High Performance Direct Torque Control of Electrical Aerodynamics Load Simulator using Fractional Calculus

Nasim Ullah[1], Wakeel Khan[2] and Shaoping Wang[3]

[1,2,3]School of Automation Science and Electrical Engineering, 37 XueYuan road, HaiDian District, Beihang University, Beijing China, [1]nasim@mse.buaa.edu.cn, [2]wakeel_10@asee.buaa.edu.cn, [3]shaopingwang@vip.sina.com

Abstract: Electrical load simulator system (ELSS) is a test rig used to apply medium range aerodynamics loads on flight actuation system in real time experiments. A novel high performance fractional order adaptive robust torque control law is proposed for Electrical Load Simulator which is subjected to extra torque disturbance, friction, and parametric uncertainties. Adaptive fuzzy system is used to estimate extra torque disturbance and parametric uncertainty is estimated using discontinuous projection based adaptive control. A friction observer is used to compensate nonlinear friction. Stability of closed loop is derived using Lyapunov method .The proposed method ensures transient performance of ELSS system subjected to none zero initial conditions. Frequency testing and extra torque elimination tests are performed using PID, integer order sliding mode control and the proposed controller. The efficiency of proposed controller is verified using extensive numerical simulations.

Keywords: Electrical load simulators; Fractional calculus; Backstepping control; Fuzzy logic system

1 Introduction

Electrical load simulator system is important laboratory-based hardware in the loop (HIWL) test rig that is used to exert aerodynamics loads on control surfaces of a flight vehicle according to flight conditions. The laboratory setup consists of a loading motor which is directly connected to the flight actuation system through a stiff shaft. During torque loading experiment movement of flight actuator is a strong disturbance for ELS loading motor which induce extra torque [1]. Different integer order control techniques are proposed in literature to compensate extra torque disturbance. A velocity synchronization control is proposed for electro hydraulic load simulator in [1]. The same technique is proposed for eliminating influence of extra torque in electrical load simulators [2]. In the above cited work, velocity of actuator is approximated using its nominal model, but practically parametric uncertainty can degrade control performance of ELS system. To rectify the same problem some robust control techniques are presented in literature such as disturbance observer based control [3], H-infinity control [4] and variable structure sliding mode control [5]. The robust control techniques in [3-5] ensure good tracking performance with known dynamics and disturbance bounds. Direct torque control is proposed for ac dynamometer [6]. In practice a direct torque control is hard to realize based on measured states due to measurement noise. A novel speed and mechanical torque estimation algorithm is proposed for ac dynamometer [7]. Design of a high performance dynamometers is proposed in [8] and a nonlinear predictor based controller is proposed in [9, 10].

It is hard to design and realize a high performance control for systems which is subjected to nonlinear friction. To overcome the problem, different control techniques are proposed in literature. Several techniques such as adaptive fuzzy compensation for robot manipulators in [11], friction state predictor [12], robust state observer [13] and modified Lugre model based friction compensation in [14] are successfully applied to compensate nonlinear friction. Friction compensation using fuzzy logic system is efficient but tuning process of membership function and fuzzy rules is very tedious. Similarly friction observers are effective as long as identified models and their parameters are accurate. For electro hydraulic load simulators, friction modeling and its compensation methods are discussed in [15].

Back stepping is a recursive nonlinear control method which has been successfully applied to many nonlinear systems. The control method is very effective in situation if system parameters are uncertain. In order to formulate high performance control for servo drive, several controllers are proposed using backstepping method. A high performance torque controller is proposed in [16], adaptive position control using fuzzy and neural network in [17, 18] and integral backstepping methods are proposed and validated [19-22]. A robust IMC–PID controller is formulated using H-infinity and model matching approach [23]. The above work shows excellent tradeoff between robustness and performance but the major limitation is that both robustness and performance are not decoupled totally. To ensure robustness of PI and PID controllers, tuning process is introduced for integral type servo system which is subjected to parametric uncertainties [24].

To achieve performance objectives, fractional order control offers more degree of freedom as compared to integer order. The first fractional order controller “CRONE” was proposed in 1996 [25]. Later on researcher extended the idea and developed PID and adaptive fractional PID controllers [26]. Several fractional order sliding mode controllers are presented in literature [27-30]. An integer order robust gain scheduled speed controller is proposed in [35], which ensures stability and performance of the closed loop over a wide range of operation. For good control performance, a robust digital controller with iterative tuning is proposed in [36].

Based on the above literature survey, this work is focused on developing a fractional order adaptive fuzzy backstepping torque control for electrical load simulator. Fuzzy logic system is used to estimate lumped disturbance due to extra torque and uncertainty in friction compensation. To estimate uncertain parameters of load simulator, adaptive laws are derived using Lyapunov function method. Detailed numerical simulations are presented to prove effectiveness of the proposed control method.

