Multiple Components Fixed Point Iteration in the Adaptive Control of Single Variable 2nd Order Systems

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Abstract: In various fields of applications as robotics, life sciences, chemistry, typical non-linear systems are controlled for the description of which only imprecise and often partial system models are available and the model-based approaches have to be completed by either robust or adaptive solutions for the compensation of the effects of the models' imperfectness and imprecisions. An alternative approach transformed the control task into a fixed point iteration and tried to solve it in real-time on the basis of Banach's fixed point theorem. This approach does not require complete state estimation. However, it needs feeding back noisy signals that may corrupt the quality of such adaptive controllers. In the preliminary investigations in [1] this multiple variable solution was utilized for the control of a single variable 1st order system so that it yielded some noise filtering possibility. In the present paper this method is extended for the control of single input - single output 2nd order dynamical systems by applying it to shorter time series of the state variable. The operation of this method is investigated via numerical simulations for the control of a strongly nonlinear, oscillating dynamical system, the van der Pol oscillator that further was modified with the introduction of time-delay effects. The simulations are promising.

Keywords: Modeling Errors; Measurement Noise; Adaptive Control; Fixed Point Iteration-based Adaptive Control; Banach Space; van der Pol Oscillator; Time Delay;

1 Introduction

In the adaptive control of nonlinear systems the model linearization around some "point of operation" can work well if the controlled system's actual state dwells in the close vicinity of this point. In more general cases, e.g. in robotics, the dynamic properties of the system drastically vary with the state variables and such a simple approach is not satisfactory. In such cases the prevailing controller design method is Lyapunov's 2^{nd} or "direct" method he elaborated for the investigation of the stability of motion of nonlinear systems in his PhD Thesis [2]. In the sixties of the past century the translation of his work (originally written in Russian) became known in the western world (e.g. [3]), and after the nineties it became the "number 1 design method" for adaptive controllers. Some relevant early work is for example [4]. The main point in this ingenious design methodology is that though it is impossible to obtain the solution of the equations of motion of the controlled system in "closed analytical form", various stability properties can be defined and guaranteed without knowing all the "details" of the solution. For this purpose sophisticated estimation techniques have been developed.

In spite of the development of the numerical techniques Lyapunov's method is quite relevant in our days since for the description of the behavior of the unstable systems the "validity" of the numerical solutions is always limited in time. With regard to the application of Lyapunov's "estimation techniques", for a given particular problem a particular Lyapunov function has to be constructed, and this function has to be kept between upper and lower limits determined by some "type κ" functions of the norm of the trajectory tracking error. In this manner "global" or "uniformly global" stability properties can be guaranteed. Furthermore, if it can be guaranteed that the time-derivative of the Lyapunov function is "negative enough", "global asymptotic" or "uniformly global asymptotic stability" can be guaranteed. In control technology normally quadratic Lyapunov functions are used for which the asymptotic stability can be proved by the use of Barbalat's lemma (e.g. [5]). In the control systems that suffer from time-delay effects the Lyapunov-Krasovskii functional can be used in the controller design (e.g. [6]) that mathematically corresponds to some extension of the Lyapunov function.

In control applications the Lyapunov function-based design can be criticized from certain practical points of view [7] as follows:

a) it works by meeting *satisfactory conditions* instead of *necessary and satisfactory* ones, therefore "too restrictive" properties have to be proved for its use,

b) in the Lyapunov function-based design approach the main "design intent" is guaranteeing only stability or asymptotic stability without paying appropriate attention to the initial phase of the motion; for instance, in life sciences these "transients" may have lethal consequences, therefore they deserve more attention than that they obtain in this "conventional" design methodology; more precisely, in the resulting control signal some "fragments" of the Lyapunov function normally are present; these fragments contain numerous free parameters for which wide ranges of settings can guarantee the stability (even asymptotic stability), while they considerably influence the initial phase of the motion, too; to achieve appropriate operation these free parameters cannot be "trivially tuned", so their setting may need the application of evolutionary methods (e.g. [8]);

c) the Lyapunov function normally uses each component of the state variable, so for the application of this methodology these components must be either directly measured or at least estimated; reliable estimation is possible only in the possession of satisfactory number of independent sensor signals; if no such sensors are available –that is a typical situation in life sciences and other technical fields–, such estimations can be made by the use of certain "system models" that themselves are not very precise or reliable; with regard to this problem it is just enough to refer to the different insulin – glucose system models that contain various numbers of compartments (e.g. [9]) or the latest modeling and measuring efforts developed for turbo jet engines (e.g. the utilization of the near magnetic field [10], thermal imaging [11]) for the control of which rather "practical" than mathematically too sophisticated control approaches as the "Situational Control" [12] or robust methods [13] can be applied.

