Dynamic Multi-Robot Coalition Formation: Precision Agriculture Case Study

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Abstract: The use of multiple autonomous robots to accomplish a complex task is a highly relevant topic for intelligent systems and collective robotics. In this paper, a game-theoretic framework for the self-organization of a group of heterogeneous self-interested robots is described. The proposed approach enables both the tasks allocation and dynamic reward distribution to maximize the expected total gain, which ensures the effectiveness of multi-robot coalitions. The solution is based on the theory of fuzzy cooperative games with core. The precision farming scenario is used as an example of a complex task. In this scenario, several robots belonging to two different classes interact with each other to distribute field processing tasks to meet the given marginal cost of each task, corresponding to the coalitions' payoffs. Simulation results show the convergence of the solution when searching for the coalition structures capable of providing a given payoff. That allows to assert the applicability of the theory of fuzzy coalition games for the self-organization in collective robotics.

Keywords: collective robotics; self-organization; coalition formation; fuzzy cooperative game; precision farming

1 Introduction

Organization of multi-robot teams is one of the most relevant areas of research in the field of collective robotics. This is due to the development of robotic systems, in which robots can perform rather complex operations and independently make decisions about further actions. It is a common knowledge that universal robots capable of performing a large number of operations highly increase the cost of their development and operation [1]. Economically feasible scenario also improving the quality of individual operations is the use of specialized robots joined together to solve a complex task. In this case, the winnings are distributed among the robots (and, accordingly, among their owners) in accordance with the task assignments and the contribution of each of them.

Swarm, flock and coalition interaction schemes are most frequently considered in collective robotics [2, 3]. In swarm models, all robots are equipped with the same equipment. The tasks are executed in parallel by several robots coordinating their actions with each other. When using the flock model of interaction, a leader stands out among the robots, who coordinates the subordinate robots. The coalition model of interaction is the most complex since each of the robots is independent from the others and is equipped with unique tools for performing a specific task. Such a model requires decomposition of a complex task, allocation of subtasks among self-interested robots according to their capabilities, and reward distribution according to the expected gains. The coalition in this case can be considered a union of self-interesting agents, which in the negotiation process make a decision on a joint solution to the problem and the distribution of the gain [1].

The difficulty of forming a coalition of autonomous robots lies in the need to consider many parameters when choosing individual robots to join a coalition. Furthermore, the problem is to allocate in a fair way the payoff of the grand coalition among the players. In collective robotics, it is usually impossible to accurately assess (at the moment of coalition formation), what part of the work should be performed by the robot and what benefit the robot can gain from participating in the coalition and decisions have to be made under uncertainty [4]. Such intrinsic fuzziness of the problem adds additional complexity to coalition formation task.

In the proposed framework, tasks are dynamically assigned to coalition participants during the coalition game with fuzzy core obtaining efficient coalition structures based on participants' fuzzy expectations. As a case study illustrating the proposed approach, precision farming is considered [5]. The choice of this area is due to the development of autonomous agricultural machinery and the growing demand by farmers for a lease or pay-as-you-go rental schemes based on the precise estimation of equipment ordering since the cost of buying a property is often economically impractical for small farms [6].

The rest of the paper is organized as follows. The following section provides an overview of existing coalition-formation methods. The third section describes the formulation of the problem and a general description of the approach. The fourth section contains the formulation of a cooperative game with fuzzy core and a solution method using a genetic algorithm. Section five contains the results of the simulation experiment. The last section presents a discussion over the experiment results and conclusions. Thus, this approach is developed in this paper.

2 Related Work

The distribution of tasks between a group of robots is a time challenge, as evidenced by quite a few studies on this topic. The proposed approaches depend upon the number of robots, types of robots used to solve the problem, specific tasks, time constraints, structural features of the interaction of robots, to mention few. One can consider the distribution of tasks between robots by assigning one task to one robot, several tasks to one robot, or several tasks to several robots. To solve tasks in the most beneficial way, robots can form groups [7, 8]. This process is called coalition formation. Forming several coalitions for solving several problems is the most general type of coalition formation [1, 9].

Three groups of methods for forming coalitions can be considered: centralized, based on self-organization and auction. In the centralized approach, one agent collects all available information about the state of the environment and robots and centrally decides on the coalition structure. Coalition formation tasks, task ordering and allocation, paths planning are often solved by linear programming methods and genetic algorithms and computationally are very expensive [1, 3, 10, 11].

An example of a coalition self-organization is the swarm intelligence. Its main feature is the absence of a leader [12]. All coalition members are equal and coordinate their actions only with respect to the closest members of the coalition. A problem of this type of organization is the poor awareness of the coalition members about the current state of the task solution and the common goal. It significantly complicates the solution of complex tasks that require decisions regarding the general context of the task, not just the local contexts of coalition members.

Finally, the auction approach is based on the use of communication between robots in the process of negotiating the distribution of tasks and the results of their execution between all members of the coalition [9, 13]. The methods used in this approach include machine learning (e.g., support vector regression), negotiations algorithms, as well as cooperative games [14, 15]. Their advantage is the absence of a single point of failure since the decision is made in a distributed manner, based on information received from the participants. However, decision making requires more time and energy consumption compared to centralized systems and more complex communication algorithms compared to swarms.

The game-theoretic approach to task allocation and coalition formation among self-organizing robots have been studied recently in [16-18]. Hedonic coalition games have been applied to homogeneous and heterogeneous robotic swarms [16, 17]. Coalition formation is considered a partitioning problem optimizing individual utilities in non-overlapping coalitions. The authors prove their partitioning being Nash-stable. Closely related to the hedonic coalition games, the approach proposed in [18], is also based on the independent decision making procedures of individual robots, which repeatedly revise their task selections and obtained rewards in changing environment conditions.