2 Problem Formulation

PMSM motor is used as loading device in ELS system. The dynamics of ELS system in *d*-*q* reference frame can be written as

$$\begin{aligned} u_d &= i_d R_s + L_{sd}\frac{di_d}{dt} - PL_{sq} i_q w_m \\ u_q &= i_q R_s + L_{sq}\frac{di_q}{dt} + PL_{sd} i_d w_m + P\Psi_m w_m \\ T_e &= \frac{3P}{2}[\Psi_m i_q + (L_{sd} - L_{sq}) i_d i_q] = J\frac{dw_m}{dt} + \beta w_m + T_f + T_L \end{aligned} \tag{1}$$

In Eq. (1) $[i_d \; i_q]$ represents *d*-axis and *q*-axis currents, w_m represents angular speed ofloading motor, $[u_d \; u_q]$ represents *d*-axis and *q*-axis voltages, $[L_{sq} \; L_{sd}]$ represents inductances, R_s is winding resistance, $[P \;\; \Psi_m]$ represents number of pole pairs and magnetic flux of rotor, $[J \;\; \beta]$ represents total inertia and damping coefficient and $[T_e \; T_f \; T_L]$ represent the electromagnetic , friction and loading torque respectively. Assuming that inertia and damping coefficient of torque sensor are very small, and then the reduced dynamics can be written as

$$T_L = K_s(\theta_m - \theta_a) \tag{2}$$

Here $[\theta_m \;\; \theta_a]$ represents angular positions of ELS loading motor and flight actuator respectively and K_s is the total stiffness of torque sensor and connecting shaft. Dynamics of electrical load simulator and its detailed mathematical formulations are given in [31]. State space representation of ELS system is derived in [31] and re- written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -Ax_2 + Bu - Cf(T_{extra}) - CT_f \end{aligned} \tag{3}$$

Here $[x_1\ x_2\ ]$ represents system states, u is the control effort, $f(T_{extra})$ is the extra torque disturbance, T_f is friction torque. From Eq. (3) parameters of state equation are defined in [31] as $A = \frac{k_t k_b}{JR_s}$, $B = \frac{K_s k_t}{JR_s} + \frac{\beta}{J}$ and $C = \frac{K_s}{J}$. Here k_t represents motor torque constant, $k_b = P\Psi_m$ is back emf constant, J is total inertia of system. All other parameters are defined above. Practically parameters of ELS system are uncertain. To include the effect of uncertain parameters in state model, define $\phi_1 = A$, $\phi_2 = B$ and $F = Cf(T_{extra}) + \varepsilon_f$. Here ε_f is the friction compensation error due to uncertainty in friction model. Eq. (3) can be represented as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\phi_1 x_2 + \phi_2 u - F \end{aligned} \tag{4}$$

Remark 2.1 *In Eq. (4)* $F = C\,f(T_{extra}) + \varepsilon_f$ *and* ε_f *is the friction compensation error. Friction torque is compensated using Lugre model to be discussed later. Lugre model based compensation control may not be perfect due to parametric uncertainty in Lugre model, so its effect is included in the state model. The unknown component* F *is the lumped disturbance which is to be estimated using fuzzy logic system.*

Assumption 1: *The extent of the parametric uncertainty is known and bounded such that*

$$\phi \in \Omega\,\{\phi : \phi_{\min} \le \phi \le \phi_{\max}\} \tag{5}$$

Here ϕ *represents unknown parameter vector,* $\phi_{\min}$ *and* $\phi_{\max}$ *are bounds of uncertain parameters and* Ω *represents set of uncertain parameters.*

Assumption 2: *State vector* $[x_1\ x_2\ ]$ *is available to formulate control law and noise free.*

Remark 2.2 *In practical situations, state vector* $[x_1\ x_2\ ]$ *may contain measurement noise. In this work* ***Assumptions 2*** *is made to validate the effectiveness of fractional order control law to be derived later. The state estimation problem will be addressed in future research using algebraic method.*

The control objective is to get ELS torque motor to track a desired reference loading command vector $[x_r\ \dot{x}_r\ ]$. State errors vector $[\ z_1\ z_2\ ]$ can be defined as

$$\begin{cases} z_1 = x_1 - x_r \\ z_2 = x_2 - \dot{x}_r \end{cases} \tag{6}$$

The objective of this work is to design adaptive robust fuzzy fractional order backstepping controller for torque tracking loop of ELS loading system.

2.1 Fractional Calculus & Fuzzy Logic System

Definition 1. Fractional operator is defined as ${}_aD_t^{\lambda}$ [26]

$$
{}_aD_t^{\lambda} \cong D^{\lambda} = \begin{cases} \dfrac{d^{\lambda}}{dt^{\lambda}} & R(\lambda) > 0 \\ 1 & R(\lambda) = 0 \\ \displaystyle\int_a^t (d\tau)^{-\lambda} & R(\lambda) < 0 \end{cases} \tag{7}
$$

Here a and t are the limits of operation, λ is the order of fractional operator and R is set of real numbers.

Definition 2. Riemann–Liouville fractional order difference- integral of function $f(t)$ is given by [26].

$$
{}_aD_t^{\lambda} f(t) = \frac{d^{\lambda}}{dt^{\lambda}} f(t) = \frac{1}{\Gamma(m-\lambda)} \frac{d^m}{dt^m} \int_a^t \frac{f(\tau)}{(t-\tau)^{\lambda-m+1}} d\tau \tag{8}
$$

Here Γ is the gamma function and $m-1<\lambda\le m\in N$

Definition 3. The Caputo, ^{s} fractional order difference- integral of function $f(t)$ is given by [26].

$$
{}_aD_t^{\lambda} f(t) = \begin{cases} \dfrac{1}{\Gamma(m-\lambda)} \displaystyle\int_a^t \frac{f^m(\tau)}{(t-\tau)^{\lambda-m+1}} d\tau & ; m-1<\lambda<m \\ \dfrac{d^m}{dt^m} f(t) & ; \lambda = m \end{cases} \tag{9}
$$

Rieman-Liouville and Caputo definitions are very much similar; the only difference lies in dealing the initial conditions. In Rieman-Liouville definition, initial conditions are fractional order while for Caputo definition it is of integer order.