In an alternative technique suggested in [14], the control task was transformed into iteratively finding the solution of a fixed point problem as follows. By the use a purely kinematic design, the appropriate *"Desired"* time-derivative of the generalized coordinate of the system under control (it is referred to as the "relative order of the control task") is calculated by the use of lower order derivatives and the time-integral of the tracking error. Since this derivative abruptly can vary with the control signal, by its use a "slowly meandering" fixed point can be constructed that can be so tracked that during each digital control cycle only one step of the adaptive iteration can be completed. The method's scheme is outlined in Fig. 1. In the box named *"Kinematic Block"* an arbitrary design can be applied for the desired time-derivative of the generalized coordinate. In the case of a first order system the system's response $r(t) \equiv \dot{q}(t)$, and the desired $\dot{q}^{Des}(t)$ has to guarantee that $\left|q^N(t) - q(t)\right| \to 0$ as $t \to \infty$, *if it is realized*. For instance, by using the *integrated tracking error* a PI-type feedback can be constructed by using a constant $\Lambda > 0$ as

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + \Lambda\right)^2 e_{int}(t) \equiv \left(\frac{\mathrm{d}}{\mathrm{d}t} + \Lambda\right)^2 \int_{t_0}^{t} \left[q^N(\xi) - q(\xi)\right] \mathrm{d}\xi \equiv 0 \;\; \Rightarrow \tag{1a}$$

$$\dot{q}^{Des}(t) = \Lambda^2 e_{int}(t) + 2\Lambda e(t) + \dot{q}^N(t) \;\; , \tag{1b}$$

in which $e(\xi) \stackrel{def}{=} q^N(\xi) - q(\xi)$ is the tracking error, and its integral from t_0 (the commencement of the control action) to the actual time t is given within (1a). It worths noting that further integration of the integrated error in (1) in principle can be used together with a higher power of the operator $\left(\frac{\mathrm{d}}{\mathrm{d}t} + \Lambda\right)$ that could lead to "more fluctuations" in the transient part of the error. However, taking into account that the sequence of multiple integrals can be expressed by a single formula as the *Riemann-Liouville n-fold integral* (e.g. [15]), and that on this basis, by the generalization of this formula, various *fractional order integrals and derivatives* can be introduced that can produce nice monotonic error relaxation, fractional order controllers can be constructed. (A survey on the history of fractional calculus can be found in [16].) The fractional order controllers recently became very popular in robotics (e.g. [17]), in the control of flexible systems (e.g. [18]), in vibration damping (e.g. [19]) and generally in relation with feedback problems (e.g. [20]).

Returning to the scheme depicted in Fig. 1, if the available model is not precise, the $\dot{q}^{Des}(t)$ value is deformed into $\dot{q}^{Def}(t) \neq \dot{q}^{Des}(t)$ to achieve or at least better approximate the $\dot{q}(t) = \dot{q}^{Des}(t)$ situation. This case corresponds to the more precise realization of the tracking strategy formulated in (1). The aim is to approach it via iteration in which $u(t)$ immediately generates the actual $\dot{q}(t)$. Normally in the *"Delay"* boxes only 1 cycle time lag can be calculated that means that the system adaptively learns by computing the deformation in control cycle $n+1$ by considering the applied deformed value and the response obtained for it in cycle n. The iteration can be commenced with $\dot{q}^{Def}(t_{ini}) = \dot{q}^{Des}(t_{ini})$. If the controlled system has well known delay effects, the appropriate delays in these boxes can be modified accordingly, however, because in this case the controller learns from more or less obsolete observed data, some precision degradation in the control can be expected [21].

Mathematically the iterative deformations are calculated on the basis of Banach's fixed point theorem [22] according to which *"in a linear, normed, complete metric space" (i.e. Banach space* $\mathbb{B}$) the sequence generated by the contractive map $\Phi : \mathbb{B} \mapsto \mathbb{B}$ as $\{x_0, x_1 = \Phi(x_0), \dots, x_{n+1} = \Phi(x_n), \dots\}$ has a unique limit point $x_\star$ so that $x_n \to x_\star \in \mathbb{B}$. This limit point is a fixed point of $\Phi(x)$, that is $\Phi(x_\star) = x_\star$. By definition $\Phi(x)$ is contractive if $\exists 0 \leq K < 1$ so that $\forall x, y \in \mathbb{B}$ $\|\Phi(x) - \Phi(y)\| \leq K\|x - y\|$. Into the block *"Adaptive Deformation"* an appropriate function must be placed that realizes this contractive map. For higher order control essentially the same structure can be used. For instance, in the case of 2$^{\text{nd}}$ order systems as fully actuated robots, PD- or PID-type feedback structures suggested in Fig. 1 can be applied in the *"Kinematic Block"* in the case of the *Resolved Acceleration Rate Control* (e.g. [23]).

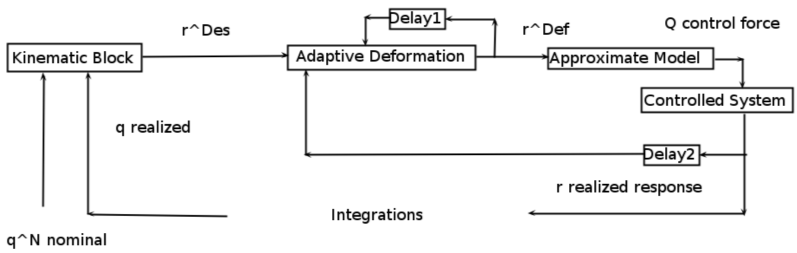

Figure 1
The structure of the *"Fixed Point Transformation-based Adaptive Controller"* (the adaptive deformation can be realized by the use of various fixed point transformations, and the system's response r can be an arbitrary order time-derivative of the generalized coordinates of the controlled system)

However, in our case, the higher order derivative is fed back, too. Such feedback also appears in the *"Acceleration Feedback Controllers"* (e.g. [24]) but in a quite different manner. These cited examples testify that in spite of the expectation that feeding back second time-derivatives may introduce too much noise into the control process, these methods can work. For instance, the above structure was successfully applied in [25] in the realization of the adaptive control of a cheap speed-controlled electric motor manufactured by a Chinese company without having the precise mathematical model of the motor. The adaptive loop was realized by a simple Arduino embedded microcontroller without using any sophisticated noise filtering technique. For testing adaptivity the rotating axle of the motor was picked by the fingers of the designer so varying the viscous damping in the system in real-time.

