# The Analysis of Two-Phase Condensation Heat Transfer Models Based on the Comparison of the Boundary Condition

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*Abstract: The article aims to present, analyze and select two-phase condensation heat transfer coefficients for the refrigerant-side in the condenser of the heating and cooling system of a heat pump. The heat transfer models are analyzed for condensation in horizontal smooth tubes. The mathematical models published by various authors refer to the range of heterogeneous condensation. The final aim of the analysis is to select the optimal model from the examined two-phase condensation heat transfer models. The selection of the condensing refrigerant two-phase heat transfer model is based on the boundary condition. The applied method of analysis is numerical-graphical.*

*Keywords: heat pump; condenser; R134a; heat transfer; boundary condition*

*Nomenclature*

| | |
|---|---|
| D | inside diameter of tube [m] |
| x | vapor quality [-] |
| G | mass velocity [kg/m$^2$s] |
| T | temperature [K] |
| p | pressure [Pa] |
| p* | reduced pressure [-] |
| Re | Reynold`s number [-] |
| Pr | Prandt number [-] |
| F | two phase multiplier |
| Fr | Froude number [-] |
| Z | Shah`s correlating parameter [-] |
| $X_{tt}$ | Martinelli`s correlating parameter [-] |
| C | Constant [-] |

*Greek Symbols*

| | |
|---|---|
| $\alpha$ | heat transfer coefficient[W/m$^2$K] |
| $\lambda$ | thermal conductivity [W/mK] |
| $\rho$ | density [Ns/m$^2$] |
| ε | void fraction [-] |
| $\eta$ | dynamic viscosity [Ns/m$^2$] |

*Subscript*

| | |
|---|---|
| g | vapor |
| f | liquid |
| kf | two phase |

# 1 Introduction

The condenser heat exchanger plays a significant role in the structure and operation of the heat pump as it affects the system's coefficient of performance (COP).

The motivation for the composition of this article was the selection of the optimal two-phase heat transfer mathematical model of refrigerant among the analyzed mathematical models in the horizontal tube of the condenser. The structure and dimensions of the condenser have a significant impact on the heat transfer intensity. The parameters that influence heat transfer are the heat transfer coefficients of the refrigerant and of the heated water.

The earliest determination of the condensation heat transfer coefficient of the two-phase refrigerant flowing in horizontal smooth tubes was carried out by Boyko and Kruzhilin [1] and Akers et al. [2]. For the stationery condensation, there are a large number of mathematical models for different conditions and refrigerants, authored by Cavallini and Zecchin [3, 4], Shah [5], M. K. Dobson, J. C. Chato [6], and J. R. Thome et al. [7]. The common characteristics of these heat transfer models are that under the same conditions they give a high difference in values of heat transfer coefficient. M. M. Awad et al. [8] in their mathematical model used the heat transfer correlations.

A number of researchers have dealt with this field of science and implemented heat transfer coefficients in numerous mathematical models of heat pump; see J. Nyers et al. [9-12].

# 2 The Physical Model of Condenser with Horizontal Smooth Tubes

The condenser is a heat exchanger where the higher temperature refrigerant gives heat to the lower temperature heating medium. The analyzed condenser has parallel, straight and smooth tube bundles with baffles. The refrigerant flows inside tubes, while the heated water flows in the shell tube.

The process in the condenser on the refrigerant side is made up of three sections:

A the Superheated vapor section

B the Condensation section

C the Subcooled liquid section

After compression, the vapor is single-phase and superheated. In the superheated section, the value of vapor quality equals 1.0.

The vapor is in contact with the tube wall, whose temperature is lower than the saturation temperature, and therefore the heat transfers to the wall and vapor condenses. Heat is then transferred from the tube wall to the water by conduction.

In the condensation section, the condensate and the vapor have a heterogeneous flow, which is characterized by intensive turbulence.

At the beginning of the condensation section, the flow pattern is annular, because in the core the velocity of the vapor is much higher than the velocity of the liquid. In the annular flow regime at the liquid-vapor interface, the dominant force is tangential stress, with the gravitational force playing a less important role. As condensation continues, the velocity of the vapor phase reduces and the dominant force shifts from tangential force to gravitational force. The liquid phase accumulates at the bottom of the tube. Condensation takes place mainly at the top of the tube because the liquid layer is thin, and therefore heat resistance is smaller.