Lemma 2.1 If integral of fractional derivative ${}_aD^{\lambda}{}_t$ of a function $f(t)$ exits, then [30]

$$
{}_aD^{-\lambda}{}_t({}_aD^{\lambda}{}_t f(t)) = f(t) - \sum_{J=1}^{k} [{}_aD_t^{\lambda-J} f(t)]_{t=a} \frac{(t-a)^{\lambda-J}}{\Gamma(\lambda-J+1)} \tag{10}
$$

Here $k-1\le\lambda<k$

Lemma 2.2 The fractional integral operator ${}_aD_t^{-\lambda}$ with $\lambda > 0$ is bounded such that [30]

$$\| {}_aD^{-\lambda}{}_t(f) \|_p \leq K \| f \|_p \ ; \ 1 \leq p \leq \infty \ ; \ 1 \leq K \leq \infty \tag{11}$$

Stability of fractional order control theory is the emerging research area. Stability of fractional order systems has been discussed by several authors. In [37], Matignon states that

Theorem 1: The system of the form ${}_0D_t^{\lambda} x = Ax,\ x(0) = x_0$ is asymptotically stable if $\left|\arg(eig(A))\right| > \lambda\pi/2$ and each component of the states decays towards 0 like $t^{-\lambda}$. Also this system is stable if either it is asymptotically stable, or those critical eigenvalues that satisfy $\left|\arg(eig(A))\right| = \lambda\pi/2$ have geometric multiplicity one.

To approximate a continuous unknown function, fuzzy logic system is proposed. The output of SISO fuzzy logic system with centre average defuzzifier, product inference and singleton fuzzifier is given by following relation [11].

$$y_j = \frac{\sum_{l=1}^{M} u_{A_i}(x_i) y_j^{-l}}{\sum_{l=1}^{M} u_{A_i}(x_i)}, \ j = 1, 2.........m \tag{12}$$

Here x_i is the input parameter vector, y_j is the output parameter vector, M represents the total number of rules and $u_{A_i^l}(x_i)$ is the membership function vector. Equation (12) can be simplified as

$$y_j = \theta_j \xi(x), \ j = 1, 2.........m \tag{13}$$

Here θ_j is the parameter vector which is adaptive term, $\xi(x)$ is fuzzy basis function vector and y_j^{-l} is a free parameter.

Lemma 2.3 [17] *Let $f(x)$ be a continuous function defined on a compact set ∂ then for a any constant scalar $k > 0$, there exit a fuzzy logic system in form of (13) such that*

$$Sup_{x \in \partial} \mid f(x) - y_j(x) \mid \leq k$$

2.2 Approximation of Fractional Operator

In this work fractional operator is approximated using Oustaloup's recursive method as given in [34]. Let fractional operator is represented as;

$$W(s) = s^{\lambda} \quad ; \lambda \in R^{+} \; ; \; \lambda \in [-1 \; 1] \tag{14}$$

Let the function $W(s)$ is approximated using a rational function of the form;

$$\hat{W}(s) = C_0 \prod_{k=-N}^{N} \frac{s + w_k}{s + w'_k} \tag{15}$$

The above function is approximated for a frequency range of $[w_b \;\; w_h]$ using the following relations:

$$\begin{cases} w'_k = w_b \left[\dfrac{w_h}{w_b} \right]^{\frac{k+N+0.5(1-\lambda)}{2N+1}} \\ w_k = w_b \left[\dfrac{w_h}{w_b} \right]^{\frac{k+N+0.5(1+\lambda)}{2N+1}} \\ C_0 = \left[\dfrac{w_h}{w_b} \right]^{\frac{-\lambda}{2}} \displaystyle\prod_{k=-N}^{N} \frac{w_k}{w'_k} \end{cases} \tag{16}$$

Here $[w_b \;\; w_h]$ represents high and low frequencies.

2.3 Lugre Model Friction Compensation

In this work Lugre model is proposed for compensating friction torque. The compensation control is given [32]

$$\begin{cases} T_f = \sigma_0 \hat{z} + \sigma_1 \dot{\hat{z}} + \sigma_2 w_m \\ \dot{\hat{z}} = w_m - \dfrac{\sigma_0 \, | v |}{g(v)} \hat{z} \\ g(v) = f_c + (f_c - f_s) e^{[\frac{-w_m}{v_s}]^2} \end{cases} \tag{17}$$

Here $g(v)$ is the stribeck effect, v_s is the stribeck velocity, w_m represents angular velocity of loading motor, f_c is coulomb friction, f_s is static friction, $\hat{z}$ is the estimated average bristle defection, σ_0 is the stiffness of the bristles, σ_1 is the damping term and σ_2 is the viscous friction coefficient.

3 Fractional Order Adaptive Robust Torque Controller

Let a non singular fractional order sliding surface is defined as

$$s = D^{1-\lambda} z_1 + c_1 z_2^{\gamma} \tag{18}$$

Here $c_1 > 0$, λ is order of fractional operator and $\gamma = \frac{P}{q}$. A two step controller using backstepping sliding method is proposed.