In the block *"Adaptive Deformation"* various functions can be placed. In [25] the single variable function introduced in [14] was applied. Later multiple variable functions were introduced by Dineva [26], and an abstract rotations-based software block was suggested in [27] by Csanádi et al. In this approach special orthogonal matrices were applied that were computed according to a generalization of the Rodrigues formula invented in 1840 [28] in the following manner. In the case of a vector transformation $b \in \mathbb{R}^n$ to vector $a \in \mathbb{R}^n$ ($\|a\| \neq \|b\|$), via introducing a *"buffer dimension"* the vectors $B \in \mathbb{R}^{n+1}$ and $A \in \mathbb{R}^{n+1}$ can be introduced with the components: for $s \in \{1, \dots, n\}$, $B_s = b_s$, $A_s = a_s$, and $B_{n+1} = \sqrt{R^2 - \|b\|^2}$, $A_{n+1} = \sqrt{R^2 - \|a\|^2}$. Evidently, $\|A\| = \|B\|$, therefore in the $n+1$ dimensional space these vectors can be rotated into each other. Furthermore, in the $n+1$ dimensional space the rotation operator that rotates B into A can be easily constructed (the details were published in [27], and a simple interpolation between the two positions can be obtained by a factor $\lambda_a \in [0,1]$ by rotating with $\lambda_a \varphi$ instead of φ that is the angle between the two vectors.

The noise sensitivity of the method generally can be reduced by various noise filtering techniques that can apply smoothed signals in the *"Kinematic Block"* for the calculation of the "Desired" time-derivatives, and for smoothing the observed response, too. In the present paper the idea that was investigated for a 1$^{\text{st}}$ order single variable system in [1] is extended to the adaptive control of a 2$^{\text{nd}}$ order one with the calculation of the *"desired"* 2$^{\text{nd}}$ order derivatives from pre-filtered terms. The paper is structured as follows: in Section 2 the dynamic model of the controlled system is detailed. Section 3 reveals the simulation results. The paper is closed with the conclusions and the acknowledgement section.

2 The Dynamic Model of the van der Pol Oscillator

In 1927 van der Pol modeled the nonlinear oscillations of an externally excited triode [29]. This model later served as a popular paradigm of nonlinear systems in control technology because it had an unstable equilibrium point in the state $q(0) = 0$, $\dot{q}(0) = 0$. If some small external perturbation kicks the system's state out of this equilibrium point, it produces nonsinusoidal oscillations that have to be "curbed" by the controller. In this paper this model is considered to be a "mathematical construction" only with a nondimensional "generalized coordinate" $q(t)$, and physical interpretation is given only to the variable of time t measured in $[s]$ units. In similar manner, the control signal $u(t)$ remains without physical interpretation. The original van der Pol model is further modified by the introduction of some delay time τ in the equations of motion given in (2). Certain parameters have some "physical analogy" like the *"inertia"*, the *"spring stiffness"* and the *"damping parameter"*. The *"separator parameter"* a determines the border between excitation and damping the strength of which is determined by parameter b_1. Parameter b_2 corresponds to the traditional viscous damping model.

$$\ddot{q}(t) = \frac{-kq(t-\tau) + b_1\left(q(t-\tau) - a^2\right)\dot{q}(t-\tau) - b_2\dot{q}(t-\tau) + u(t)}{m} \quad , \tag{2}$$

with the parameter values given in Table 1.

Table 1
The system parameters (the "Exact" values), and the "Approximate" values utilized in the simulations

System Model	**Dynamic Parameters**		
	Parameter	***Exact Val.***	***Approx. Val.***
k	"Spring stiffness"[a]	150.0	100.0
m	"Inertia"[a]	1.5	1.0
τ	Delay time (s)	6×10^{-3}	10×10^{-3}
a	Separator [a]	1.2	1.0
b_1	Excitation/Damping parameter [a]	1.5	1.0
b_2	"Damping parameter" [a]	2.5	2.0
δt	Cycle time (s)	0.001	Not applicable

[a]The units are compatible with (2).

For using this model, *in the simulations* the exact q, $\dot{q}$ values were considered by applying Euler integration utilizing the $\ddot{q}$ value provided by the *exact dynamic model of the system.* For the controller the noisy observed value of $q(t)$ denoted by $\tilde{q}(t)$ was used as an input. It was simulated by the Julia language code as

"`q_noisy_mem[i]=q[i]+NoiseAmpl*2*(0.5-rand())`"

in which even noise distribution was applied. This noisy value was filtered as $\overline{\tilde{q}(t)}$ by minimizing the following quadratic error in the digital control step i according to a_0, a_1, and a_2, and using the filtered values as in (3b) and (3c).