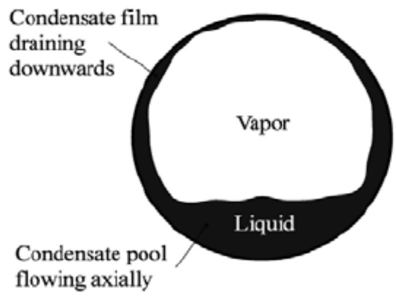


Figure 1
Condensation in fully annular flow in horizontal tube

In the continuation of the condensation, the vapor continually condenses, and in the cross section area the surface of liquid increases, and the flow pattern changes to slug and plug pattern.

The size of the vapor slug reduces further and a bubbly flow pattern develops. At the end of condensation section, the vapor quality reduces to zero and the flow in the tube becomes a single-phase flow (liquid flow).

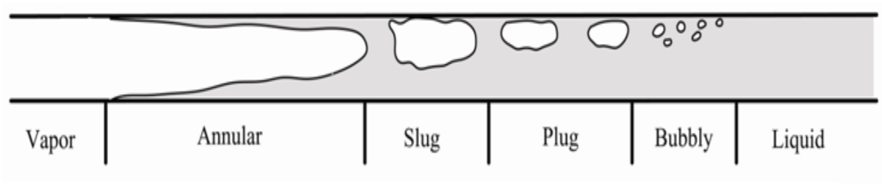


Figure 2
Flow pattern map for condensation in a horizontal tube

A sub-cooling section is only formed in exceptional circum stances, maybe in the cross-flow condensers. In the case of shell-tube condensers, the sub-cooling is realized in a separate heat exchanger (subcooler). In the subcooler, the refrigerant is in the single-phase (liquid phase).

# 3 Annular Flow Condensation

One of the most important flow regimes is annular flow, which is characterized by a phase interface separating a thin liquid film from the gas flow in the core region. This flow regime is the most investigated one both analytically and experimentally because of its practical significance.

Much of the condensation process occurs in the annular flow regime. Therefore, many of the existing in-tube condensation correlations are based on the annular flow regime. These correlations are classified into three categories, that is, shear-based correlations, boundary layer-based correlations, and two-phase multiplier-based correlations. The majority of smooth-tube heat transfer correlations are of the two-phase multiplier-based variety.

# 4 The Two-Phase Multiplier-based Heat Transfer Correlations in Condensation Section

The simplest method of heat transfer prediction in the annular flow regime is the two-phase multiplier approach. Two-phase multiplier-based correlations were pioneered for predicting convective evaporation data (Dengler and Addoms 1956) and were adapted for condensation by Shah (1979).

The two-phase multiplier based heat transfer correlations typically result in the following form:

$$\alpha_{kf} = \alpha_f \cdot F_{kf}$$

Where:

$$\alpha_f = f \quad (C, \mathrm{Re}^n, \mathrm{Pr}^m) \text{ and } F_{kf} = f \quad \left( x, \frac{\rho_f}{\rho_g}, \frac{\eta_f}{\eta_g}, Fr_f \right) \qquad (1)$$

The two-phase multiplier can depend on more dimensionless groups than those indicated in Eq. 1; the shown groups are the most prevalent.

Several examples of two-phase multiplier-based condensing correlations are available, including those of Akers et al. (1959), Boyko and Kruzhilin (1967), Cavallini and Zecchin (1974), Tang (1998) and Dobson and Chato (1998).

## 4.1 The Cavallini and Zecchin Correlation

Cavallini and Zecchin developed a semi-empirical equation that has a simple form.