The payoff distribution should guarantee the stability of the coalition structure when no one player has an intention to leave a coalition because of the expectation to increase its payoff. The predictions or recommendations of payment distribution are embodied in different solution concepts, the Shapley value, the core, and the kernel, being the former the most popular approach [19]. Games with core belong to the class of games with a solution set and represent a mechanism for analyzing the possible set of stable outcomes of cooperative games with transferable utilities. The concept of a core is attractive since it tends to maximize the sum of coalition utilities in the so-called C-stable coalition structure.

The definition of the core may be crisp or fuzzy. The crisp core has two associated problems, which made it quite unpopular for practical applications [20]. Firstly, the computational complexity of finding the optimal structure is high since for the game with n players at least $2^n - 1$ of the total $n^{\frac{n}{2}}$ coalition structures should be tested. Secondly, for particular classes of the game and real-world situations, a core can be empty. Since the benefit distribution among the coalition members has proved to be fuzzy, uncertain, and ambiguous, the concept of fuzzy cooperative games with core was introduced [24, 39]. In fuzzy cooperative games (FCGs), the uncertainty can be processed by means of the introduction of a fuzzy benefit concept through the bargaining process to the conclusion about the corresponding fuzzy distribution of individual benefits among the coalition members [23, 24]. The introduction of the fuzzy core helped solve the main problems of the games with crisp core.

The advantage of the core compared to the hedonic games, mentioned above, is the way payoff distributions are analyzed to guarantee the stability of the game. In the above cases, the robots construct a dominance relationship and try to improve their gain by analyzing different options until finding a Nash stable partition. The core itself is the set of outcomes forming the equilibrium states. The search for a solution is thus reduced to choosing from the number of possible imputations forming the C-core that provide the maximum possible gains for robots included in the effective coalitions structure at the maximum degree of membership.

3 Theoretical Background

3.1 Basic Concepts of a Cooperative Game

A cooperative game is a variable-sum game, in which players are allowed to discuss their strategies before the game and agree to act together [23]. In other words, players can form coalitions. The main task in the game is to divide the overall payoffs between the coalitions and their members. Cooperative game theory offers results showing the structure of possible interaction and the conditions, under which it is achieved. In many cases, there is a wide class of achievable interaction models and finite payoff distributions, and it is important to choose the best or most unbiased ones [19].

A cooperative game (*Robot, v*) is defined by a) the set of $Robot = \{1,2,\dots,n\}$, where each subset of *Robot* is called a coalition *K*, and b) a characteristic function $v\colon 2^n \rightarrow \mathbb{R}$, defined on the set of real values and giving each coalition its expected payoff (the so-called coalition gain). The value of the coalition *v(K)* is interpreted as the net gain from the cooperation of robots. The empty set (∅) and the set *Robot* are also called an empty and grand coalitions, respectively. The set of all subsets *Robot* or coalitions of agents is an exponent set (Boolean or degree set):

$$\wp\colon 2^n = ,1,2,\dots,n,1,2,\dots,n-1,n,1,2,3,\dots,,n-2,n-1,n,\dots,,1,2,\dots,n \qquad (1)$$

The structure S of coalitions $K_1,\dots,K_m, where\ K_j \subset Robot, j = 1,2,\dots,m,$ is defined as a set $k = \{K_1,\dots,K_m\}$, for which in case of non-overlapping coalitions $K_1 \cup \dots \cup K_m = Robot, K_i \cap K_j = \emptyset, \mathrm{i} \neq \mathrm{j}$. In other words, the structure of coalitions S consists of a complete mutually exclusive partition of the set *Robot* into subsets.

The solution of a cooperative game with transferable utility is a coalition configuration (S, x) formed by partitioning S of the *Robot* set into coalitions and an effective distribution of payoffs x, which assigns to each robot in the *Robot* a certain benefit from the gain of the coalition, whose member she is in a given coalition structure S. It is usually assumed that any coalition can be formed, either singular or complete (including all robots from *Robot*). It should be noted, however, that due to the combinatorial complexity of decision search, many methods involve limiting the number or size of coalitions in order to guarantee, for example, the polynomial complexity of the decision search process [20]. The set of payments $X = (x_i)_{i \in Robot} \in \mathbb{R}^n$ is called sharing. In a game with transferable utility (when winnings can be transferred between players) the following condition is fulfilled:

$$\forall K \subset Robot <=> \textstyle\sum_{i \in K} x_i \leq v(K) .$$

To reduce the admissible set of partitions, various dominance criteria can be considered. Such a criterion can be defined by means of the notion of the accompanying "core" of the game. This class of cooperative games is called games with core [7]. The set of payoff distributions, known as a core, with respect to a given structure of coalitions S is the set of configurations of coalitions with not necessarily unambiguous distributions of payoffs, which ensures that any subgroup of robots is not motivated to abandon the given coalition structure.