$$S \stackrel{def}{=} \sum_{s=1}^{L}\left(\sum_{\ell=0}^{2} a_\ell s^\ell - \tilde{q}(i-L+s)\right)^2 \tag{3a}$$

$$\overline{\tilde{q}}(i) = \sum_{\ell=0}^{2} a_\ell L^\ell \;\; ,\ldots, \;\; \overline{\tilde{q}}(i-L+1) = \sum_{\ell=0}^{2} a_\ell 1^\ell \tag{3b}$$

$$\dot{\overline{\tilde{q}}}(i) \approx \frac{\overline{\tilde{q}}(i) - \overline{\tilde{q}}(i-1)}{\delta t} \quad . \tag{3c}$$

In the possession of the filtered $\left\{\overline{\tilde{q}}(i),\ldots,\overline{\tilde{q}}(i-1)\right\}$ and the "desired" $\ddot{q}^{Des}(i)$ ***for the desired next step*** $q^{NextDes}(i+1)$ the approximation in (4)was applied.

$$\ddot{q}^{Des}(i) \approx \frac{q^{NextDes}(i+1) - 2\overline{\tilde{q}}(i) + \overline{\tilde{q}}(i-1)}{\delta t^2} \text{ leading to} \tag{4a}$$

$$q^{NextDes}(i+1) = \ddot{q}^{Des}(i)\delta t^2 + 2\overline{\tilde{q}}(i) - \overline{\tilde{q}}(i-1) \quad . \tag{4b}$$

From the "deformed buffer" the "deformed 2nd time-derivative" was approximated as

$$\ddot{q}^{Def}(i) \approx \frac{q^{Def}(L) - 2q^{Def}(L-1) + q^{Def}(L-2)}{\delta t^2} \quad . \tag{5}$$

In the sequel simulation results will be presented.

3 Simulation Results

The numerical estimations applied in Section 2 are mathematically justified if the buffer length L is small enough in the sense that during the time interval of length $L\delta t$ the variation of the state in the free motion of the system to be controlled, that of the nominal trajectory to be tracked, and the variation of the unknown external disturbances "of no stochastic origin" are not very significant. If the controller can work under such conditions the effects of the stochastic noises can be investigated in a next step. Since for the calculation of the 2nd time-derivatives at least 3 measured values are necessary, the *minimal reasonable buffer length* that can make some filtering is about 5.

3.1 Investigation of The Effects of The Buffer Length in Noise-free Cases

Figures 2 and 3 reveal that the modeling errors and the external disturbances of non-stochastic origin with the PID-type control of $\Lambda = 2s^{-1}$ result in poor trajectory tracking in the case of the non adaptive controller. With the adaptive parameters $R = 10^4$ and $\lambda_a = 0.9$ the "adaptive counterparts" of these diagrams are given in Figs. 4 and 5. Figure 6 reveals more finer details on the operation of the adaptive controller: the adaptive deformation is considerable and almost perfectly guarantees the realization of $\ddot{q}^{DesFilt}$. The $L = 5$ buffer length evidently does not cause considerable obsolescence in the calculation of the adaptive deformation.

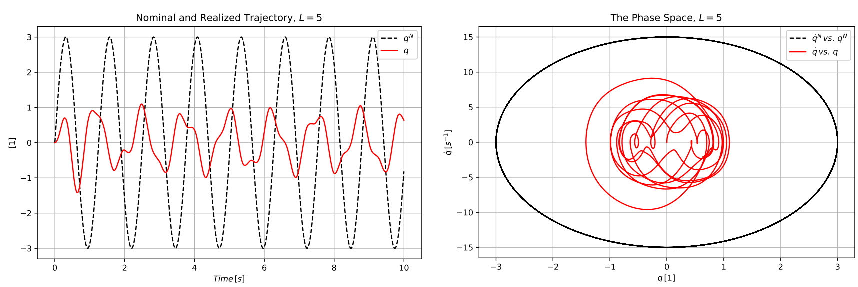

Figure 2
The trajectory and phase trajectory tracking of the non adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 5$

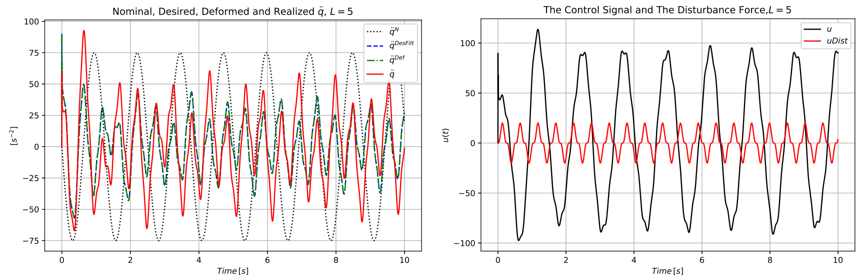

Figure 3
The 2nd time-derivatives and the control forces of the non adaptive controller for a PID control with $\Lambda = 2\,s^{-1}$ with the buffer length $L = 5$

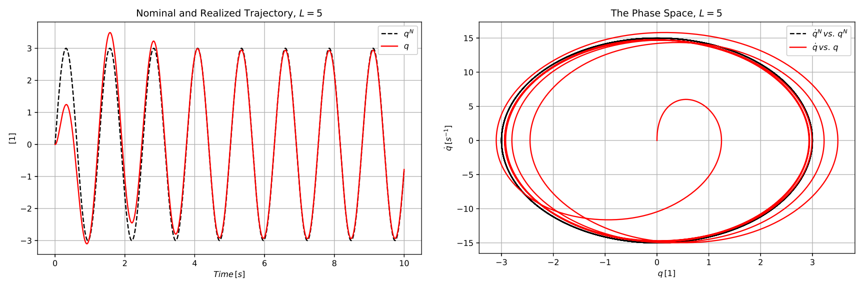

Figure 4
The trajectory and phase trajectory tracking of the adaptive controller for a PID control with $\Lambda = 2\,s^{-1}$ with the buffer length $L = 5$