The mathematical model for heat transfer by Cavallini and Zecchin correlation is:

$$\alpha_{kf} = \frac{\lambda_f}{D} \cdot 0.05 \cdot R_e^{0.8} \cdot \Pr_f^{0.33} \tag{2}$$

The equivalent Reynolds number is:

$$\mathrm{Re} = \mathrm{Re}_g \cdot \left(\frac{\rho_f}{\rho_g}\right)^{0.5} \cdot \left(\frac{\eta_g}{\eta_f}\right) + \mathrm{Re}_f \tag{3}$$

where $\mathrm{Re}_v$ and $\mathrm{Re}_l$ are the Reynolds number of the liquid and vapor phase, respectively, which can be calculated by Eq. 4.

$$\mathrm{Re}_l = \frac{G \cdot (1-x) \cdot d_i}{\mu_l}, \ \mathrm{Re}_g = \frac{G \cdot x \cdot d_i}{\mu_l} \tag{4}$$

In Eq. 2 the application range of Cavallini and Zecchin`s correlation was summarized as follows:

$$d_i = 8mm, 30 < T_{sat} < 50^\circ C,$$

$$10 < \rho_l / \rho_v < 2 \cdot 10^3, \ 10 < \mu_l / \mu_v < 2 \cdot 10^3,$$

$$0.8 < \Pr_l < 20, \ 1.2 \cdot 10^3 < \mathrm{Re}_l$$

Flow regime: Annular flow

Refrigerants tested: R-11, R-12, R-21, R-22, R-113, R-114, R-134a, R-410A, R-407C

## 4.2 Shah Correlation

The Shah correlation takes into account the pressure of the refrigerant, in addition to the quality of the mixture. This can also be used to find the local condensation heat transfer coefficient. The heat transfer coefficient is a product of liquid heat transfer coefficient given by the Dittus-Boelter equation and an additional term.

In 1979, Shah presented the following correlation:

$$\frac{\alpha_{kf}}{\alpha_f} = 1 + \frac{3.8}{Z^{0.95}} \tag{5}$$

The liquid heat transfer coefficient is calculated by using the Dittus-Boelter equation:

$$\alpha_f = 0.023 \cdot \mathrm{Re}_f^{\,0.8} \cdot \mathrm{Pr}^{0.4} \cdot \frac{\lambda_f}{D} \tag{6}$$

The mathematical model for heat transfer by the Shah correlation is:

$$\alpha_{kf} = \alpha_f \cdot \left[ (1-x)^{0.8} + \frac{3.8 \cdot x^{0.76} \cdot (1-x)^{0.04}}{p^{*0.38}} \right] \tag{7}$$

The application range of Shah's correlation was summarized as follows:

$$7 < d_i < 40,\ 10 < P_{sat} < 9.87 MPa,$$

$$21 < T_{sat} < 310^\circ C,\ 10.8 < G_{cr} < 1.599 \, {}^{kg}\!/_{m^2 s},$$

$$0.5 < \mathrm{Pr}_l,\ 350 < \mathrm{Re}_l,$$

Flow regime: Annular flow

Refrigerants tested: R-718, R-11, R-12, R-22, R-113, methanol, ethanol, benzene, toluene, and ethylene

## 4.3 The Boyko and Kruzhilin Correlation

The Boyko and Kruzhilin correlation is an adaptation of the Mikheev correlation. The correlation is simple to use, generally conservative, and sufficiently accurate. This correlation takes into account the heat transfer coefficient in single-phase flow, the density of the two-phase flow and vapor quality.

The mathematical model for heat transfer by the Boyko and Kruzhilin correlation is:

$$\alpha_{kf} = \alpha_f \cdot \left( 1 + x \cdot \left( \frac{\rho_l}{\rho_g} - 1 \right) \right)^{0.5} \tag{8}$$

The liquid heat transfer coefficient is obtained as follows:

$$\alpha_f = 0.021 \cdot \mathrm{Re}_f^{\,0.8} \cdot \mathrm{Pr}^{0.43} \cdot \frac{\lambda_f}{D} \tag{9}$$

The application range of Boyko and Kruzhilin's correlation was summarized as follows:

$$1500 < \mathrm{Re} < 15000$$

Flow regime: Annular flow

Refrigerants tested: steam, R22

## 4.4 Akers Correlation

Akers et al. (1959) developed a two-phase multiplier-based correlation that became known as the "equivalent Reynolds number" model. This model defines the all-liquid mass flow rate that provides the same heat transfer coefficient as an annular condensing flow.