Step1. Let fist virtual control t is defined as α_1. Differentiate Z_1 in Eq. (6)

$$\dot{Z}_1 = \dot{x}_1 - \dot{x}_r \tag{19}$$

To calculate virtual control α_1, the Lyapunov function is $V_1 = \frac{1}{2} z_1^{\,2}$. 1st derivative of V_1 yields

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1(\alpha_1 - \dot{x}_r)$$

$$\alpha_1 = -k_1 z_1 + \dot{x}_r \tag{20}$$

If $k_1 > 0$ then $\dot{V}_1 < 0$

Step2. Differentiating z_2 in Eq. (6), one obtains;

$$\dot{Z}_2 = \dot{x}_2 - \dot{\alpha}_1 = -\phi_1 x_2 + \phi_2 u - F - \dot{\alpha}_1 \tag{21}$$

In Eq. (21) parameters ϕ_1 and ϕ_2 are unknown so we cannot formulate control law directly. To estimate the unknown parameters Eq. (18) is modified as

$$\dot{Z}_2 = -\tilde{\phi}_1 x_2 + (\tilde{\phi}_1 - \phi_1) x_2 + \tilde{\phi}_2 u + (\phi_2 - \tilde{\phi}_2) u - F - \dot{\alpha}_1 \tag{22}$$

Differentiate Eq. (18)

$$\dot{s} = D^{1-\lambda} \dot{z}_1 + c_1 \gamma z_2^{\gamma-1} \dot{z}_2 \tag{23}$$

Combine Eqs. (22) and (23)

$$\dot{s} = D^{1-\lambda}\dot{z}_1 + c_1\gamma z_2^{\gamma-1}(-\tilde{\phi}_1 x_2 + (\tilde{\phi}_1 - \phi_1)x_2 + \tilde{\phi}_2 u + (\phi_2 - \tilde{\phi}_2)u - F - \dot{\alpha}_1) \tag{24}$$

The Lyapunov function V_2 is

$$V_2 = \frac{1}{2}(s^2 + \sum_{i=1}^{n}\eta_1\tilde{\theta}_i^2 + \eta_2(\tilde{\phi}_1 - \phi_1)^2 + \eta_3(\phi_2 - \tilde{\phi}_2)^2) \tag{25}$$

Here η_1, η_2 and η_3 represent learning rates of fuzzy system and parameters estimation algorithms. Differentiate Eq. (25)

$$\dot{V}_2 = s\dot{s} + \sum_{i=1}^{n}\eta_1\tilde{\theta}_i\dot{\tilde{\theta}}_i + \eta_2(\tilde{\phi}_1 - \phi_1)\dot{\tilde{\phi}}_1 + \eta_3(\phi_2 - \tilde{\phi}_2)(-\dot{\tilde{\phi}}_1)) \tag{26}$$

Combine Eq. (24) and (26)

$$\begin{aligned}\dot{V}_2 = s[D^{1-\lambda}\dot{z}_1 + c_1\gamma z_2^{\gamma-1}(-\tilde{\phi}_1 x_2 + (\tilde{\phi}_1 - \phi_1)x_2 + \tilde{\phi}_2 u + (\phi_2 - \tilde{\phi}_2)u - F - \dot{\alpha}_1)] + \\ \sum_{i=1}^{n}\eta_1\tilde{\theta}_i\dot{\tilde{\theta}}_i + \eta_2(\tilde{\phi}_1 - \phi_1)\dot{\tilde{\phi}}_1 + \eta_3(\phi_2 - \tilde{\phi}_2)(-\dot{\tilde{\phi}}_1))\end{aligned} \tag{27}$$

Using Eq. (27) a fractional order torque control law is given by

$$u = \frac{1}{\tilde{\phi}_2}[\tilde{\phi}_1 x_2 + \tilde{F} + \dot{\alpha}_1 - \frac{1}{c_1\gamma}z_2^{1-\gamma}D^{1-\lambda}\dot{z}_1 - Q_1\,\mathrm{sgn}(s)] \tag{28}$$

Here c_1, Q_1 and γ are positive constants greater than zero.

3.1 Stability Proof and Convergence Analysis

To prove stability of closed loop system, combine Eq. (25) and Eq. (24), one obtains

$$\begin{aligned}\dot{V}_2 = c_1\gamma s z_2^{\gamma-1}[(\tilde{\phi}_1 - \phi_1)x_2 + (\phi_2 - \tilde{\phi}_2)u - (F - \tilde{F})] + \sum_{i=1}^{n}\eta_1\tilde{\theta}_i\dot{\tilde{\theta}}_i + \eta_2(\tilde{\phi}_1 - \phi_1)\dot{\tilde{\phi}}_1 \\ +\eta_3(\phi_2 - \tilde{\phi}_2)(-\dot{\tilde{\phi}}_1)) - c_1\gamma s z_2^{\gamma-1}Q_1\,\mathrm{sgn}(s)\end{aligned} \tag{29}$$

Fuzzy error e_f is defined as [11]

$$\begin{aligned}e_f &= F - \hat{F} \\ \tilde{\theta}_i\xi_i(\dot{\theta}_i) &= \hat{F} - \tilde{F}\end{aligned} \tag{30}$$

