Stability of the Classifications of Returns to Scale in Data Envelopment Analysis: A Case Study of the Set of Public Postal Operators

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Abstract: A significant theme in data envelopment analysis (DEA) is the stability of returns to scale (RTS) classification of specific decision making unit (DMU) which is under observed production possibility set. In this study the observed DMUs are public postal operators (PPOs) in European Union member states and Serbia as a candidate country. We demonstrated a sensitivity analysis of the inefficient PPOs by DEA-based approach. The development of this analytical process is performed based on real world data set. The estimations and implications are derived from the empirical study by using the CCR RTS method and the most productive scale size concept (MPSS). First, we estimated the RTS classification of all observed PPOs. After that, we determined stability intervals for preserving the RTS classification for each CCR inefficient PPO under evaluation. Finally, scale efficient inputs and output targets for these PPOs are designated.

Keywords: data envelopment analysis; returns to scale; stability; scale efficient targets; public postal operators

1 Introduction

Data envelopment analysis (DEA) is a non-parametric technique for evaluating the relative efficiency of multiple-input and multiple-output of a decision making units (DMUs) based on the production possibility set. DEA method is introduced by Charnes *et al*. [1] and extended by Banker *et al*. [2]. There are various DEA models that are widely used to evaluate the relative efficiency of DMUs in different organizations or industries. Additionaly, DEA is recognized as a powerful analytical research tool for modeling operational processes in terms of performance evaluations, e.g. [3], competiteveness, e.g. [4] and decision making e.g. [5]. A taxonomy and general model frameworks for DEA can be found in [6, 7].

The stability of the classifications of returns to scale (RTS) is an important theme in DEA and was first examined by Seiford and Zhu [8]. There are several DEA approaches considering this topic. One approach is the stability analysis of a specific DMU which is under evaluation [see 9, 10]. Another approach is the stability of a specific DMU which is not under evaluation [see 11, 12]. Additionally, some authors used free disposal hull (FDH) models (unlike the convex DEA models, FDH models are non-convex) for estimating RTS [see 13, 14, 15].

The stability of RTS and the methods for its estimating in DEA provides important information on the data perturbations in the DMU analysis. These information provide discussions that can be developed in performance analysis. This enables to determine the movement of inefficient DMUs on the frontier in improving directions. In [8, 16], the authors developed several linear programming formulations for investigating the stability of RTS classification (constant, increasing or decreasing returns to scale). These authors considered data perturbations for inefficient DMUs. The authors of [17] indicated that sometimes a change in input or output or simultaneous changes in input and output are not possible. In the papers of Jahanshahloo *et al*. [10] and Abri [18] developed an approach for the sensitivity analysis of both inefficient and efficient DMUs from the observations set.

The current article proceeds as follows: In Section 2, the determination of RTS in the CCR models is reviewed. Additionally, in this Section are introduced output-oriented RTS classification stability and scale efficient targets inputs and outputs of DMUs. In Section 3, we applied methods from Section 2 on real world data set of public postal operators (PPOs). Finally, conclusions are given in Section 4.

2 Methods

2.1 RTS Classification

In the DEA literature there are several approaches for estimating of returns to scale (RTS). Seiford and Zhu in [19] demonstrated that there are at least three equivalent RTS methods. The first CCR RTS method is introduced by Banker [20]. The second BCC RTS method is developed by Banker *et al*. [2] as an alternative approach using the free variable in the BCC dual model. The third RTS method based on the scale efficiency index is suggested by Fare *et al*. [21]. The CCR RTS method is based upon the sum of the optimal lambda values in the CCR models of DEA, and is used in this study to the RTS classifications of observed PPOs.

The CCR is original model of DEA for evaluating the relative efficiency for a group of DMUs proposed by Charnes *et al*. [1]. The CCR stands for Charnes, Cooper and Rhodes which are the last names of this model creators. Suppose there are a set (A) of DMUs. Each DMU_j ($j \in A$) uses m inputs x_{ij} ($i = 1,2,3,...,m$) to produce s outputs y_{rj} ($r = 1,2,3,...,s$). The CCR model evaluates the relative efficiency of a specific DMU_o, $o \in A$, with respect to a set of CCR frontier DMUs defined $E_o = \{ j \mid \lambda_j > 0$ for some optimal solutions for $DMU_o\}$. One formulation of a CCR model aims to minimize inputs while satisfying at least the given output levels, i.e., the CCR input-oriented model (see the M1 model). Another formulation of a CCR model aims to maximize outputs without requiring more of any of the observed input values, i.e., the CCR output-oriented model (see the M1' model). The CCR models assume the constant returns to scale production possibility set, i.e. it is postulated that the radial expansion and reduction of all observed DMUs and their nonnegative combinations are possible and hence the CCR score is called overall technical efficiency. If we add $\sum_{j \in E_o} \lambda_j = 1$ in the M1 and M1' models, we obtain the BCC input-oriented and the BCC output-oriented models, respectively proposed by Banker *et al*. [2]. The name BCC is derived from the initial of each creator's last name (Banker-Charnes-Cooper). The BCC models assume that convex combinations of observed DMUs form the production possibility set and the BCC score is called local pure technical efficiency. It is interesting to investigate the sources of inefficiency that a DMU might have. Are they caused by the inefficient operations of the DMU itself or by the disadvantageous conditions under which the DMU is operating? For this purpose the scale efficiency score (SS) is defined by the ratio, $SS = \frac{\theta^*_{CCR}}{\theta^*_{BCC}}$. This approach depicts the sources of inefficiency, i.e. whether it is caused by inefficient operations (the BCC efficiency) or by disadvantageous conditions displayed by the scale efficiency score (SS) or by both.