The mathematics model for heat transfer by the Akers correlation is:

$$\alpha_{kf} = \frac{\lambda_f}{D} \cdot 0.05 \cdot R_e^{0.8} \cdot \Pr_f^{0.33} \tag{10}$$

where:

the equivalent mass velocity is: $$G_{ekv.} = G \cdot \left[ (1-x) + x \cdot \left( \frac{\rho_f}{\rho_g} \right)^{0.5} \right] \tag{11}$$

The multiplier factors function of the Reynolds number is:

$C = 0.0265$ and $n = 0.8$ if $\mathrm{Re}_f \rangle 50000$

$C = 5.03$ and $n = \frac{1}{3}$ if $\mathrm{Re}_f \langle 50000$

Flow regime: Annular flow

Refrigerants tested: R-12, Propane, Methanol

## 4.5 Dobson et al.

Dobson et al. (1998) proposed separate correlations for the wavy and annular flow regimes. For the annular flow regime, they correlated condensation data by assuming that the ratio of the two-phase Nusselt number to the Nusselt number predicted by a single-phase correlation is exclusively a function of the Martinelli-parameter. Utilizing the Dittus-Boelter correlation to predict the single-phase Nusselt number, regression analysis of annular flow data yielded the following form:

$$Nu = 0.023 \cdot \mathrm{Re}_l^{0.8} \cdot \Pr_l^{0.4} \cdot g(X_{tt}) \tag{12}$$

Where: $g(X_{tt}) = 1 + \frac{2.22}{X_{tt}^{0.889}}$

The function $g$ at the end of equation 12 represents the two-phase multiplier, and the values of the constant and the exponent are similar to the values from convective evaporation correlations [Wattelet et al., 1992].

The mathematical model for heat transfer by the Dobson et al. correlation for annular flow is:

$$\alpha_{kf} = \frac{\lambda_f}{D} \cdot 0.023 \cdot \mathrm{Re}_l^{0.8} \cdot \mathrm{Pr}_l^{0.4} \cdot \left[1 + \frac{2.22}{X_{tt}^{0.89}}\right] \tag{13}$$

where the application range of Dobson et al.'s correlation was summarized as follows:

$$d_i = 4.57mm,\ 35 < T_{sat} < 60^{\circ}C, \quad 75 < G_{cr} < 500\, {kg}/{m^2 s}, \quad 0.1 < x < 0.9,$$

Flow regime: Annular flow

Refrigerants tested: R-22, R-134a, R-410A

# 5 Comparison of Heat Transfer Correlations

Many correlations that are available come with no explicit range of parameters over which they can be expected to give accurate results. Therefore, a design engineer requiring a suitable heat transfer correlation typically encounters a series of seemingly contradictory reports about which correlation is "best".

The heat transfer process in annular two-phase flow is similar to that in single-phase flow of the liquid, and thus their ratio may be characterized by a two-phase multiplier. This concept uses the same rationale as the Lockhart-Martinelli (1949) two-phase multiplier, developed for the prediction of two-phase frictional pressure drop.

This work investigated the comparison of the two-phase correlations on the basis of the above-mentioned similarities. The deviation of values of the two phase heat transfer correlation was examined in the values of boundary condition, i.e. single-phase heat transfer coefficient values of liquid. The liquid phase heat transfer coefficient was examined the reference value.

The single-phase heat transfer coefficients are typically predicted by the Dittus and Boelter (1930) [13] correlation, which results in the following form:

$$\alpha_{kf} = \frac{\lambda_f}{D} \cdot 0.023 \cdot R_e^{0.8} \cdot \mathrm{Pr}_f^{0.33} \tag{14}$$

Shah [14] also compared the condensation heat transfer correlations. Shah's basis of comparison was his own model, namely the two-phase heat transfer correlation.