Combining Eq. (29) and (30), the following fractional order adaptive laws are derived

$$\begin{aligned}\dot{\tilde{\phi}}_1 &= -\eta_2^{-1} c_1 \gamma s x_2 z_2^{\gamma-1} \\ \dot{\tilde{\phi}}_2 &= \eta_3^{-1} c_1 \gamma s u z_2^{\gamma-1} \\ \dot{\tilde{\theta}}_i &= -\eta_1^{-1} c_1 \gamma s \xi_i(\dot{\theta}_i) z_2^{\gamma-1}\end{aligned} \tag{31}$$

Combine Eqs. (29) & (31) and simplify

$$\dot{V}_2 = s[-c_1 \gamma Q_1 z_2^{\gamma-1}\,\mathrm{sgn}(s)] \tag{32}$$

Eq. (32) is always negative, if $Q_1 > 0 : Q_1 > \varsigma$, γ : $1 < \gamma < 2$ and $c_1 > 0$. Here ς is the system uncertainty. So

$$\dot{V}_2 = -c_1 \gamma Q_1 \,|\, z_2 \,|^{\gamma-1} |\, s \,| \le 0 \tag{33}$$

If $\dot{V}_2 < 0$ exits then reaching condition of sliding surface is satisfied and $s = 0$.

So Eq. (18) is written as $D^{\lambda} y = -c_1 y^{\frac{1}{\gamma}}$. Here $y = z^{\gamma}$ and $y^{\frac{1}{\gamma}} = z$ According to **Theorem 1** $A = -c_1$, and $\left|\arg(eig(A))\right| = \pi$. Now $\left|\arg(eig(A))\right| > \lambda\pi/2$ is constantly established. State errors of above modified Eq. (18) converges towards 0 like $t^{-\lambda}$ if the following conditions hold, i.e. $c_1 > 0$ and $0 < \lambda < 1$.

To prove error convergence property, it is necessary to prove $t_r \le t_s < \infty$ [30]. Here t_r represents reaching time. At $t = t_r$, $s = 0$, so Eq. (18) can be written as

$$\begin{cases} D^{-\lambda} D^1 z_1 = -c_1 z_2^{\gamma} \\ D^{-\lambda} z_2 = -c_1 z_2^{\gamma} \end{cases} \tag{34}$$

Eq. (34) can be written as

$$D^{-\lambda}[D^{\lambda} z_2] = -c_1 [D^{\lambda} z_2^{\gamma}] \tag{35}$$

Using Lemma 2.1, Eq. (35) is written as

$$z_2 - [{}_{t_r}D_t^{\lambda-1}(z_2]_{t=t_r} \frac{(t-t_r)^{\lambda-1}}{\Gamma(\lambda)} = -c_1 D^{\lambda} z_2^{\gamma} \tag{36}$$

At $t = t_r$ left hand side of Eq. (36) under fractional integration is equal to zero. i.e.

$$[{}_{t_r}D_t^{\lambda-1}(z_2]_{t=t_r} \frac{(t-t_r)^{\lambda-1}}{\Gamma(\lambda)} = 0 \tag{37}$$

Using Eq. (36) and (37), one obtains

$$z_2 = -c_1 D^{\lambda} z_2^{\gamma} \tag{38}$$

Multiply Eq. (38) by D^1; the resultant equation is given as

$$D^2 z_1 = -c_1 D^{1+\lambda} z_2^{\gamma} \tag{39}$$

Now multiply Eq. (39) by D^{-2} and apply Lemma 2.1, one obtains;

$$z_1(t) - [{}_{t_r} D_t^{2-1} z_1]_{t=t_r} \frac{(t-t_r)^{2-1}}{2} - z_1(t_r) = -c_1 D^{\lambda-1} z_2^{\gamma} \tag{40}$$

If $0 \le \lambda < 1$, then $D^{\lambda-1}$ represents fractional integrator hence Lemma 2.2 can be applied to right hand side of Eq .(40) as

$$-c_1 D^{\lambda-1}(z_2^{\gamma}) \le -c_1 K \| z_2^{\gamma} \| \tag{41}$$

Combine Eq. (40) and (41);

$$\| z_1(t) - [{}_{t_r} D_t^1 z_1]_{t=t_r} \frac{(t-t_r)}{2} \| - \| z_1(t_r) \| \le -c_1 K \| z_2^{\gamma} \| \tag{42}$$

If $z_1(t = t_s) = 0$ and $z_2(t = t_s) = 0$ then it is necessary to prove $t_r \le t_s < \infty$. Eq. 42 is written as

$$\| [{}_{t_r} D_t^1 z_1]_{t=t_r} (t_s - t_r) \| \le 2 \| z_1(t_r) \| \tag{43}$$

Simplifying Eq. (43) as

$$t_r \le -\frac{2 \| z_1(t_r) \|}{\| z_2 \|_{t=t_r}} + t_s \tag{44}$$

From Eq. (44) it is concluded that tracking errors convergence occurs in finite time.

Remark 3.1. Discontinuous projection operator is used to simulate the adaptive laws proposed in Eq. (31). The projection operator is defined as

$$\dot{\tilde{p}} = proj_{\tilde{p}_i}(\tau \upsilon_i) \tag{45}$$

Here $\tau > 0$ is the adaptation gain matrix and υ_i represents the adaptive algorithm as derived in Eq. (31). The projection operator is defined as [16]

$$proj_{\tilde{p}_i}(\bullet) = \begin{cases} 0 & If \quad \tilde{p}_i = p_{\max} \ and \ \bullet > 0 \\ 0 & If \quad \tilde{p}_i = p_{\min} \ and \ \bullet < 0 \\ \bullet & Otherwise \end{cases} \tag{46}$$

In Eq. (46), $\tilde{P}_i$ is the estimated parameters vector, $p_{\min}$ is the lower limit of uncertain parameters and $p_{\max}$ represents the upper bound of parameters.