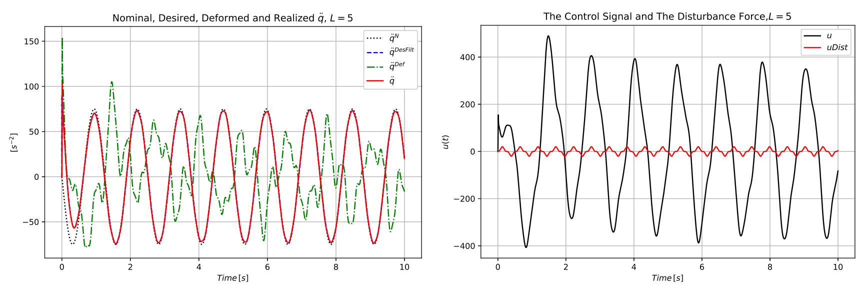

Figure 5
The 2nd time-derivatives and the control forces of the adaptive controller for a PID control with $\Lambda = 2\,s^{-1}$ with the buffer length $L = 5$

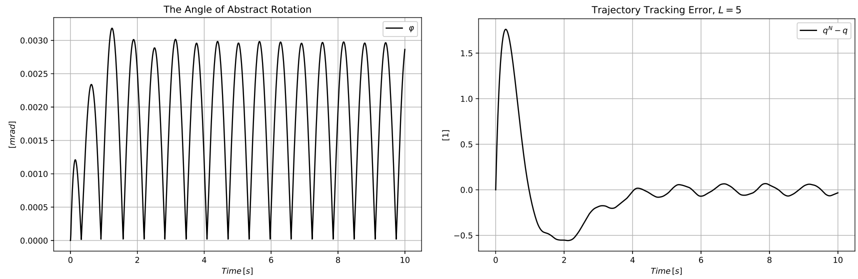

Figure 6
The angle of the abstract rotation and the trajectory tracking error of the adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 5$

If the buffer length is increased to $L = 25$, according to Fig. 7 the behavior of the nonadaptive controller shows drastic modification. The same can be stated for the adaptive controller according to the Fig. 8. So the buffer length $L = 25$ is evidently too large for these dynamical signals.

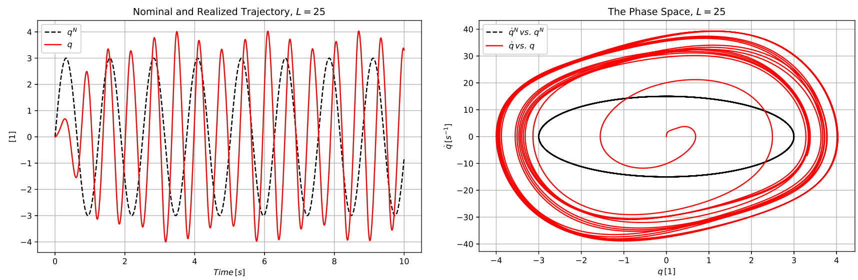

Figure 7
The trajectory and phase trajectory tracking of the non adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 25$

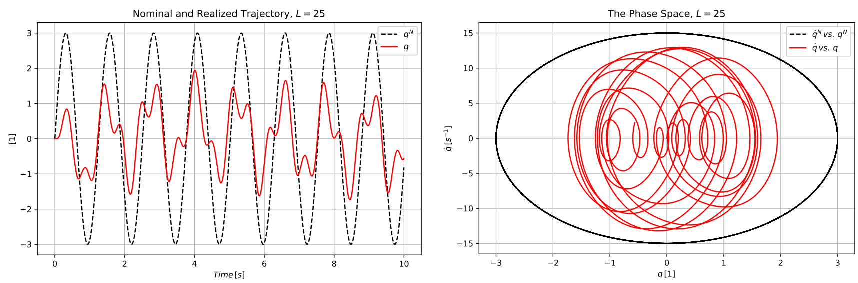

Figure 8
The trajectory and phase trajectory tracking of the adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 25$

Decreasing the buffer length to $L = 10$ provides rather acceptable result for the adaptive controller as it is revealed by Fig. 9, while according to Fig. 10 the non adaptive controller remains quite inappropriate.