# 6 Initial Condition and Values

The mathematical models are simulated by the use of the software tool MathCAD. The initial conditions and values for the simulation of the stationary condensation are:

Refrigerant: R134a

Mass velocity: $G = 100 - 500 \quad \left[\frac{kg}{m^2 s}\right]$

Vapor quality ranged: x = 1 - 0 [-]

Pressure in the inlet of condenser: $p_k$= 15 [bar]

Inner diameter of tube: d= 6 [mm]

Below, the reader can find graphs drawn for the considered condensation heat transfer correlations, showing the deviation of the condensation heat transfer coefficients from the reference value and average of local condensation heat transfer in function of vapor quality and different mass velocity.

# 7 Results and Discussions

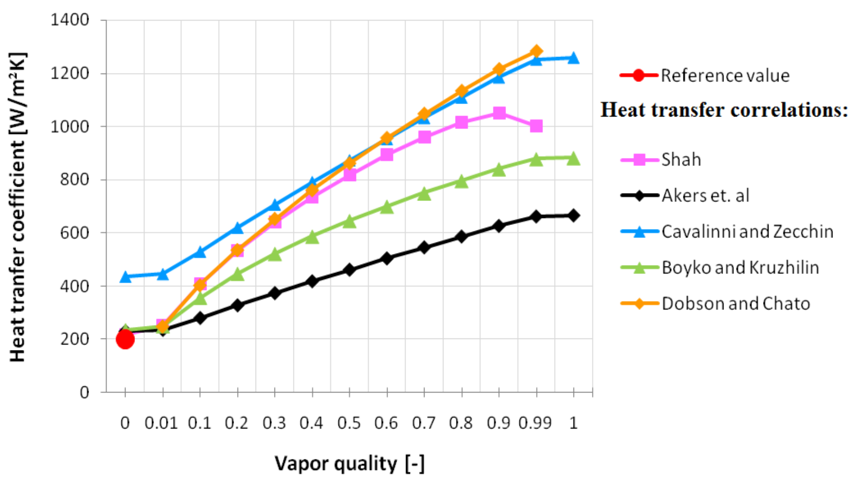


Figure 3

Condensation heat transfer coefficients flow in tube with reference value shown, $G = 100 \left[\frac{kg}{m^2 s}\right]$

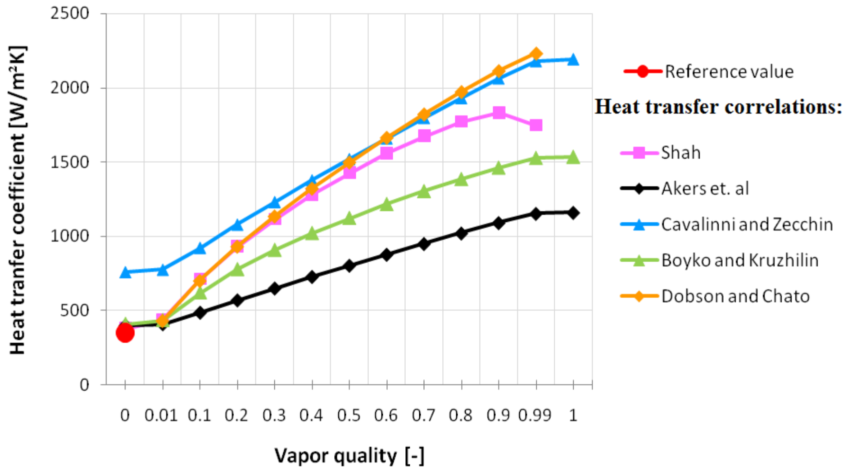


Figure 4

Condensation heat transfer coefficients flow in tube with reference value shown, $G = 200 \left[\frac{kg}{m^2 s}\right]$