3.2 Basic Notions of a Cooperative Game with Core

In the following, we will use the basic definition of the core of the cooperative game (Robot, v) given in [25] by M. Mareš:

$$C = \{X = (x_i)_{i \in Robot}: \sum_{i \in robot} x_i \leq v(Robot), \forall K \subset robot: \sum_{i \in K} x_i \geq v(K)\} \quad (2)$$

The core is the set of all non-dominated partitions. The first argument of the set C indicates that the sum of all payments does not exceed the value of the game I. The second argument determines the property of rationality of the robots. Individually rational distributions of the total payment attribute to each agent at least the profit he can make without cooperation in any coalition. In turn, if the value of the total gain of the players is equal to the gain of the coalition, it indicates group rationality. In coalition configurations with the so-called pareto-optimal distribution of payoffs, no agent will get more payoff in any other possible distribution for a given game and coalition structure. It follows from the definition of the core that the set of distributions included in the core satisfies the coalitional rationality condition. In turn, this condition includes more particular conditions of individual rationality (when subsets of individual players are considered) and group rationality (when a subset is a grand coalition uniting all players). The principle of optimality in cooperative games is closely related to the concept of stability. In games with core, a set of payoff distributions where no agent has an incentive to leave its coalition *K* from *S* because of the assigned payment x_i is called a C-stable solution. Only those coalition structures are C-stable that maximize the welfare of all:

$$S^* = \arg \max_{S=\{K_1,\dots,K_m\}} \sum_{K \in S} v(K) \quad (3)$$

A coalition is called C-stable if it has a non-empty core [26]. Coalition structures that fully distribute the gains of a grand coalition are called efficient.

The choice of the above definition of the core is due to the following advantages for collective robotics:

- Allows considering the results of robots' interactions as a utility function.
- Considers a fuzzy membership function of coalition payments, whereas in traditional coalition games, introduced by Aubin [23], the fuzzy nature of the game is the fuzzy membership of the players.
- The convexity property of the game guarantees a non-empty solution set [25].
- Reduces the high computational cost of interactions between agents, typical for distributed negotiation-based models.

The generalized model of the game was proposed by the authors, which helped solving the problems of the computational complexity of finding the optimal structure and of the empty core, and enabled its use in practical applications of supply chains partner selection [32]. Unfortunately, it can´t be used as is for the problems where payoff distributions depend upon task allocations. During the game, robots must dynamically check different possible task allocations since the further payoff distributions explicitly depend upon the current allocation. So, we can consider the task allocation problem to be dynamic, though this definition differs from the traditional one, in which the assignment of robots to sub-tasks is continuously adjusted in response to changes in the task environment or group performance [27].

4 Cooperative Game Model with Fuzzy Core with Task Assignments for Precision Farming

There are many application areas that require the use of a coalition of robots to solve a complex task, including disaster medicine, precision farming, remote and local explore of space objects [28]. Robots, participating in the task execution, receive a reward. To decide whether participate or not in a task, the robots analyze if the amount of the reward meets their expectations. Below, a cooperative game model with fuzzy core for task allocation is proposed, illustrated by the use case of precision farming. The main distinctive feature of coalition formation in this context is that payment distribution in the game depends on task assignments to each robot.

4.1 Precision Farming as a Fuzzy Coalition Formation Problem

The problem of precision farming is described in [29]. There is a field with various geological and ecological characteristics of soils, suitable for growing several crops that require different growth conditions. The goal is to get the best possible gain from the field within certain time and financial constraints. The field is processed by several types of robots (scouts, planters, tractors, transports) equipped with devices for plowing, loosening, planting, watering, fertilizing, and harvesting crops. Robots' capabilities are functions of specific parameters (cruise velocity, cultivator's length), which define their productivity for each task (hectares per hour). The robots belong to different owners who are willing to provide them for processing the field in exchange for a reward (Figure 1).

In general, the processing of the field can be divided into three stages: i) exploration of the field to determine its condition, ii) preliminary processing and fertilization of the field, followed by planting crops suitable for the parameters of the field, iii) harvesting. Processing the field has the fixed cost. For illustrative purposes, we consider the second stage in this paper. The task itself consists in plowing and seeding the field, for which two types of robots are used: a tractor and a planter.

The coalition of all robots involved in the task processing is called *grand coalition.* In this case, the field processing is defined as a problem of coalition formation with task allocation, where each task assignment can be seen in terms of hectares to be processed (using specific robot's capability) within each subtask. The purpose of the game is to find a task allocation strategy that provides a mutually beneficial payoffs distribution. In other words, we are looking for the efficient and stable coalition structure to process a field under temporal and financial constraints.

In the general case of coalition formation with dynamic task allocation, a coalition can be understood as a group of robots, joined together to perform the subtask T_i of task T. A robot's strategy is defined as a binary variable φ, such that:

$$\varphi(T_i, k, j) = \begin{cases} 1, if\ the\ robot\ j\ from\ coalition\ k\ executes\ T_i \\ 0, otherwise \end{cases}$$

The coalition's utility $v(K_{T_i})$ is distributed between the coalition members according to the vector of payment distribution $x_{T_i} = \{x_{T_i}\}\{x_{T_i}^1, \dots, x_{T_i}^{|K_{T_i}|}\}$, where $x_{T_i}^j$ – is a payment to robot $j \in Robot$, and $x_{T_i}^{|K_{T_i}|}$ is a payment to the coalition.