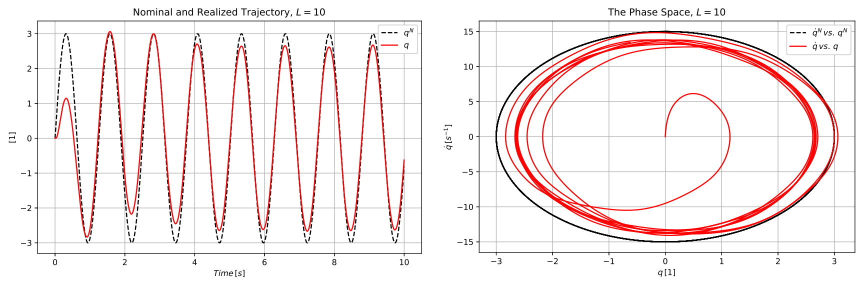

Figure 9
The trajectory and phase trajectory tracking of the adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 10$

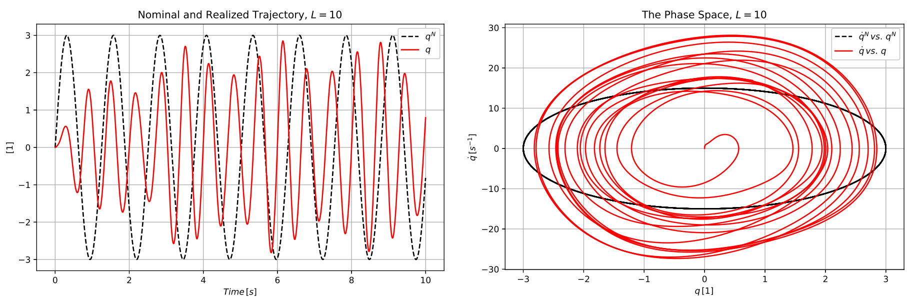

Figure 10
The trajectory and phase trajectory tracking of the non adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 10$

Following these preparations it seems to be expedient for considering the additional effects of measurement noises for the buffer lengths $L = 5$ and $L = 10$.

3.2 Investigation of The Effects of Measurement Noises

In these investigations the noise amplitude was selected to be 0.02. Figure 11 reveals that the non-adaptive controller is quite inappropriate, while according to Fig. 12 the adaptive solution behaves quite well.

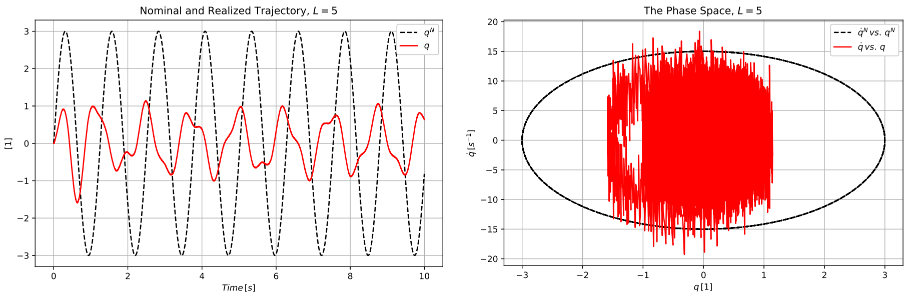

Figure 11
The trajectory and phase trajectory tracking of the non adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 5$ in the case of measurement noises

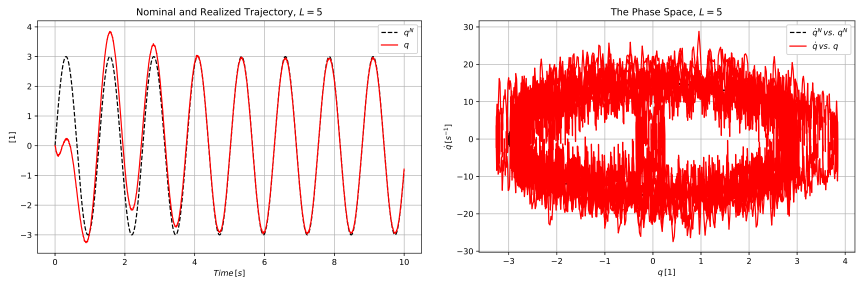

Figure 12
The trajectory and phase trajectory tracking of the adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 5$ in the case of measurement noises

To the buffer length $L = 10$ Figs. 13, 14, and 15 belong. The adaptive controller evidently was found to be more precise than the non adaptive one, in spite of the noisy feedback.

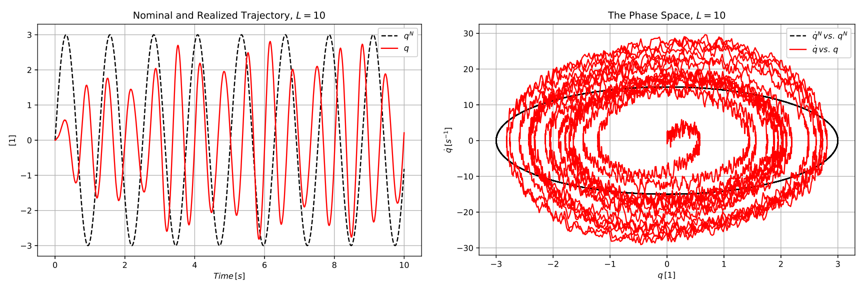

Figure 13
The trajectory and phase trajectory tracking of the non adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 10$ in the case of measurement noises

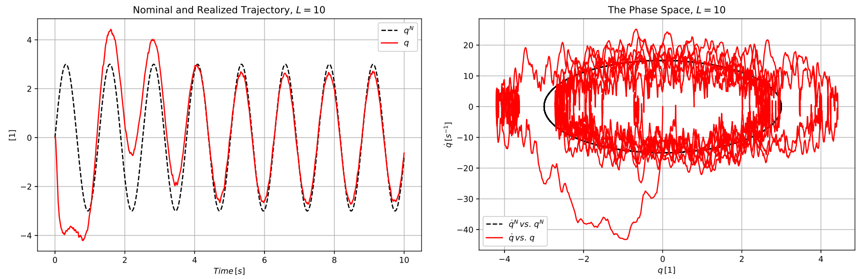

Figure 14
The trajectory and phase trajectory tracking of the adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 10$ in the case of measurement noises