M1 model

$$\theta^* = \min \theta \tag{1}$$

Subjec to:

$$\sum_{j \in E_o} \lambda_j x_{ij} \leq \theta x_{io}, \ \ i = 1,2,3,...,m$$

$$\sum_{j \in E_o} \lambda_j y_{rj} \geq y_{ro}, \ \ r = 1,2,3,...,s$$

$$\lambda_j \geq 0, \ \ j \in E_o$$

M1' model

$$\phi^* = \max \phi \tag{2}$$

Subject to:

$$\sum_{j \in E_o} \lambda_j x_{ij} \leq x_{io}, \ \ i = 1,2,3,...,m$$

$$\sum_{j \in E_o} \lambda_j y_{rj} \geq \phi y_{ro}, \ \ r = 1,2,3,...,s$$

$$\lambda_j \geq 0, \ \ j \in E_o$$

If $E_o = A$, then the M1 model is original form of the input-oriented CCR model. The DMU_j ($j \in E_o$) are called CCR efficient and form a specific CCR efficient aspect. These DMU_j ($j \in E_o$) appear in optimal solutions where $\lambda_j > 0$. The fact that $\lambda_j = 0$ for all $j \notin E_o$ in the M1 model when evaluating DMU_o enables performing the CCR model in form of M1 model or M1' model. By using the M1 or M1' model, we can estimate the RTS classification based on the following theorem by Banker and Thrall [22]:

Theorem 1. Let λ_j^* ($j \in E_o$) be the optimal values in M1 or M1' model, returns to scale at DMU_o can be determined from the following conditions:

(i) If $\sum_{j \in E_o} \lambda_j^* = 1$ in any alternate optimum then constant returns-to-scale (CRS) prevails.

(ii) If $\sum_{j \in E_o} \lambda_j^* > 1$ for all alternate optima then decreasing returns-to-scale (DRS) prevails.

(iii) If $\sum_{j \in E_o} \lambda_j^* < 1$ for all alternate optima then increasing returns-to-scale (IRS) prevails.

Seiford and Thrall in [23] derived the relationship between the solutions of the M1 model and M1' model. Suppose λ_j^* ($j \in E_o$) and θ^* is an optimal solution to M1 model. There exists a corresponding optimal solution λ_j^{**} ($j \in E_o$) and ϕ^* to the M1' model such that $\lambda_j^* = \frac{\lambda_j^{**}}{\phi^*}$ and $\phi^* = \frac{1}{\theta^*}$.

A change of input levels for DMU_o in the M1 model or a change of output levels in the M1' model does not change the RTS nature of DMU_o. These models yield the identical RTS regions. However, they can generate different RTS classifications. In this study we chose the M1 model to determine the RTS classification.

2.2 Stability of the RTS Classifications

The stability of the RTS classifications provides some stability intervals for preserving the RTS classification of a specific DMU_o. It enables to consider perturbations for all the inputs or outputs of DMU_o. Input-oriented stability of RTS classifications allows output perturbations in DMU_o, and output-oriented stability of RTS classifications enables input perturbations.

In this study stability intervals of each CCR inefficient PPO under evaluation are derived from output-oriented RTS classification stability because we aim to consider input increases and decreases for each CCR inefficient PPO. Lower and upper limit of stability intervals determined by using two linear programming models (see the M2 and M2' models) where ϕ^* is the optimal value to the M1' model when evaluating DMU_o.

M2 model

$$\eta_o^* = \frac{1}{\min \sum_{j \in E_o} \lambda_j} \tag{3}$$

Subjec to:

$$\sum_{j \in E_o} \lambda_j x_{ij} \le x_{io}, \; i = 1,2,3,...,m$$

$$\sum_{j \in E_o} \lambda_j y_{rj} \ge \phi^* y_{ro}, \; r = 1,2,3,...,s$$

$$\lambda_j \ge 0, \;\; j \in E_o$$

M2' model

$$\mu_o^* = \frac{1}{\max \sum_{j \in E_o} \lambda_j} \tag{4}$$

Subjec to:

$$\sum_{j \in E_o} \lambda_j x_{ij} \le x_{io}, \; i = 1,2,3,...,m$$

$$\sum_{j \in E_o} \lambda_j y_{rj} \ge \phi^* y_{ro}, \; r = 1,2,3,...,s$$

$$\lambda_j \ge 0, \;\; j \in E_o$$

By using the M2 and M2' models, we can define lower and upper limit of stability intervals of DMUs based on the following theorems by Seiford and Zhu [8]:

Theorem 2. Suppose DMU_o exhibits CRS.

If $\gamma \in R^{CRS} = \left\{ \gamma : \min\left\{1, \mu_o^*\right\} \le \gamma \le \max\left\{1, \eta_o^*\right\} \right\}$. The CRS classification continues to hold, where γ represents the proportional change of all inputs, $\hat{x}_{io} = \gamma x_{io}$ $(i = 1,2,3,...,m)$, and η_o^* and μ_o^* are defined in the M2 and M2' models, respectively.

Theorem 3. Suppose DMU_o exhibits DRS. The DRS classification continues to hold for $\xi \in R^{DRS} = \left\{ \xi : \eta_o^* < \xi \le 1 \right\}$, where ξ represents the proportional decrease of all inputs, $\hat{x}_{io} = \xi x_{io}$ $(i = 1,2,3,...,m)$, and η_o^* is defined in the M2 model.

Theorem 4. Suppose DMU_o exhibits IRS. Then the IRS classification continues to hold for $\zeta \in R^{IRS} = \left\{ \zeta : \mu_o^* < \zeta \le 1 \right\}$, where ζ represents the proportional change of all inputs, $\hat{x}_{io} = \zeta\, x_{io} \quad (i = 1,2,3,...,m)$, and μ_o^* is defined in the M2' model.