Block diagram of proposed control scheme is shown in Figure 1

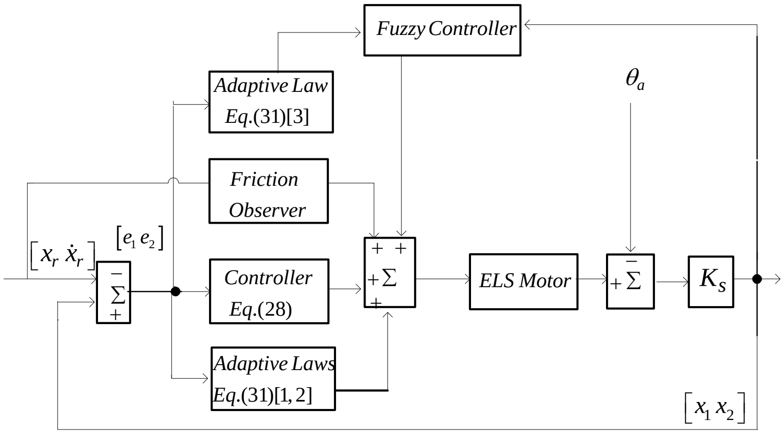

Figure 1
Fractional order controller for ELS system

4 Results and Discussions

To verify performance of proposed controller, parameters of ELS system and controller are tabulated in Table 1. There are many tests which can be done to qualify the performance of load simulator namely static loading, frequency test, gradient loading and extra torque elimination. Main focus of this article is to verify performance of load simulator for frequency testing and extra torque elimination.

Reference command of ELS torque motor is $T_r = 10 * Sin(2\pi * 10 * t)$ with frequency 10 Hz. Torque tracking performance is compared in Fig. 2. From results obtained it is clear that transient error introduced as a result of nonzero initial conditions is effectively compensated in case of fractional order controller at $\lambda = 0.4$. The maximum transient error with integer order control is $15Nm$ for time interval $0 \le t \le 0.03$ sec. Using proposed fractional order control the same error is reduced to $10Nm$ at fractional power $\lambda = 0.2$ and $5Nm$ at fractional power $\lambda = 0.4$. Tracking error comparison is given in Fig. 3. As from previous analysis of Fig. 2, best transient performance are achieved at $\lambda = 0.4$ which is shown as region A in Fig. 3. At the same time region B of Fig. 3 shows steady state performance. At $\lambda = 0.4$ a negligible steady state error is introduced, however this error is not very big and acceptable. At $\lambda = 0.2$ and $\lambda = 0.3$ the proposed control offer better transient performance and their respective steady state errors are also comparable to integer order control. Similar results are obtained and presented in Fig. 4. Control signal simulations are shown in Fig. 5. Practically computer output is restricted to $\pm 10 Volts$. As shown in the simulations fractional order method generates high values of control signal in transient time but at the same time practically it is restricted to $\pm 10 Volts$. Moreover using proposed method chattering phenomena is minimized in steady state. Inspite of high control action in transient time , the calculated rms fractional order control at $\lambda = 0.4$ is almost equal to integer order control. For integer order, rms control effort is 6.95 volts and for fractional order it is 7 volts. So fractional order control effort is almost comparable to its integer counterpart, while it offers the advantage of chattering minimization. A detailed discussion about chattering reduction using fractional order control is discussed in [33]. Using proposed method, control signal is saturated for $t \le 0.01$ sec. To apply actual control effort, saturation compensation control is proposed as presented in [38]. For $t \le 0.01$ saturation compensation control is effective as shown in Fig. 6. The estimated control effort due to saturation phenomena is prominent for $t \le 0.01$ sec and after $t > 0.01$ sec the estimated value is not very big. The reason is very obvious because the control effort without saturation compensation saturates for $t \le 0.01$ sec and after $t > 0.01$ sec, it is within the maximum limits.

Simulation results of fuzzy estimated lumped disturbance $\tilde{F}$ is shown in Fig. 7a. Friction compensation control is shown Fig. 7b. The estimated state parameters are shown in Fig. 8a & b. From simulations it is clear that the estimated parameters converge to their true values without overshoots and oscillations. Adaptive laws derived in Eq. 31 are used for online parameters estimation. Eq. 31 contains sliding surface s which is fractional order. As fractional operator is adjustable so the proposed parameters estimation laws give more degree of freedom to adjust convergence speed and overshoots as compared to its integer counterparts.