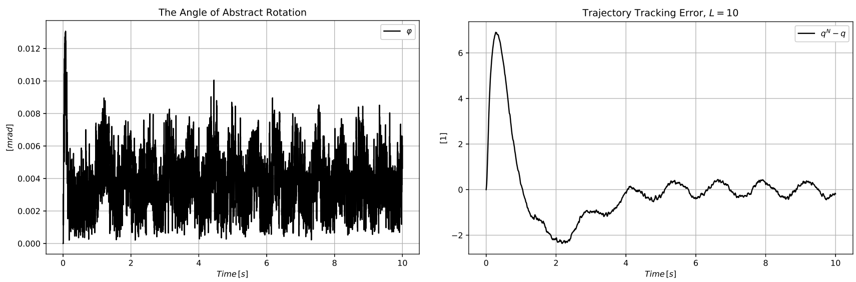

Figure 15
The angle of the abstract rotation and the trajectory tracking error of the adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 10$ in the case of measurement noises

For a better comparison it makes sense to consider the 2nd time-derivatives and the control signals. Figures 16 and 17 reveal that both the 2nd time-derivatives and the control signals varied in the same order of magnitude.

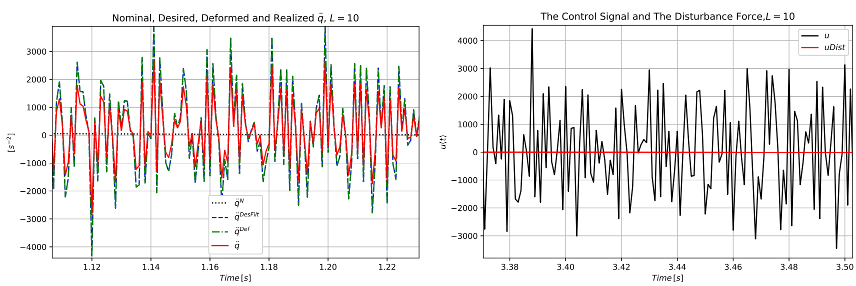

Figure 16
The 2nd time-derivatives and the control forces of the non adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 10$ in the case of measurement noises (zoomed in excerpts)

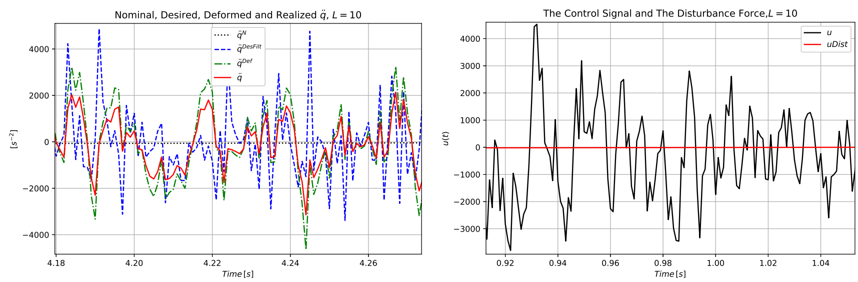

Figure 17
The 2nd time-derivatives and the control forces of the adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 10$ in the case of measurement noises (zoomed in excerpts)

It can be guessed that the division by δt^2 in the estimation of the 2nd time-derivatives may cause huge values, and consequently, huge control signal values. It was interesting to see what happens if the time-resolution is reduced by using $\delta t = 10^{-2}\,s$ and simultaneously the buffer length is reduced to $L = 5$. According to Figs. 18, 19, and 20 the rougher time-resolution made the adaptive controller better because considerably smaller control signal were applied in the calculations. Figures 21, and 22 testify that the non adaptive controller provides less precise tracking though the control signals and the 2nd time-derivatives are of the same order of magnitude in the adaptive and the nonadaptive cases.

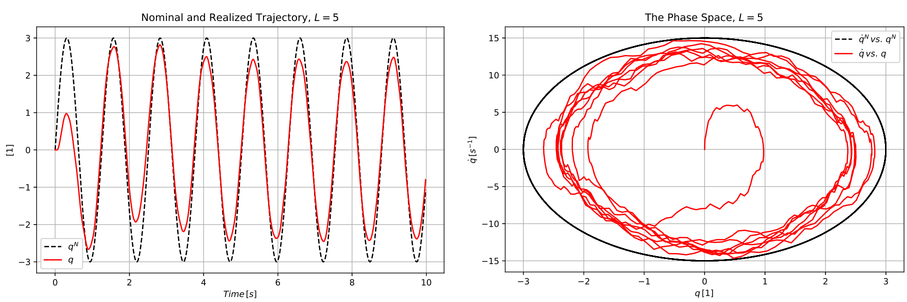

Figure 18
The trajectory and phase trajectory tracking of the adaptive controller for a PID control with $\Lambda = 2s^{-1}$ with the buffer length $L = 5$ in the case of measurement noises using $\delta t = 10^{-2}\,s$