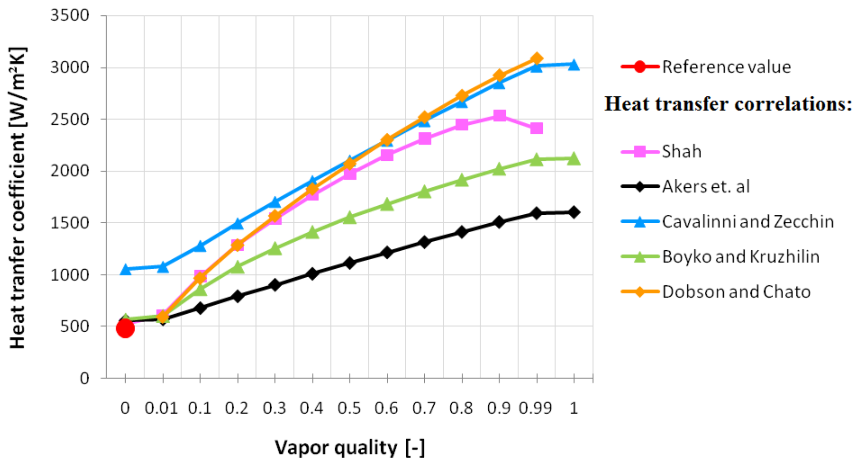


Figure 5

Condensation heat transfer coefficients flow in tube with reference value shown, $G = 300 \left[\frac{kg}{m^2 s}\right]$

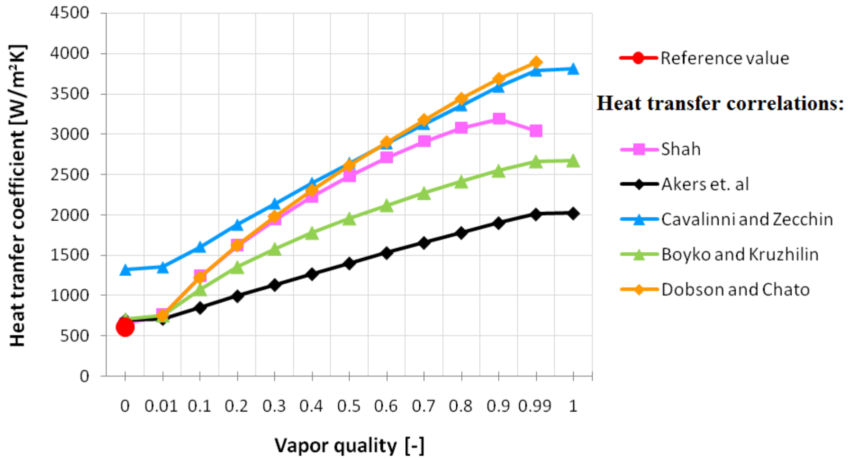


Figure 6

Condensation heat transfer coefficients flow in tube with reference value shown, $G = 400 \left[\frac{kg}{m^2 s}\right]$

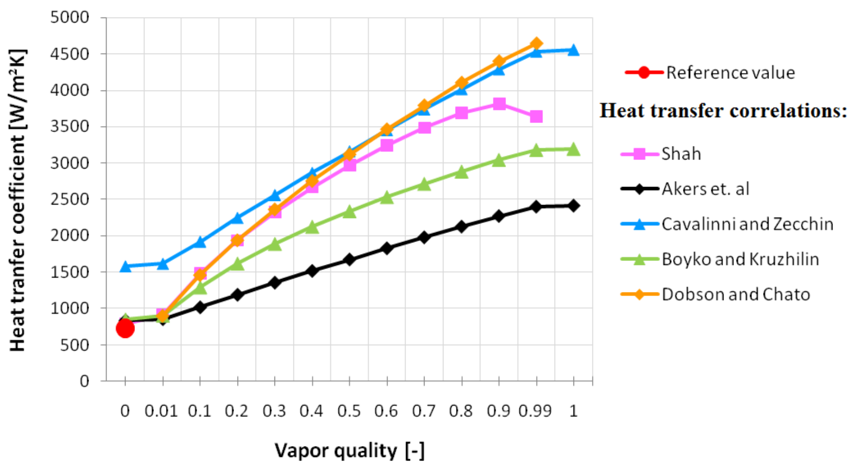


Figure 7

Condensation heat transfer coefficients flow in tube with reference value shown, $G = 500 \left[\frac{kg}{m^2 s}\right]$