2.3 Scale Efficient Targets

Scale efficient targets (inputs and outputs) for DMUs can be derived by using the most productive scale size concept proposed by Banker [20]. This concept in DEA is known as acronym MPSS (see the M3 and M3' models). Both models are based on output-oriented CCR model. The M3 model produces the largest MPSS targets ($MPSS_{max}$), and the M3' model the smallest ($MPSS_{min}$).

M3 model

$$\vartheta^* = \min \sum_{j \in E_0} \lambda_j \tag{5}$$

Subject to:

$$\sum_{j \in E_o} \lambda_j x_{ij} \le x_{io}, \;\; i = 1,2,3,...,m$$

$$\sum_{j \in E_o} \lambda_j y_{rj} \ge \phi^* y_{ro}, \;\; r = 1,2,3,...,s$$

$$\lambda_j \ge 0, \;\; j \in E_o$$

M3' model

$$\upsilon^* = \max \sum_{j \in E_0} \lambda_j \tag{6}$$

Subject to:

$$\sum_{j \in E_o} \lambda_j x_{ij} \le x_{io}, \;\; i = 1,2,3,...,m$$

$$\sum_{j \in E_o} \lambda_j y_{rj} \ge \phi^* y_{ro}, \;\; r = 1,2,3,...,s$$

$$\lambda_j \ge 0, \;\; j \in E_o$$

The largest MPSS for DMU_o (x_{io} , y_{ro}) are $\widehat{x}_{io} = \frac{x_{io}}{\phi^* \vartheta^*}$ and $\widehat{y}_{ro} = \frac{y_{ro}}{\vartheta^*}$, and the smallest MPSS for DMU_o are $\breve{x}_{io} = \frac{x_{io}}{\phi^* \upsilon^*}$ and $\breve{y}_{io} = \frac{y_{ro}}{\upsilon^*}$. Seiford and Thrall in [23] demonstrated that $\mathrm{MPSS}_{\mathrm{max}}$ and $\mathrm{MPSS}_{\mathrm{min}}$ remains the same under both orientations.

3 Results and Discussion

In this study we observed the sample of 27 DMUs. The observed DMUs are public postal operators (PPOs) in the countries of European Union (Austria, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Great Britain, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden) and the PPO in Serbia. We employed 3 inputs (the number of full-time staff (x_1), the number of part-time staff (x_2) and total number of permanent post offices (x_3)) and one output (the number of letter-post items, domestic services (y_1)) for evaluating the stability of the RTS classifications and scale efficient targets of PPOs. There are two types of reasons for selecting these particular input and output. The first and essential reason is that chosen input parameters (human capital and infrastructure) imply the largest part of the total costs for public postal operator functioning. On the other hand, the output that refers to the letter post produces the largest part of revenue. The second reason lies in the fact that we had an intention to use available data from the same database which was a constraint in the selection of input and output. Input and output data are listed in Table 1.

Table 1
Data of 27 public postal operators[1]

PPO No.	PPO Name	x_1	x_2	x_3	y_1
PPO_1	Austria	17233	3882	1880	6215000000
PPO_2	Bulgaria	8689	3796	2981	19159655
PPO_3	Cyprus	714	1034	1082	58787116
PPO_4	Czech Republic	28232	8020	3408	2574778260
PPO_5	Denmark	12800	6200	795	800000000

[1] source: Universal Postal Union (2013), http://pls.upu.int/pls/ap/ssp_report.main?p_language=AN&p_choice=BROWSE

PPO_6	Estonia	2290	502	343	25837400
PPO_7	Finland	20077	7508	978	837000000
PPO_8	France	204387	25900	17054	14900000000
PPO_9	Germany	512147	0	13000	19784000000
PPO_{10}	Great Britain	117206	38558	11818	18074291171
PPO_{11}	Greece	9060	28	1546	446505500
PPO_{12}	Hungary	28592	5368	2746	857056665
PPO_{13}	Ireland	7825	1584	1156	614320000
PPO_{14}	Italy	133426	11025	13923	4934317901
PPO_{15}	Latvia	2438	2055	571	28886614
PPO_{16}	Lithuania	2336	4226	715	36599075
PPO_{17}	Luxembourg	950	547	116	110800000
PPO_{18}	Malta	490	123	63	35123154
PPO_{19}	Netherlands	13141	46590	2600	3777000000
PPO_{20}	Poland	77548	16534	8207	822176000
PPO_{21}	Portugal	11608	315	2556	868548000
PPO_{22}	Romania	32630	1319	5827	292635204
PPO_{23}	Slovakia	9650	5081	1589	425743495
PPO_{24}	Slovenia	6344	161	556	1013027273
PPO_{25}	Spain	65924	0	3183	5123200000
PPO_{26}	Sweden	19222	2918	1924	2231000000
PPO_{27}	Serbia	14659	280	1507	243130583

Given data were obtained from Universal Postal Union for the year 2011. Considering the 27 European Union member states, there is only one PPO that was not included in the research. It is PPO in Belgium for which there were no official data on the website of Universal Postal Union in the moment of this research. Beside that PPO in Serbia as a candidate country was included in observed production possibility set consisting of PPOs in European Union member states.

By reviewing the literature on Thomson Reuters Web of Science[2], considering years from 1996 to 2014, we have not found the examples of using a RTS in DEA in postal sector. This was an inspiration for the authors to demonstrate the applicability of this analytical process in this field.

