C = Ck + Cv} = \begin{bmatrix} 1.5 & 2.5 & 3.7 & 5.7 & 4.1 & 7.7 & 3.8 & 9.5 \\ 1.5 & 3.0 & 3.2 & 5.7 & 5.7 & 8.1 & 3.8 & 9.5 \\ 1.5 & 3.0 & 3.2 & 6.9 & 4.1 & 7.6 & 3.8 & 10.8 \\ 1.5 & 2.5 & 3.7 & 5.7 & 4.1 & 7.6 & 4.0 & 10.6 \\ 1.5 & 3.0 & 3.2 & 5.7 & 5.7 & 7.6 & 5.0 & 10.8 \end{bmatrix} \tag{22b}$$

The elements [i, j] of the matrix **C•Tr** (23a and 23b) are the test values of the sectors that form the actual trajectories, which product is the Hadamard product of the two matrices:

$$\mathbf{C \cdot Tr} = \begin{bmatrix} 1.5 & 0. & 3.7 & 0. & 0. & 7.7 & 0. & 0. \\ 1.5 & 3.0 & 0. & 0. & 5.7 & 8.1 & 0. & 0. \\ 1.5 & 3.0 & 0. & 6.9 & 0. & 0. & 0. & 10.8 \\ 1.5 & 0. & 3.7 & 0. & 0. & 0. & 4.0 & 10.6 \\ 1.5 & 3.0 & 0. & 0. & 5.7 & 0. & 5.0 & 10.8 \end{bmatrix} \tag{23a}$$

$$\mathbf{C \cdot Tr} = \begin{bmatrix} 1.5 & 0. & 3.7 & 0. & 0. & 7.7 & 0. & 0. \\ 1.5 & 0. & 3.2 & 0. & 0. & 8.1 & 3.8 & 9.5 \\ 1.5 & 0. & 3.2 & 6.9 & 4.1 & 0. & 3.8 & 0. \\ 1.5 & 2.5 & 0. & 5.7 & 0. & 0. & 4.0 & 0. \\ 1.5 & 0. & 3.2 & 0. & 0. & 7.6 & 0. & 10.8 \end{bmatrix} \tag{23b}$$

The vector **Sc** (24a and 24b) is obtained by summing the rows of the matrix **C•Tr**, so its coordinates determine the test values of the possible trajectories:

$$\mathbf{Sc} = \begin{bmatrix} 12.9 \\ 18.3 \\ 22.2 \\ 19.8 \\ 26.0 \end{bmatrix} \tag{24a}$$

$$\mathbf{Sc} = \begin{bmatrix} 12.9 \\ 26.1 \\ 19.5 \\ 13.7 \\ 23.1 \end{bmatrix} \tag{24b}$$

The vectors **Sc** (24a and 24b) contain the values of each test trajectory according to the complex tests.

Conclusions

In the course of this research, we used the transition probability matrix prescribed for the connection type tracks A and B. These are summarized of the following two tables:

Table 1

The elements of the transition probability matrix $\left[\alpha_{i,j}\right]$ in *Fig. 1a* are determined by the distribution values of the large-scale network model

i/j	1	2	3	4	5	6	7	8
1								
2	$\alpha_{21} = P(2\|1)$							
3	$\alpha_{31} = P(3\|1)$							
4		$\alpha_{42} = P(4\|2)$						
5		$\alpha_{52} = P(5\|2)$						
6			$\alpha_{63} = P(6\|3)$		$\alpha_{65} = P(6\|5)$			
7			$\alpha_{73} = P(7\|3)$		$\alpha_{75} = P(7\|5)$			
8				$\alpha_{84} = P(8\|4)$			$\alpha_{87} = P(8\|7)$	

Table 2

The elements of the transition probability matrix $\left[\alpha_{i,j}\right]$ in *Fig. 1b* are determined by the distribution values of the large-scale network model

i/j	1	2	3	4	5	6	7	8
1								
2	$\alpha_{21} = P(2\|1)$							
3	$\alpha_{31} = P(3\|1)$							
4		$\alpha_{42} = P(4\|2)$			$\alpha_{45} = P(4\|5)$			
5			$\alpha_{53} = P(5\|3)$					
6			$\alpha_{63} = P(6\|3)$				$\alpha_{67} = P(6\|7)$	
7				$\alpha_{74} = P(7\|4)$				
8						$\alpha_{86} = P(8\|6)$		

The aim of our research was to develop an efficient and fast algorithm, that resulted in the determination of all the different trajectories, starting from any sector of the test track and ending in one or more sectors of an arbitrary choice.

In our opinion, in the case of interconnected curves the subject of the analysis is not only the value of the individual curves (in addition to the value of the traffic situations generated on the track and the value due to the geometric properties of the routing), but also, the value of which type of curve is connected to which other type of curve and how it impacts the driving, i.e. the value of the transition type.

For this purpose, our research used the connection matrix of the large-scale road network model.

The work is used for the qualification, development and design of test tracks, as it can also be used to perform complex evaluations of the defined trajectories. The calculations consider both the static and dynamic state characteristics of the sectors. They also take into account the geometric values of the sectors, the values of the situations generated on them, the values of the connections of the sectors and the direction of traversal of the sectors, which can also be freely varied. The field of application of the research is the definition and selection of the most valuable test trajectories, in summary, it helps the development of the capabilities of the test track and supports its successful operation. In the case of the examined trajectories, knowing the maximum number of sectors "m", the vector containing the routes can be determined in one step using the m-th power of the transition probability matrix. By analyzing the distributivity markers with a character analysis program, the trajectories can be automatically determined as described. The presented method accurately discusses and examines the geometry of the trajectories in an exact way. The presented matrix transformation expansion method for generating trajectories is extremely fast and very efficient in terms of real-time computational resources, which is a great advantage over the Xcity method that requires extraordinary computational power. The discussed method for evaluating test tracks already built or under design can be effectively applied both for the purposes of testing and for the selection of the most valuable trajectories defined by the objective function. The method can be similarly applied to the selection of optimal solutions for the exploration and evaluation of the possibilities of not yet constructed but pre-designed tracks. This provides an important opportunity for the preliminary evaluation of a large number of tracks, designed and prepared using computers, in the design phases and for further planning as needed. The above procedure takes advantage of a very useful property of the macroscopic model [8-13], that for, any complex transport network, the mathematical model, can take into account, all of the connections between the sector elements that make up the trajectories.

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