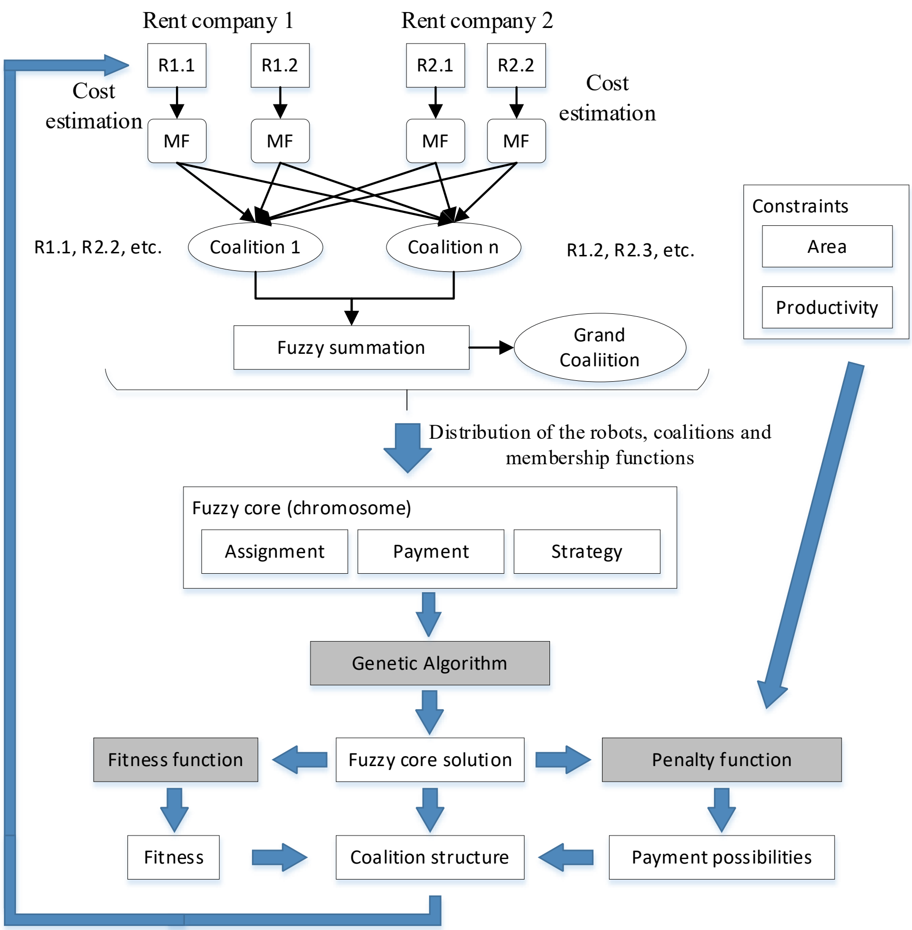

Figure 1

Structural diagram of the coalition formation for precision farming (based on [30])

4.2 Definition of a Fuzzy Cooperative Game with Dynamic Task Assignments

In the case of multi-robot teams with task allocation, the fuzzy payments for each robot depend on the solution of the allocation problem. A FCG is defined as a pair $(Robot, w)$, where $Robot$ is nonempty and finite set of players; subsets of $Robot$ joining together to fulfil a task T_i are called coalitions K, and w is called a characteristic function of the game, being $w: 2^n \rightarrow \Re^+$ a mapping connecting every coalition $K \subset Robot$ with a fuzzy quantity $w(K) \in \Re^+$, with a membership function $\mu_K: R \rightarrow [0,1]$. A modal value of $w(K)$ corresponds to the characteristic function of the crisp game $v(K)$: $\max \mu_K(w(K)) = \mu_K(v(K))$. For an empty coalition $w(\emptyset) = 0$. A fuzzy core for the game $(Robot, w)$ with the imputation $X = (x_{ij})_{i\in I, j\in Robot} \in \Re^+$ is a fuzzy subset C_F of $\Re^+$:

$$C_F = \left\{ x_{ij} \in \Re^+ : \nu \succ= \left(w(Robot), \sum_{\substack{i\in I,\\ j\in Robot}} y_{ij} x_{ij} \varphi_{ij} \right), \min_{\substack{K_i \in k\\ j\in Robot}} \left(\nu \succ= \left(\sum_{j\in K_i} y_{ij} x_{ij} \varphi_{ij}, w(K_i) \right) \right) \right\}, \quad (4)$$

where y_{ij} is the assignment of a task T_i to a robot j, $x_{ij} = f(y_{ij})$ is the fuzzy payment of a robot j participating in a coalition i, $i = 1, 2, \ldots, I, j = 1, 2, \ldots, N$, $\bar{k} = [K_1, K_2, \ldots, K_l]$ is the ordered structure of effective coalitions; $\succ=$ is a fuzzy partial order relation with a membership function $\nu \succ= : R \times R \to [0,1]$, and φ_{ij} is a binary variable such that:

$$\varphi_{ij} = \begin{cases} 1, if\ robot\ j\ participates\ in\ a\ coalition\ i; \\ 0, otherwise. \end{cases}$$

This variable can be considered a result of some robot's strategy on joining a coalition. A fuzzy partial order relation is defined as follows (for more details see [31]). Let *a, b* be fuzzy numbers with membership functions μ_a and μ_b respectively, then the possibility of partial order $a \succ= b$ is defined as $\nu \succ= (a, b) \in [0,1]$ as follows:

$$\nu \succ= (a, b) = \sup_{\substack{x,y\in R\\ x\geq y}} \left(\min(\mu_a(x), \mu_b(y)) \right). \quad (5)$$

The core C_F is the set of possible distributions of the total payment achievable by the coalitions, and none of coalitions can offer to its members more than they can obtain accepting some imputation from the core. The first argument of the core C_F indicates that the payments for the grand coalition are less than the characteristic function of the game. The second argument reflects the property of group rationality of the players, that there is no other payoff vector, which yields more to each player. The membership function $\mu_{C_F}: R^n \to [0,1]$, is defined as:

$$\mu_{C_F}(x) = \min \left\{ \nu \succ= \left(w(Robot), \sum_{\substack{i\in I\\ j\in Robot}} y_{ij} x_{ij} \varphi_{ij} \right), \min_{\substack{K_i \in k\\ j\in Robot}} \left(\nu \succ= \left(\sum_{j\in K_i} y_{ij} x_{ij} \varphi_{ij}, w(K_i) \right) \right) \right\} \quad (6)$$

With the possibility that a non-empty core C_F of the game $(Robot, w)$ exists:

$$\gamma_{C_F}(Robot, w) = \sup\left(\mu_{C_F}(x) : x \in \Re^n \right) \quad (7)$$

The solution of a cooperative game is a coalition configuration (S, x) which consists of (i) a partition S of $Robot$, the so-called coalition structure, (ii) the task assignment for each S member y_{ij}, and (iii) an efficient payoff distribution *x,* which assigns each robot in *Robot* its payoff out of the utility of the coalition it is a member in the given coalition structure *S*. As previously shown in [32], any real argument, like y_{ij} in this case, can be included in the fuzzy set of solutions, such that all the definitions and theorems proved for the game are also fulfilled for the core defined by (4). In the following theorem it is proved that the fuzzy set of coalition structures forming the core is a subset of the fuzzy set formed by the structure of effective coalitions.