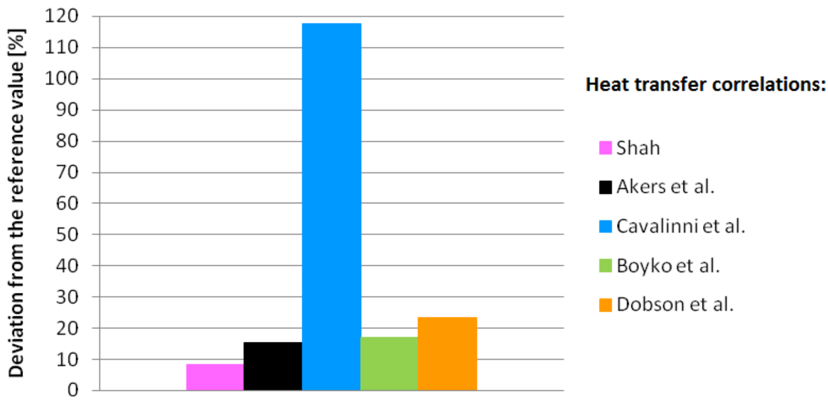


Figure 8

The deviation of the condensation heat transfer coefficients from the reference value, when the vapor quality x=0

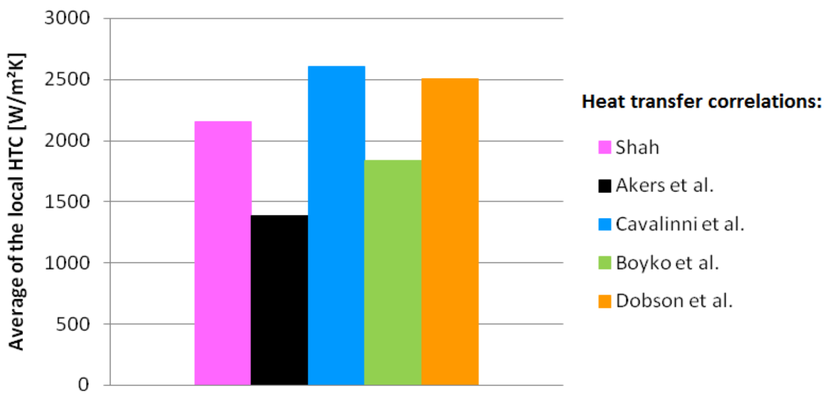


Figure 9

The average condensation heat transfer coefficient, $G = 400 \left[\frac{kg}{m^2 s}\right]$

## Conclusions

The condensation heat transfer coefficients linearly increases in the function of the vapor quality of the refrigerant.

For vapor quality x=0.01 (x approximately 0), the values of the condensation heat transfer coefficient obtained from the analyzed models are approximately equal to the values of the models which have been used exclusively for the liquid phase x=0, published by several authors. The Cavalinni et al. condensation heat transfer coefficient is an exception to this rule.

In the case of pure liquid phase, i.e. x=0, the examined condensation heat transfer correlation provide real values, whereas the Dobson et al. correlation is an exception. The Dobson et al. condensation heat transfer correlation is not suitable for determining the single phase heat transfer coefficient.

The heat transfer coefficient of the Shah condensation correlation differs in the smallest amount from the reference value out of the examined correlations. This deviation is only 8.36%, while the heat transfer coefficient of the Cavalinni et al. correlation provides the highest level of deviation. The author examined the heat transfer coefficients deviation from the reference value at x=0 vapor quality and different mass velocity of refrigerant. The Dobson correlation values are an exception to this, where the vapor quality was x=0.01.

In the case of the pure vapor phase, i.e. x=1 vapor quality, the examined condensation heat transfer correlation presents real values of heat transfer coefficient; however, the Shah and Dobson correlation values are exceptions to this rule.

The majority of the condensation heat transfer correlations provide the highest condensation heat transfer coefficient near x = 0.99, Shah's correlation being an exception.

For vapor quality x=0.99, (x approximately 1),the calculated values of the condensation heat transfer coefficients by using the investigated models are much higher than by the value of single heat transfer coefficients suggested by other authors.

The deviation between reference value and calculated heat transfer coefficients is constant with an increase in the mass velocity of the refrigerant.

The average values of the local heat transfer coefficients also provide significant dispersion.

Based on the above analysis and facts, Shah's model is considered as an optimum.

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