All calculations in the study are performed by using the software DEA Excel Solver developed by Zhu [24]. It is a Microsoft Excel Add-In for solving data envelopment analysis (DEA) models.

[2] http://apps.webofknowledge.com/

By using the M1 and the Theorem 1 we derived RTS classification of observed PPOs. The M1 model evolved according to the selected input and output and applied to the sample from Table 1, e.g. the PPO in Czech Republic is:

$$\theta^* = \min \theta$$

Subjec to:

$$\begin{aligned}&17233\lambda_1 + 8689\lambda_2 + 714\lambda_3 + 28232\lambda_4 + 12800\lambda_5 + 2290\lambda_6 + 20077\lambda_7\\&+ 204387\lambda_8 + 512147\lambda_9 + 117206\lambda_{10} + 9060\lambda_{11} + 28592\lambda_{12} + 7825\lambda_{13}\\&+ 133426\lambda_{14} + 2438\lambda_{15} + 2336\lambda_{16} + 950\lambda_{17} + 490\lambda_{18} + 13141\lambda_{19} + 77548\lambda_{20}\\&+ 11608\lambda_{21} + 32630\lambda_{22} + 9650\lambda_{23} + 6344\lambda_{24} + 65924\lambda_{25} + 19222\lambda_{26}\\&+ 14659\lambda_{27} \le 28232\theta\end{aligned}$$

$$\begin{aligned}&3882\lambda_1 + 3796\lambda_2 + 1034\lambda_3 + 8020\lambda_4 + 6200\lambda_5 + 502\lambda_6 + 7508\lambda_7\\&+ 25900\lambda_8 + 0\lambda_9 + 38558\lambda_{10} + 28\lambda_{11} + 5368\lambda_{12} + 1584\lambda_{13} + 11025\lambda_{14}\\&+ 2055\lambda_{15} + 4226\lambda_{16} + 547\lambda_{17} + 123\lambda_{18} + 46590\lambda_{19} + 16534\lambda_{20} + 315\lambda_{21}\\&+ 1319\lambda_{22} + 5081\lambda_{23} + 161\lambda_{24} + 0\lambda_{25} + 2918\lambda_{26} + 280\lambda_{27} \le 8020\theta\end{aligned}$$

$$\begin{aligned}&1880\lambda_1 + 2981\lambda_2 + 1082\lambda_3 + 3408\lambda_4 + 795\lambda_5 + 343\lambda_6 + 978\lambda_7 + 17054\lambda_8\\&+ 13000\lambda_9 + 11818\lambda_{10} + 1546\lambda_{11} + 2746\lambda_{12} + 1156\lambda_{13} + 13923\lambda_{14} + 571\lambda_{15}\\&+ 715\lambda_{16} + 116\lambda_{17} + 63\lambda_{18} + 2600\lambda_{19} + 8207\lambda_{20} + 2556\lambda_{21} + 5827\lambda_{22}\\&+ 1589\lambda_{23} + 556\lambda_{24} + 3183\lambda_{25} + 1924\lambda_{26} + 1507\lambda_{27} \le 3408\theta\end{aligned}$$

$$\begin{aligned}&6215000000\lambda_1 + 19159655\lambda_2 + 58787116\lambda_3 + 2574778260\lambda_4\\&+ 800000000\lambda_5 + 25837400\lambda_6 + 837000000\lambda_7 + 14900000000\lambda_8\\&+ 19784000000\lambda_9 + 18074291171\lambda_{10} + 446505500\lambda_{11} + 857056665\lambda_{12}\\&+ 614320000\lambda_{13} + 4934317901\lambda_{14} + 28886614\lambda_{15} + 36599075\lambda_{16}\\&+ 110800000\lambda_{17} + 35123154\lambda_{18} + 3777000000\lambda_{19} + 822176000\lambda_{20}\\&+ 868548000\lambda_{21} + 292635204\lambda_{22} + 425743495\lambda_{23} + 1013027273\lambda_{24}\\&+ 5123200000\lambda_{25} + 2231000000\lambda_{26} + 243130583\lambda_{27} \ge 2574778260\end{aligned}$$

$$\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{27} \ge 0$$

The optimal solution for this PPO is:

$$\theta^* = \theta^*_{CCR} = 0.25288$$

$$\phi^* = \frac{1}{\theta^*} = \frac{1}{\theta^*_{CCR}} = \frac{1}{0.25288} = 3.95444$$

$\lambda_1^* = 0.41428,$ other $\lambda_j^* = 0 \Rightarrow \sum_{j=1}^{27} \lambda_j^* < 1 \Rightarrow$ the PPO in Czech Republic exhibits IRS. Since $\lambda_1^* > 0,$ the reference for this PPO is the PPO in Austria.

The BCC score (θ_{BCC}^*) is obtained by adding the following condition in the M1 model:

$\lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_{27} = 1$

The BCC score for this PPO is:

$\theta_{BCC}^* = 0.26107$

In the same manner, the M1 model should be evolved for all other 26 PPOs. The CCR, BCC and returns to scale characteristics of each PPO are listed in Table 2.