Finally performance of proposed control is compared with its integer version and feed forward PID when ELSS is subjected to nonzero initial conditions. Fig. 9 compares transient tracking response using feed forward PID control, Integer order TSMC control and proposed control method. The reference command of ELS torque motor is $T_r = 10 * Sin(2\pi * 10 * t)$ with frequency 10 Hz. The initial conditions of state vector are $\begin{bmatrix} x_{10} & x_{20} \end{bmatrix}^T = \begin{bmatrix} 3.3 & 55 \end{bmatrix}^T$. Parameters of PID control are $K_P = 20.2$, $K_I = 8.5$ and $K_D = 0.01$ taken from [2]. From Figure 9 it is concluded that using proposed method transient tacking error due to none zero initial conditions is effectively compensated at fractional power 0.5. Using feed forward PID control and integer order TSMC control transient tracking error is approximately $3.5 Nm$. Steady state response is shown in Figure 10. From simulation results is concluded that as compared to Feed forward PID, integer order TSMC the proposed method compensate steady state error effectively. Although steady state performances of all three controllers are almost comparable, the proposed controller performs better to suppress transient errors. Since ELSS is used to qualify a crucial part of flight control system, so both transient and steady state control performance of ELSS system should be guaranteed. Fractional order control requires more computational burden but with advent of modern DSP processors and FPGA, s it is easy to implement algorithms with high processing requirements. This will increase overall cost of the implementations but in case of aerodynamics load simulators, good control performance is vital and cannot be compromised.

Table 1
Controller and ELS system Parameters

ELSS Parameters		Controller Parameters	
J	$0.04\ Kg/m^2$	γ	1.2
R_s	7.5Ω	P	1.5
k_t	$5.732\frac{N.m}{A}$	η_1,η_2,η_3	0.0001, 0.25, 0.0125
β	$0.244\frac{N.m-s}{rad}$	λ	1−0.6
K_s	$950\frac{N.m}{rad}$	Q_1	2.5
k_b	$5.732\frac{N.m}{V}$	k_1	10
T_s, T_c	$3N.m, 2.7N.m$	c_1	80
$\sigma_0,\sigma_1,\sigma_2$	$200\frac{N.m}{rad}, 2.5, 0.02$	x_{10} x_{20}	15, 400