Definition 1. A coalition K is called effective if it cannot be removed from the coalition structure by a sub-coalition $L \subset K$. The possibility that a coalition K is effective is defined as:

$$\sup_{x \in R^n}(\min(\mu_k(x), \mu_l^*(x) \colon L \subset K)). \tag{8}$$

Theorem. Let $(Robot, w)$ be a fuzzy coalition game. Then, for some structure of effective coalitions its possibility is at least equal to the possibility of forming a core.

Proof. From (6), when all φ_{ij} are equal to 1, we have a coalition structure belonging to the core; otherwise, we have a coalition structure corresponding to the generalized model. In addition, $\nu \succ= \left(\sum_{j \in K_i} y_{ij} x_{ij}, \sum_{j \in K_i} y_{ij} x_{ij} \varphi_{ij}\right)$ is satisfied with a positive possibility, and, therefore, the possibility of the structure of effective coalitions for the generalized model is higher than for the basic model.

The algorithm of fuzzy number summation for obtaining coalition membership functions represents an important element of the model. The sum operation is based on Zadeh extension principle [33] for fuzzy numbers *a* and *b* (which are convex sets normalized in R):

$$\mu_{a(*)b}(Z) = \sup_{z = x * y} \min\left(\mu_a(x), \mu_b(y)\right) \tag{9}$$

where * can designate the sum ⊕ or the product • of fuzzy numbers. Each fuzzy set is decomposed into two segments, a non-decreasing and non-increasing one. The operation * is performed for every group of n segments (one segment for each fuzzy set) that belong to the same class (non-decreasing or non-increasing one). Thus, a fuzzy set is generated for every group of *n* segments. The summation result is derived as superposition of these sets, which gives the membership function as the sum of *n* fuzzy numbers.

4.3 Solution of the FCG Model with Core

A genetic algorithm (GA) in the context of fuzzy logic has been used. This is equivalent to the binary coding of a fuzzy C-core with a target function of the upper minimum of the membership functions. The algorithm is based on a population of chromosomes, each one serving as a feasible solution of the game. A chromosome consists of an array of real numbers representing robot assignments, payments to the robots and coalitions, binary variables defining participation of each robot in a coalition and payment possibilities (Figure 2).

Task assignments				Fuzzy payments							Binary variable (strategy)				Payment possibilities							
$y_{1,1}$	$y_{1,2}$	…	$y_{m,n}$	$x_{1,1}$	$x_{1,2}$	…	$x_{m,n}$	x_{c1}	…	x_{ck}	x_{gc}	$\varphi_{1,1}$	$\varphi_{1,2}$	…	$\varphi_{m,n}$	$w_{1,1}$	$w_{1,2}$	…	$w_{m,n}$	w_{c1}	…	w_{ck}	w_{gc}

Figure 2

A chromosome representation with real encoding, where m – number of tasks, n – number of robots, k – number of coalitions

The fitness function is set by the core of the game and fuzzy expectations of the robots and minimizes the difference between the gain of the game and the sum of individual payments to the robots, according to Eq. 3 and fulfilling the productivity constraints: $y_{ij} \leq b_j^i, i \in I, j \in Robot$ and the following strict area constraints: $\sum_{j \in K_i} y_{ij} = Y_i$, where $i \in I$, Y_i is the plot corresponding to the T_i. To guarantee the feasibility of the solution in terms of area constraints, the normalization can be used in this case: $\bar{y}_{ij} = y_{ij} / \sum_{j \in K_i} y_{ij} * Y_i$, where Y_i is the plot corresponding to the T_i.

The problem with such solution is that productivity constraints initially considered on the chromosome level, should be included in the fitness function as another hard constraint.

For the unconstraint problem, the following quadratic penalty functions are defined:

a) Area constraints penalty: $pen_a = \left(\sum_{j \in K_i} y_{ij}\varphi_{ij} - Y_i\right)^2$,

b) The first part of the core constraint penalty: $pen_c = \left(\sum_{\substack{i \in I \\ j \in Robot}} x_{ij}\varphi_{ij} - w(K_i)\right)^2$

c) Productivity constraints $y_{ij} \leq b_j^i, i \in I, j \in Robot$ are considered on the chromosome level and thus, are excluded from the fitness function

Calculation of fitness involves several steps according to the **Algorithm 1** (unconstraint problem). The GA used heuristic initialization and applicable genetic operators following the elitist strategy. The solution algorithm was implemented using R language. The following packages have been used: *genalg* implementing the GA, *forcats* - for levels identification of the categorical variables, *compositions* - for compositional data analysis, *fuzzyreg* - to calculate the fuzzy sum of the coalition payments, and *animation* - for visualization of the fitness function.