Table 2
Analytical results derived from the M1 model

PPO No.	RTS Region	BCC	CCR				Scale Score (SS)
		Score (θ_{BCC}^*)	Score (θ_{CCR}^*)	Reference	$\sum_{j \in E_o} \lambda_j^*$	Input-oriented RTS	$SS = \frac{\theta_{CCR}^*}{\theta_{BCC}^*}$
PPO_1	II	1.00000	1.00000		1.00000	Constant	1.00000
PPO_2	I	0.05639	0.00611	PPO_1	0.00308	Increasing	0.10842
PPO_3	I	0.77607	0.22830	PPO_1	0.00946	Increasing	0.29417
PPO_4	VI	0.26107	0.25288	PPO_1	0.41428	Increasing	0.96862
PPO_5	I	0.36212	0.30440	PPO_1	0.12872	Increasing	0.84059
PPO_6	I	0.24353	0.03182	PPO_1, PPO_{24}	0.00445	Increasing	0.13067
PPO_7	I	0.30549	0.25888	PPO_1	0.13467	Increasing	0.84744
PPO_8	III	0.80392	0.30588	PPO_1, PPO_{24}, PPO_{25}	2.70674	Decreasing	0.38048
PPO_9	III	1.00000	0.94551	PPO_{25}	3.86165	Decreasing	0.94551
PPO_{10}	III	1.00000	0.46263	PPO_1	2.90817	Decreasing	0.46263
PPO_{11}	I	1.00000	0.56198	PPO_{24}, PPO_{25}	0.16556	Increasing	0.56198
PPO_{12}	VI	0.11309	0.09707	PPO_1, PPO_{25}	0.13869	Increasing	0.85834
PPO_{13}	I	0.28302	0.23249	PPO_1, PPO_{24}	0.12627	Increasing	0.82146
PPO_{14}	III	0.20119	0.17023	PPO_1, PPO_{24}	2.93233	Decreasing	0.84611
PPO_{15}	I	0.20098	0.03285	PPO_1	0.00465	Increasing	0.16346
PPO_{16}	I	0.21147	0.04344	PPO_1	0.00589	Increasing	0.20543
PPO_{17}	I	0.73492	0.32340	PPO_1	0.01783	Increasing	0.44005
PPO_{18}	I	1.00000	0.19875	PPO_1	0.00565	Increasing	0.19875
PPO_{19}	I	0.80875	0.79696	PPO_1	0.60772	Increasing	0.98543

PPO_{20}	VI	0.03610	0.03069	PPO_1, PPO_{25}	0.13263	Increasing	0.85018
PPO_{21}	VI	0.48475	0.46344	PPO_1, PPO_{24}	0.84328	Increasing	0.95604
PPO_{22}	VI	0.08887	0.05130	PPO_1, PPO_{24}	0.25133	Increasing	0.57726
PPO_{23}	I	0.16045	0.12233	PPO_1	0.06850	Increasing	0.76245
PPO_{24}	II	1.00000	1.00000		1.00000	Constant	1.00000
PPO_{25}	II	1.00000	1.00000		1.00000	Constant	1.00000
PPO_{26}	VI	0.41553	0.40714	PPO_1, PPO_{24}, PPO_{25}	0.70363	Increasing	0.97983
PPO_{27}	VI	0.34138	0.11897	PPO_{24}, PPO_{25}	0.21345	Increasing	0.34851
Average		0.51441	0.34991				0.64940

Based on the results in Column 2 of Table 2 the PPOs are located in four RTS regions I, II, III and VI as shown in Figure 1. The regions IV and V are empty.

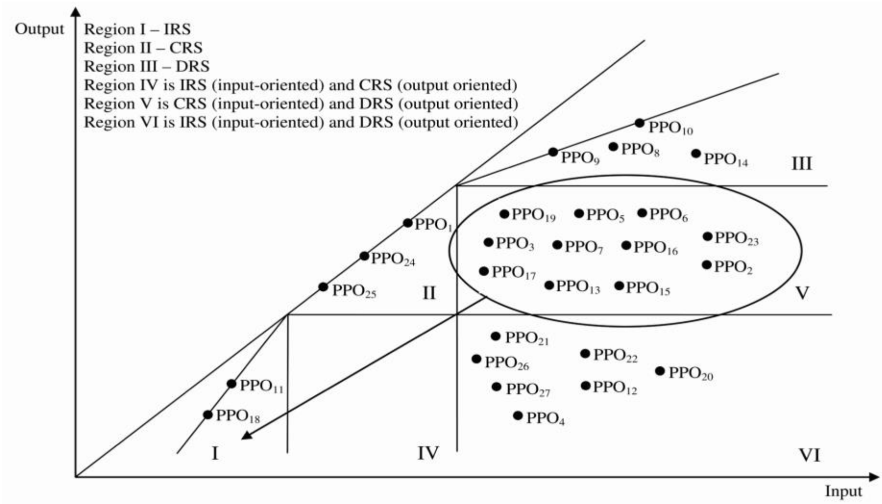

Figure 1

PPOs locating within the RTS regions

The results from Table 2 show that there are three PPOs which have the CCR score equal to 1. This score indicates overall technical efficiency when evaluated on the constant returns to scale assumption. These are PPOs in Austria, Slovenia and Spain. PPO in Austria is one of three best performers, and furthermore it is the PPO most frequently referenced for evaluating inefficient PPOs.

The BCC score provide efficiency evaluations using a local measure of scale, i.e. under variable returns to scale. In this empirical example four PPOs are accorded efficient status in addition to the three CCR efficient PPOs which retain their previous efficient status. These four PPOs are in Germany, Great Britain, Greece and Malta. For example, it can be concluded that PPO in Greece has the efficient operations ($\theta^*_{BCC} = 1$). Additionally, it can be considered that all PPOs having the BCC score above average (0.51441) have the efficient operations. These are PPOs in Austria, Cyprus, France, Germany, Great Britain, Greece, Luxembourg, Malta, Netherlands, Slovenia and Spain.