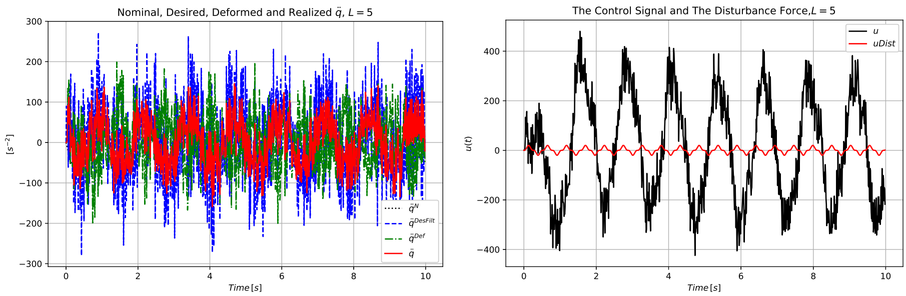

Figure 19
The 2nd time-derivatives and the control forces of the adaptive controller for a PID control with $\Lambda = 2\,s^{-1}$ with the buffer length $L = 5$ in the case of measurement noises using $\delta t = 10^{-2}\,s$

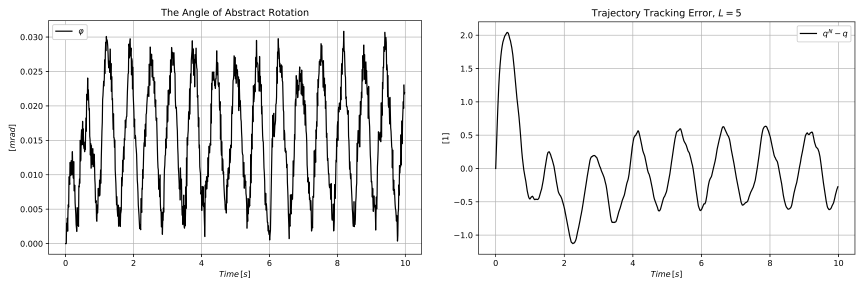

Figure 20
The angle of the abstract rotation and the trajectory tracking error of the adaptive controller for a PID control with $\Lambda = 2\,s^{-1}$ with the buffer length $L = 5$ in the case of measurement noises using $\delta t = 10^{-2}\,s$

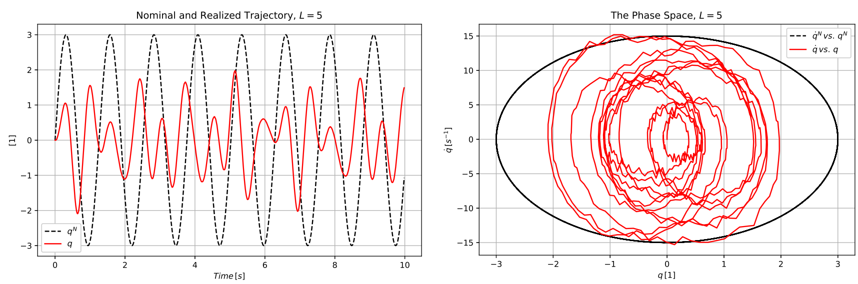

Figure 21
The trajectory and phase trajectory tracking of the non adaptive controller for a PID control with $\Lambda = 2\,s^{-1}$ with the buffer length $L = 5$ in the case of measurement noises using $\delta t = 10^{-2}\,s$

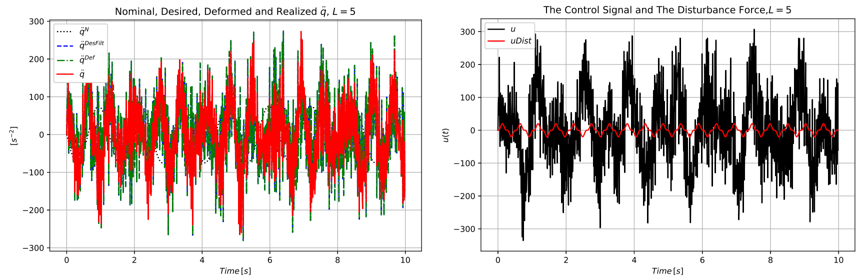

Figure 22
The 2nd time-derivatives and the control forces of the non adaptive controller for a PID control with $\Lambda = 2\,s^{-1}$ with the buffer length $L = 5$ in the case of measurement noises using $\delta t = 10^{-2}\,s$

3.2.1 Conclusions

In this paper, following the the preliminary investigations in [1] that considered the possible use of multivariable fixed point transformations in the adaptive control of single variable 1st order dynamical systems with some special noise filtering technique, the investigations were extended to 2nd order dynamical systems. For this purpose the van der Pol oscillator model was modified with the inclusion of time-delay effects.

As a conclusion it can be stated that in spite of the fact that the fixed point iteration-based method feeds back the noisy 2^{2nd} derivatives that are estimated by the controller in a finite element approximation, the operation of the adaptive solution is superior to the nonadaptive one that feeds back only lower order, therefore less noisy numerically estimated time-derivatives.

It can be concluded, too, that it is expedient tho choose the possible roughest discrete time resolution that can be allowed by the dynamics of the nominal motion to be tracked, and by the kinematically calculated PID-type error feedback correction terms to keep the occurring control signals at some low, realistic level. Any refinement in the time resolution increases the amplitude of the fluctuating control signal that is similar to the chattering effect occurring in the Robust Variable Structure / Sliding Mode Controllers.

3.2.2 Acknowledgement

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