Algorithm 1: Pseudo-code of the fitness function of the GA (with penalty functions)

 # N – wheat and rye fields' dimensions (equal area plots)

Input: Chromosome (Figure 2)

Fitness: According to Eq. (4) with penalty functions defined above

Output: current_solution # list including vector of payments x_{ij}, job assignments y_{ij}, payment possibilities and fitness

1) *Extract vectors of payments and assignments from the chromosome*
2) *For (i in 1: cnum) {*
    a. *Normalize assignments: $n_{i,j}$= field.seed/$\sum_j y_{ij}$ ($j \subset F$), for each coalition i and all tasks*
    b. *Check for area constraints and calculate pen_a*
    c. *if (!feasible_solution) return(w(GC)*2N)*

    } #end for
3) *Add vector of assignments to current_solution*
4) *Calculate payments using assignments*
5) *For (i in 1: cnum) {*
    a. *Calculate payments for the coalition*
    b. *Check for coalitional core constraint and calculate pen_c*
6) *Add vector of payments to current_solution*
7) *Calculate fitness according to (4) with penalty functions*
8) *Check the constraint for grand coalition: $N*w(GC) \geq \sum_{i \in I, j \in J} x_{ij} y_{ij}$*
9) *if (!feasible_solution) return(w(GC)*2N) else return(fitness)*

5 FCG for Precision Agriculture Case

The case study to illustrate the coalition formation with fuzzy cooperative game is formulated as a subset of general agricultural settings described above (Section 3). Let us suppose that field's exploration and plowing have already been done, resulting in two fields of equal area (200 hectares), the former to be sown with wheat and liquid fertilizers and the latter, with rye and solid fertilizers. Thus, a task is composed of two subtasks corresponding to the stage two of the described precision farming problem and can be solved by two coalitions of robots. Transport operations will be considered constraints and thus, transport robots will be excluded from the coalitions. Suppose the task should be finished within 20 hours.

5.1 Experimental Settings

Provided experiments pretended to pursue several objectives:

- To analyze the applicability of the proposed approach in the test settings close to the real ones.
- To study the shapes of applicable membership functions and their influence on the resulting solution.
- To study the convergence of the proposed solution algorithm both for the case of hard constraints and unconstrained problem with penalty function.
- To study the computational complexity of the solution algorithm.

Due to the space limits, below we describe the case with twelve robots. No one robot can fulfill the task alone and thus, they must form coalitions. On the other hand, their excessive capabilities mean that several robots can be left behind (Table 1). Robots have different productivities defined as hectares per hour for tractors and planters [34]. Area and productivity constraints for the fitness function are defined based on case study basic parameters (robots' productivities and field dimensions) as follows:

1. Area constraints:

   $Y_{1,1} + Y_{2,1} = 200$, $Y_{3,2} + Y_{4,2} = 200$, $Y_{5,1} + Y_{6,1} = 200$, $Y_{7,2} + Y_{8,2} = 200$

2. Productivity constraints:

   $Y_{1,1} <= 128$, $Y_{2,1} <= 96$, $Y_{3,2} <= 128$, $Y_{4,2} <= 96$, $Y_{5,1} <= 128$, $Y_{6,1} <= 96$, $Y_{7,2} <= 128$, $Y_{8,2} <= 96$.

In the conditions of the task, the size of the field, the cost of renting robots per hour of work, their productivity and payment that robots will receive for completing the task are fixed. As a solution to the problem, the composition of the coalition is taken, which ensures the distribution of the payoff when performing the task in accordance with the contribution of each robot.

It can be easily transformed to S and Z membership functions with the right spread outside the function's definition area in the former case, whereas with Z-shaped membership functions, left spread is outside the definition area. The membership functions for robots and coalitions were defined based on the market average values. The triangular type of function means that a robot expects the best possibility (1.0) to receive the mean payment, while the less payment is not very desirable for it and the greater payment may not be granted by the coalition. The values of left and right spread define the min and max constraints for the payments to look for. MFs of z- shape and s-shape follow different logics. In case of the z-shape, the less is the payment (from the acceptable range), the more is the possibility to receive it. On the contrary, the s-shape reflects the robot's expectations to be granted the largest possible payment. We consider this latter case to be the most appropriate for the precision agriculture case.

Table 1

Distribution of the robots' properties, coalitions and membership functions for twelve robots

Robot	Type	Function	Velocity	Coalition	Triangular MF			S-shape MF	
					MF_cent (m)	MF_lspr (a)	MF_rspr (b)	MF_lspr (a)	MF_cent (m)
1	tractor	Liquid_f	8	1	75	25	25	50	100
2	tractor	Liquid_f	7	1	60	20	20	40	80
3	tractor	Liquid_f	6	1	50	20	20	30	70
4	tractor	Solid_f	8	2	60	20	20	40	80
5	tractor	Solid_f	7	2	50	20	20	30	70
6	tractor	Solid_f	6	2	30	10	10	20	40
7	planter	wheat	8	1	55	15	15	40	70
8	planter	wheat	7	1	45	15	15	30	60
9	planter	wheat	6	1	30	10	10	20	40
10	planter	rye	8	2	55	15	15	40	70
11	planter	rye	6	2	50	10	10	40	60
12	planter	rye	4	2	30	10	10	20	40

Coalition	Triangular MF			S-shape MF	
	MF_cent (m)	MF_cent (m)	MF_cent (m)	MF_lspr (a)	MF_cent (m)
1	105	35	35	70	140
2	100,5	33,5	33,5	67	134
GC	137	0	0	137	137

In Table 1, we assign a consecutive number for each robot, consider the case of non-overlapping coalitions and use piecewise linear versions of the hat (triangular), z-shape and s-shape membership functions (MF) for payments. A symmetric triangular-shaped membership function describes a grade of membership as follows:

$$\mu_i(x) = \begin{Bmatrix} 0, & x < a \\ \frac{x-a}{m}, & a \leq x < m \\ 1 - \frac{x-b}{m}, & m \leq x < b \\ 0, & x \geq b \end{Bmatrix}, \tag{10}$$

where *m* is a central value, *a* and *b* are left and right spreads.