Based on the results of scale scores from Table 2 the following PPOs operate in the advantageous conditions: Austria, Czech Republic, Denmark, Finland, Germany, Hungary, Ireland, Italy, Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain and Sweden. Their scale scores are higher than average value (0.64940). Some of them although working in the advantageous conditions have the inefficient operations. We can notice the examples of PPOs in Czech Republic, Poland and Portugal. There are the opposite cases where PPOs work in the disadvantageous conditions but their operations are above average, for example PPOs in Cyprus and Luxembourg. Further there are PPOs operating in the disadvantageous conditions and having the inefficient operations such as PPOs in Bulgaria, Estonia, Latvia, Lithuania, Romania, Slovakia and Serbia.

By using the M2 and M2' models and the Theorem 2, 3 and 4 we derived lower and upper limit of stability intervals of PPOs. For example, the PPO in Czech Republic exhibits IRS, therefore it needs to use the Theorem 4 and the M2' model should be evolved:

$$\mu_o^* = \frac{1}{\max \lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_{27}}$$

Subjec to:

$$17233\lambda_1 + 8689\lambda_2 + 714\lambda_3 + 28232\lambda_4 + 12800\lambda_5 + 2290\lambda_6 + 20077\lambda_7$$
$$+ 204387\lambda_8 + 512147\lambda_9 + 117206\lambda_{10} + 9060\lambda_{11} + 28592\lambda_{12} + 7825\lambda_{13}$$
$$+ 133426\lambda_{14} + 2438\lambda_{15} + 2336\lambda_{16} + 950\lambda_{17} + 490\lambda_{18} + 13141\lambda_{19} + 77548\lambda_{20}$$
$$+ 11608\lambda_{21} + 32630\lambda_{22} + 9650\lambda_{23} + 6344\lambda_{24} + 65924\lambda_{25} + 19222\lambda_{26}$$
$$+ 14659\lambda_{27} \leq 28232$$

$$3882\lambda_1 + 3796\lambda_2 + 1034\lambda_3 + 8020\lambda_4 + 6200\lambda_5 + 502\lambda_6 + 7508\lambda_7$$
$$+ 25900\lambda_8 + 0\lambda_9 + 38558\lambda_{10} + 28\lambda_{11} + 5368\lambda_{12} + 1584\lambda_{13} + 11025\lambda_{14}$$
$$+ 2055\lambda_{15} + 4226\lambda_{16} + 547\lambda_{17} + 123\lambda_{18} + 46590\lambda_{19} + 16534\lambda_{20} + 315\lambda_{21}$$
$$+ 1319\lambda_{22} + 5081\lambda_{23} + 161\lambda_{24} + 0\lambda_{25} + 2918\lambda_{26} + 280\lambda_{27} \leq 8020$$

$$1880\lambda_1 + 2981\lambda_2 + 1082\lambda_3 + 3408\lambda_4 + 795\lambda_5 + 343\lambda_6 + 978\lambda_7 + 17054\lambda_8$$
$$+ 13000\lambda_9 + 11818\lambda_{10} + 1546\lambda_{11} + 2746\lambda_{12} + 1156\lambda_{13} + 13923\lambda_{14} + 571\lambda_{15}$$
$$+ 715\lambda_{16} + 116\lambda_{17} + 63\lambda_{18} + 2600\lambda_{19} + 8207\lambda_{20} + 2556\lambda_{21} + 5827\lambda_{22}$$
$$+ 1589\lambda_{23} + 556\lambda_{24} + 3183\lambda_{25} + 1924\lambda_{26} + 1507\lambda_{27} \leq 3408$$

$$6215000000\,\lambda_1 + 19159655\,\lambda_2 + 58787116\,\lambda_3 + 2574778260\,\lambda_4$$
$$+800000000\,\lambda_5 + 25837400\,\lambda_6 + 837000000\,\lambda_7 + 14900000000\,\lambda_8$$
$$+19784000000\,\lambda_9 + 18074291171\,\lambda_{10} + 446505500\,\lambda_{11}$$
$$+857056665\,\lambda_{12} + 614320000\,\lambda_{13} + 4934317901\,\lambda_{14} + 28886614\,\lambda_{15}$$
$$+36599075\,\lambda_{16} + 110800000\,\lambda_{17} + 35123154\,\lambda_{18} + 3777000000\,\lambda_{19}$$
$$+822176000\,\lambda_{20} + 868548000\,\lambda_{21} + 292635204\,\lambda_{22} + 425743495\,\lambda_{23}$$
$$+1013027273\,\lambda_{24} + 5123200000\,\lambda_{25} + 2231000000\,\lambda_{26}$$
$$+243130583\,\lambda_{27} \geq 2574778260 * 3.95444$$

$$\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{27} \geq 0$$

Inputs lower limit of stability interval of this PPO is $\mu_o^* = 0.6104066.$ According to Theorem 4, inputs upper limit of stability interval of this PPO is equal to 1. Analogously, we can define stability region for inputs of all other 26 PPOs. The analytical results are shown in Table 3.

Table 3

Stability region for inputs of PPOs

PPO No.	Stability interval	PPO No.	Stability interval
PPO_1	(1.0000000, 1.0000000)	PPO_{15}	(1.0000000, 7.0684988)
PPO_2	(1.0000000, 1.9833122)	PPO_{16}	(1.0000000, 7.3771404)
PPO_3	(1.0000000, 24.1358543)	PPO_{17}	(1.0000000, 18.1400000)
PPO_4	(0.6104066, 1.0000000)	PPO_{18}	(1.0000000, 35.1693878)
PPO_5	(1.0000000, 2.3647799)	PPO_{19}	(1.0000000, 1.3113918)
PPO_6	(1.0000000, 7.1535973)	PPO_{20}	(0.2313787, 1.0000000)
PPO_7	(1.0000000, 1.9222904)	PPO_{21}	(0.5495740, 1.0000000)
PPO_8	(0.1130049, 1.0000000)	PPO_{22}	(0.2041303, 1.0000000)
PPO_9	(0.2448462, 1.0000000)	PPO_{23}	(1.0000000, 1.7858031)
PPO_{10}	(0.1590794, 1.0000000)	PPO_{24}	(1.0000000, 1.0000000)
PPO_{11}	(1.0000000, 3.3943407)	PPO_{25}	(1.0000000, 1.0000000)
PPO_{12}	(0.6999022, 1.0000000)	PPO_{26}	(0.5786327, 1.0000000)
PPO_{13}	(1.0000000, 1.8412285)	PPO_{27}	(0.5573724, 1.0000000)
PPO_{14}	(0.0580535, 1.0000000)		