5.2 Experimental Results

In the first experiment, hard area and productivity constraints were used to solve the problem, which resulted in empty core for 10,000 iterations. For the rest of experiments, constraints were included in the penalty function as explained in Section 4.4. Table 2 shows an excerpt of the experimental results with different setting. We can observe the convergence of the method with a high probability of coalition formation. The starting parameters for this case also included 10,000 iterations, the mutation rate (M) varied from 0.05 to 0.15 with step 0.05, crossing rate (crossover) - 50%, population of 200 organisms, and the payment of the grand coalition was settled to $54,800. Convergence of the GA for 15000 iterations can be seen at Figure 3. As it can be seen, though the best solution converges shortly, it takes the average solution about 13500 iterations to get the whole population closer to the best one.

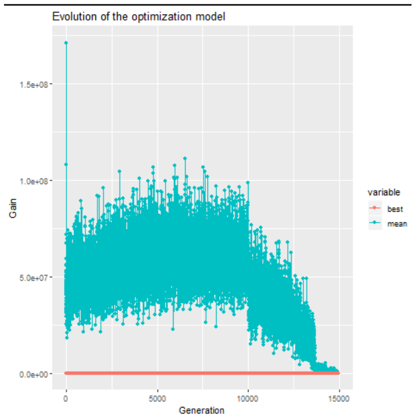

Figure 3
Convergence of the GA for twelve robots with penalty functions

As we can see, the average fitness is about 12, with the best value of 2,33 and one outlier of 58,5. This value estimates coalition performance and is composed of the differences in payments and productivity constraints. The value represents 0.02% of the total payment, which is a very good approximation of the result. A feasibility threshold for the acceptable payment possibility can be defined to exclude those solutions that have little possibilities. Nevertheless, as we can see from Table 2, most of the robots and coalitions have high levels of payment possibilities.

The limitations on the shape of a MF for the use case does not impact on the generality of the described approach. As we can see, in most of the runs, the third robots (#3, #6, #9 and #12) were excluded from the coalition structure since they have the lowest productivities (Table 1). For the HP Pavilion x360 notebook with i5 8-th generation CPU and 8 Gb of RAM, one simulation run of the GA, 10,000 iterations with unconstraint settings for 12 robots takes 22 mins.

Table 2
Use case results: the structure of effective coalitions for twelve robots. The best runs for each setting are shadowed

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10	Run 11	Run 12	Run 13	Run 14	Run 15
	M=0,1, with strategies			M=0,05, with strategies						M=0.15		M=0.1 w/o strategy			
R1	83	102	103	101	118	115	112	112	83	104	105	109	104	111	115
R2	74	98	97	99	82	85	88	88	58	96	95	81	74	21	44
R3	43	0	0	0	0	0	0	0	59	0	0	10	22	68	40
R4	106	108	121	118	106	115	122	121	103	115	114	105	118	120	119
R5	94	92	79	82	94	85	79	79	97	85	86	95	77	73	70
R6	0	0	0	0	0	0	0	0	0	0	0	0	5	7	11
R7	109	118	91	116	124	110	101	109	102	116	88	118	95	107	92
R8	91	82	109	84	76	90	99	61	98	84	72	69	85	78	98
R9	0	0	0	0	0	0	0	30	0	0	40	13	21	16	10
R10	120	111	121	118	123	117	112	123	115	125	119	122	102	100	101
R11	80	89	79	82	77	66	88	77	85	75	81	65	84	80	92
R12	0	0	0	0	0	17	0	0	0	0	0	13	15	20	7
Pay1	7195	8633	8884	8120	9943	10191	9450	9983	7924	8895	9478	9477	9308	10765	10996
Pay2	5771	7551	7484	7746	6574	6550	6687	7022	4143	7214	7297	6365	5805	1412	3322
Pay3	2967	0	0	0	0	0	0	0	3774	0	0	571	1123	3882	2131
Pay4	7669	8443	8780	9058	8081	8969	8978	9144	7272	9116	8515	8276	9325	9307	9429
Pay5	6491	6257	5310	5511	6406	5905	5425	5445	6663	5355	5610	6147	5266	5065	4846
Pay6	0	0	0	0	0	0	0	0	0	0	0	0	172	178	275
Pay7	7190	7114	6193	7577	7399	6326	5961	6454	6482	6950	5487	7614	6390	7410	5995
Pay8	4879	4697	5426	4562	4081	4930	5883	3471	5703	4943	4286	3599	4814	3945	5199
Pay9	0	0	0	0	0	0	0	1069	0	0	1459	394	581	598	372
Pay10	8017	6846	8433	7696	7707	7938	7278	7684	7746	8144	8325	8135	6706	6910	6771
Pay11	4618	5249	4276	4524	4602	3427	5141	4530	5059	4171	4351	3865	4877	4615	5235
Pay12	0	0	0	0	0	573	0	0	0	0	0	442	503	773	268
Pay13	27997	27995	27990	27998	27998	27996	27982	27995	27999	27999	27996	27997	27988	27998	27996
Pay14	26799	26792	26796	26794	26798	26799	26795	26796	26742	26789	26796	26781	26799	26797	26797
Pay15	54800	54800	54800	54800	54800	54800	54800	54800	54800	54800	54800	54800	54800	54800	54800
Strat1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Strat2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Strat3	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1.00	1.00	1.00	1.00
Strat4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Strat5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Strat6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00
Strat7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Strat8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Strat9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00
Strat10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Strat11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Strat12	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10	Run 11	Run 12	Run 13	Run 14	Run 15
	M=0,1, with strategies			M=0,05, with strategies						M=0.15		M=0.1 w/o strategy			
Poss1	0.73	0.69	0.73	0.62	0.69	0.77	0.68	0.78	0.91	0.7	0.81	0.74	0.79	0.95	0.91
Poss2	0.95	0.93	0.93	0.95	0.99	0.93	0.91	1	0.78	0.89	0.92	0.96	0.96	0.68	0.87
Poss3	0.97	0	0	0	0	0	0	0	0.86	0	0	0.66	0.55	0.67	0.58
Poss4	0.81	0.96	0.81	0.91	0.91	0.95	0.85	0.89	0.76	0.98	0.87	0.97	0.98	0.94	0.97
Poss5	0.98	0.95	0.94	0.94	0.95	0.98	0.98	0.97	0.97	0.82	0.88	0.87	0.97	0.99	0.99
Poss6	0	0	0	0	0	0	0	0	0	0	0	0	0.59	0.25	0.26
Poss7	0.86	0.67	0.94	0.84	0.65	0.59	0.64	0.64	0.78	0.66	0.74	0.82	0.92	0.98	0.84
Poss8	0.8	0.92	0.65	0.82	0.8	0.82	0.97	0.89	0.94	0.96	0.98	0.74	0.89	0.7	0.77
Poss9	0	0	0	0	0	0	0	0.78	0	0	0.85	0.53	0.41	0.93	0.84
Poss10	0.9	0.72	0.99	0.84	0.76	0.93	0.84	0.74	0.91	0.84	0.99	0.89	0.86	0.97	0.9
Poss11	0.88	0.95	0.7	0.77	0.99	0.58	0.91	0.96	0.98	0.77	0.69	0.97	0.92	0.87	0.86
Poss12	0	0	0	0	0	0.73	0	0	0	0	0	0.68	0.73	0.97	0.86
Poss13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Poss14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Poss15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
fit	4.95	9.75	11.39	7.21	2.33	3.84	20.9	7.08	58.5	7.16	6.54	20.69	11.59	3.6	6.53