The results from Table 3 indicate that PPOs in Austria, Slovenia and Spain do not need input perturbations. PPOs in Czech Republic, France, Germany, Great Britain, Hungary, Italy, Poland, Portugal, Romania, Sweden and Serbia should consider decreasing inputs. PPOs in Bulgaria, Cyprus, Denmark, Estonia, Finland, Greece, Ireland, Latvia, Lithuania, Luxembourg, Malta, Netherlands and Slovakia should consider increasing inputs.

By using the M3 and M3' models we derived scale efficient inputs and output targets for each CCR inefficient PPOs. Thus, the M3 and M3' models for PPO in Czech Republic are:

M3 model

$$\vartheta^* = \min \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_{27}$$

M3' model

$$\upsilon^* = \max \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_{27}$$

Subjec to:

$$\begin{aligned}&17233\lambda_1 + 8689\lambda_2 + 714\lambda_3 + 28232\lambda_4 + 12800\lambda_5 + 2290\lambda_6 + 20077\lambda_7\\&+ 204387\lambda_8 + 512147\lambda_9 + 117206\lambda_{10} + 9060\lambda_{11} + 28592\lambda_{12} + 7825\lambda_{13}\\&+ 133426\lambda_{14} + 2438\lambda_{15} + 2336\lambda_{16} + 950\lambda_{17} + 490\lambda_{18} + 13141\lambda_{19} + 77548\lambda_{20}\\&+ 11608\lambda_{21} + 32630\lambda_{22} + 9650\lambda_{23} + 6344\lambda_{24} + 65924\lambda_{25} + 19222\lambda_{26}\\&+ 14659\lambda_{27} \le 28232\end{aligned}$$

$$\begin{aligned}&3882\lambda_1 + 3796\lambda_2 + 1034\lambda_3 + 8020\lambda_4 + 6200\lambda_5 + 502\lambda_6 + 7508\lambda_7\\&+ 25900\lambda_8 + 0\lambda_9 + 38558\lambda_{10} + 28\lambda_{11} + 5368\lambda_{12} + 1584\lambda_{13} + 11025\lambda_{14}\\&+ 2055\lambda_{15} + 4226\lambda_{16} + 547\lambda_{17} + 123\lambda_{18} + 46590\lambda_{19} + 16534\lambda_{20} + 315\lambda_{21}\\&+ 1319\lambda_{22} + 5081\lambda_{23} + 161\lambda_{24} + 0\lambda_{25} + 2918\lambda_{26} + 280\lambda_{27} \le 8020\end{aligned}$$

$$\begin{aligned}&1880\lambda_1 + 2981\lambda_2 + 1082\lambda_3 + 3408\lambda_4 + 795\lambda_5 + 343\lambda_6 + 978\lambda_7 + 17054\lambda_8\\&+ 13000\lambda_9 + 11818\lambda_{10} + 1546\lambda_{11} + 2746\lambda_{12} + 1156\lambda_{13} + 13923\lambda_{14} + 571\lambda_{15}\\&+ 715\lambda_{16} + 116\lambda_{17} + 63\lambda_{18} + 2600\lambda_{19} + 8207\lambda_{20} + 2556\lambda_{21} + 5827\lambda_{22}\\&+ 1589\lambda_{23} + 556\lambda_{24} + 3183\lambda_{25} + 1924\lambda_{26} + 1507\lambda_{27} \le 3408\end{aligned}$$

$$\begin{aligned}&6215000000\lambda_1 + 19159655\lambda_2 + 58787116\lambda_3 + 2574778260\lambda_4\\&+ 800000000\lambda_5 + 25837400\lambda_6 + 837000000\lambda_7 + 14900000000\lambda_8\\&+ 19784000000\lambda_9 + 18074291171\lambda_{10} + 446505500\lambda_{11}\\&+ 857056665\lambda_{12} + 614320000\lambda_{13} + 4934317901\lambda_{14} + 28886614\lambda_{15}\\&+ 36599075\lambda_{16} + 110800000\lambda_{17} + 35123154\lambda_{18} + 3777000000\lambda_{19}\\&+ 822176000\lambda_{20} + 868548000\lambda_{21} + 292635204\lambda_{22} + 425743495\lambda_{23}\\&+ 1013027273\lambda_{24} + 5123200000\lambda_{25} + 2231000000\lambda_{26}\\&+ 243130583\lambda_{27} \ge 2574778260 * 3.95444\end{aligned}$$

$$\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{27} \ge 0$$

The optimal solution for this PPO is:

$$\vartheta^* = \upsilon^* = 0.41428$$

The smallest inputs for the PPO are:

$$\breve{x}_1 = \frac{x_{1o}}{\phi^* \upsilon^*} = \frac{28232}{3.95444 * 0.41428} = 17233$$

$$\breve{x}_2 = \frac{x_{2o}}{\phi^* \upsilon^*} = \frac{8020}{3.95444 * 0.41428} = 4895$$

$$\breve{x}_3 = \frac{x_{3o}}{\phi^* \upsilon^*} = \frac{3408}{3.95444 * 0.41428} = 2080$$

The largest output for the PPO is:

$$\widehat{y}_{1o} = \frac{y_{1o}}{\vartheta^*} = \frac{2574778260}{0.41428} = 6215000000$$

Analogously, the M3 and M3' models should be evolved for all other 26 PPOs. The analytical results are shown in Table 4.