Discussion and Conclusion

The study is aimed on the use of the theory of fuzzy cooperative games for the formation of coalitions of autonomous robots. For task allocation, robots exchange their individual expectations of a gain from participating in a coalition, as well as their capabilities, which are used as constraints in a cooperative game. The existence of a core allows coordinating the actions of individual members to achieve a common goal, as well as to evaluate and distribute the overall winnings. In the experiments, each robot was assigned to perform at most a single task, which led to non-overlapping coalitions. Nevertheless, coalitions can overlap, allowing robots to solve tasks for different coalitions with various game cores and receiving a reward from each of them. When changing the conditions in which the task was set, a dynamic change in the composition of the coalition is envisaged, if necessary.

A test scenario of precision farming was studied, describing a two-stage processing of the field by several robots. Obtained results show that the fuzzy definition of the requirements and the membership function of the coalition ensures the selection of participants, in the absence of a clear statement of the problem and requirements for individual members of the coalition, thus solving the problem of the empty core.

To calculate the composition of the coalition, a genetic algorithm was used as a mechanism for optimizing the solution. The population gene included variables that denote the dynamic assignments of the jobs with corresponding area to the robot, robots' strategies, the payment to the robot, and the possibility of payment. This is due to the fact that competition for payment appeared between robots and various behavioral strategies arose, which also had to be taken into account in the core.

Solving of unconstrained optimization problems (with penalty functions) in general is easier than solving a constrained optimization problem. In this case, a quadratic penalty function is defined for each general constraint, which gives greater variability and better convergence.

The distinctive features of the obtained solution include a rather small variability in payments to coalitions and grand coalitions with a very high possibility of the game. This is because the GA solves two problems in a parallel: i) the assignment task and ii) the distribution of payments. In the distribution problem, the main condition is to minimize the value of the fitness function, which in most iterations gives the maximum approximation to the total distribution of the payment. In the assignment problem, more attention is paid to constraints, and therefore more variability is formed, since, due to the presence of two constraints, there are more options for distributing tasks between coalition members. For practical purposes, the best available solution (min fitness function) among several runs or the game instance with the highest possibility of payments can be selected as the coalition structure.

The work contribution could be resumed as follows:

- Extended definition of the coalition game with fuzzy core with task allocation is proposed. The definition provides additional variables to the assignments that are added into the core. Therefore, solving in parallel coalition formation and task allocation, enables obtaining efficient and stable coalition structure as a result of the game.

- An efficient solution algorithm of metaheuristic optimization for unconstraint problem statement is proposed. The optimization is based on the genetic algorithm along with penalties for estimating variation from the expected value.

- The proposed approach is applied to the group robotics, specifically to the precision agriculture problem. Approach is used for computing a coalition for field processing task and provides coalition structures and payment distribution along with the assignments for each coalition participant.

- For the scenario from precision farming, the computation of the game solution takes less than 30 minutes with linear complexity in terms of the number of robots, coalitions and iterations that proves the scalability and robustness of the proposed framework to be used in real-life applications.

Further work will focus on two main directions. The first is to reduce computational costs and speed up the process of calculating a cooperative game with more types of robots. Achieving this goal will make it possible to make operational decisions when the situation changes dynamically and, thereby, change the composition of the coalition in real time to address these changes. This approach will provide the ability to replace robots in the event of failures. The second direction is the development of the game characteristic function and the expansion of the list of variables to take into account a larger number of factors to achieve a more accurate selection of robots based on the effectiveness of the coalition.

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