Table 4

Analytical results derived from M3 and M3' models

PPO No.	PPO Name	$\breve{x}_1$	$\widehat{x}_1$	$\breve{x}_2$	$\widehat{x}_2$	$\breve{x}_3$	$\widehat{x}_3$	$\breve{y}_1$	$\widehat{y}_1$
PPO_2	Bulgaria	8689	17233	3796	7529	2981	5912	19159655	6215000000
PPO_3	Cyprus	714	17233	1034	24956	1082	26115	58787116	6215000000
PPO_4	Czech R.	17233	28232	4895	8020	2080	3408	2574778260	6215000000
PPO_5	Denmark	12800	30269	6200	14662	795	1880	800000000	6215000000
PPO_6	Estonia	2290	16382	502	3591	343	2454	25837400	5808328848
PPO_7	Finland	20077	38594	7508	14433	978	1880	837000000	6215000000
PPO_8	France	23097	204387	2927	25900	1927	17054	5504774313	14900000000
PPO_9	Germany	125397	512147	0	0	3183	13000	5123200000	19784000000
PPO_{10}	GB	18645	117206	6134	38558	1880	11818	6215000000	18074291171
PPO_{11}	Greece	9060	30753	28	95	1546	5248	446505500	2696882321
PPO_{12}	Hungary	20012	28592	3757	5368	1922	2746	857056665	6179865198
PPO_{13}	Ireland	7825	14408	1584	2917	1156	2128	614320000	4865235527
PPO_{14}	Italy	7746	133426	640	11025	808	13923	1682726646	4934317901
PPO_{15}	Latvia	2438	17233	2055	14526	571	4036	28886614	6215000000
PPO_{16}	Lithuania	2336	17233	4226	31176	715	5275	36599075	6215000000
PPO_{17}	Luxem.	950	17233	547	9923	116	2104	110800000	6215000000
PPO_{18}	Malta	490	17233	123	4326	63	2216	35123154	6215000000
PPO_{19}	Nether.	13141	17233	46590	61098	2600	3410	3777000000	6215000000
PPO_{20}	Poland	17943	77548	3826	16534	1899	8207	822176000	6199142219

PPO_{21}	Portugal	6379	11608	173	315	1405	2556	868548000	1029965235
PPO_{22}	Romania	6661	32630	269	1319	1189	5827	292635204	1164358279
PPO_{23}	Slovakia	9650	17233	5081	9074	1589	2838	425743495	6215000000
PPO_{26}	Sweden	11122	19222	1688	2918	1113	1924	2231000000	3170687665
PPO_{27}	Serbia	8171	14659	156	280	840	1507	243130583	1139031351

Considering the results from Table 4, PPOs that need to perform input perturbations can be divided in three groups. The first group contains of PPOs with the input excess and the output deficit. Based on the results from Table 4 these PPOs are in Czech Republic, Hungary, Poland, Portugal, Romania, Sweden and Serbia. For example, PPO in Serbia has the input excess of the number of full-time staff, the number of part-time staff and total number of permanent post offices, $\widehat{x}_1 - \breve{x}_1 = 6488$, $\widehat{x}_2 - \breve{x}_2 = 124$, $\widehat{x}_3 - \breve{x}_3 = 667$, respectively. Additionally, this PPO has the output deficit of the number of letter-post items, domestic services, $\widehat{y}_3 - \breve{y}_3 = 895900768$. In the second group there are PPOs having the input excess. This means they could achieve the current output level with less inputs. The examples of this kind of PPOs are in France, Germany, Great Britain and Italy. The rest of PPOs are in the third group. The main characteristic of these PPOs is the possibility of increasing output by increased inputs. These PPOs are in Bulgaria, Cyprus, Denmark, Estonia, Finland, Greece, Ireland, Latvia, Lithuania, Luxembourg, Malta, Netherlands and Slovakia.

Obtained values from Table 4 should be considered conditionally having in mind the public expectations about postal systems, first of all the obligation to provide postal services on the whole territory of a state. Thus, in order to implement the proposed model further research should be performed for each specific country considering the legal limitations.

Conclusions

Many DEA researchers have studied the sensitivity analysis of efficient and inefficient decision making unit classifications. This study develops a RTS in DEA and the methods to estimate it in the postal sector. The sensitivity analysis is conducted for the CCR inefficient public postal operators in European Union member states and Serbia as a candidate country. The development of this analytical process is performed based on the public data obtained from the same source from Universal Postal Union.

The focus of this study was on the stability of the RTS classifications and scale efficient inputs and outputs targets of observed PPOs. It has been carried out by using the CCR RTS method and the MPSS. In order to determine lower and upper limit of stability intervals of the CCR inefficient PPOs we used output-oriented RTS classification stability when input perturbations occur in PPOs.

In order to implement the obtained results, PPOs should have in mind their legal obligations specific for the postal sector in their countries. This could be one of the possible guidelines for future research.

In this paper we used cross-section type of data. As a possible direction for the future research panel data could be used involving the efficiency measurement over time. This should be carried out in order to confirm the obtained results.

Acknowledgement

This research was supported by Serbian Ministry of Education, Science and Technological Development with project TR 36022.

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