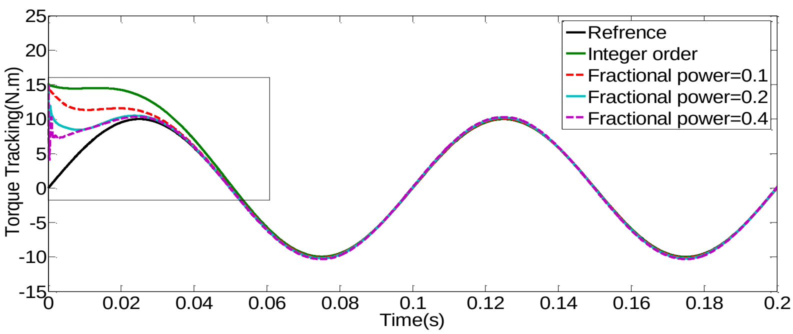

Figure 2
Tracking performance x_1

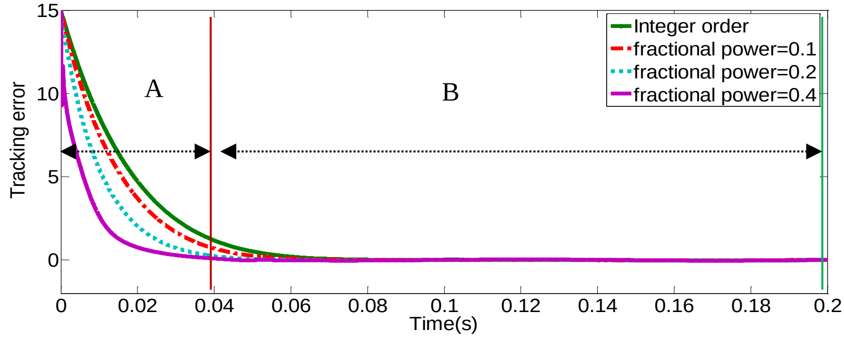

Figure 3

Tracking error comparison x_1

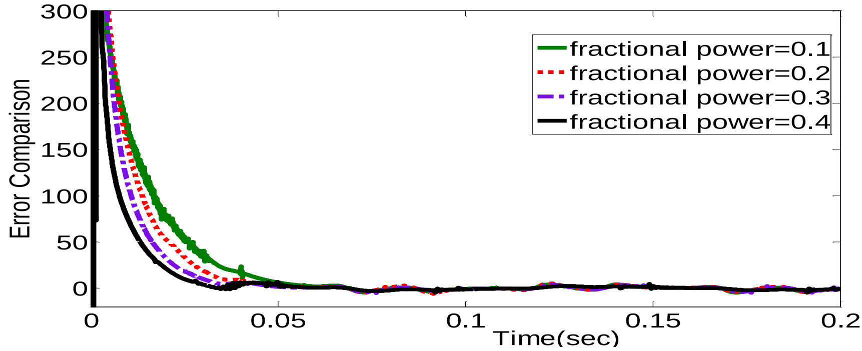

Figure 4

Tracking error comparison x_2

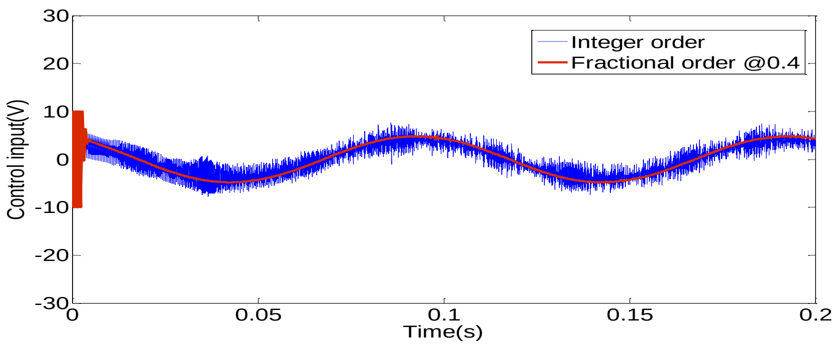

Figure 5

Control input comparison without saturation compensation

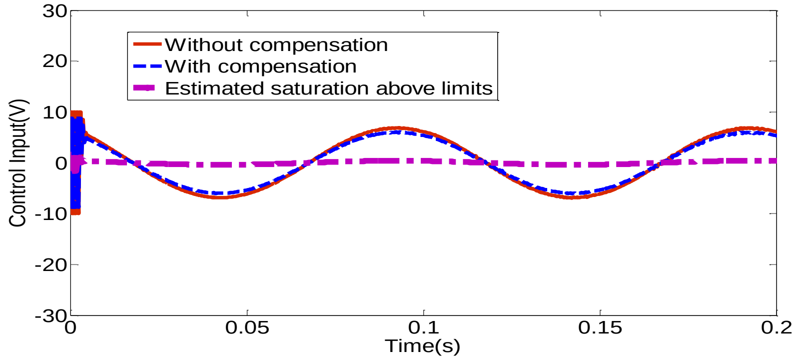

Figure 6

Control input comparison with saturation compensation

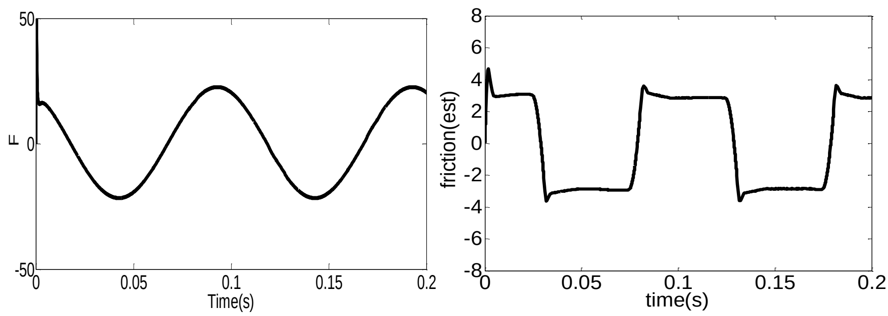

Figure 7

(a) Estimated F (b) Friction estimation

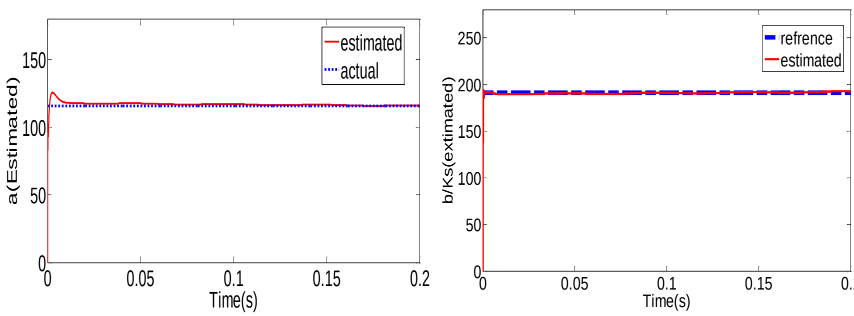

Figure 8

(a) Estimated $'a'$ (b) Estimated $'b'$

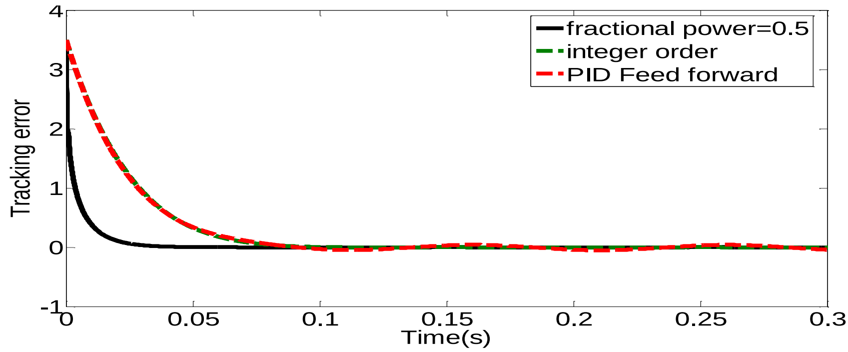

Figure 9

Error comparison under different control schemes

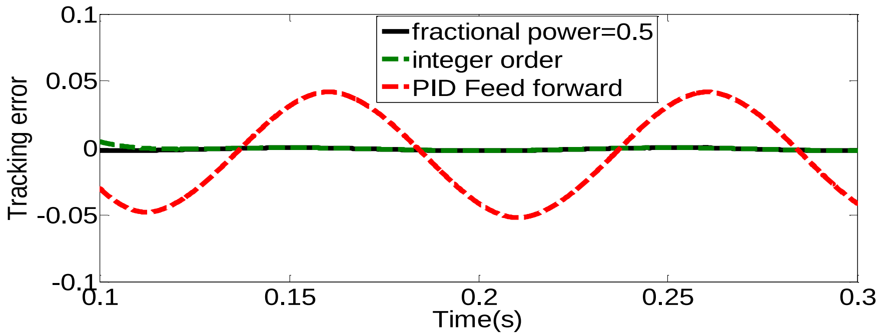

Figure 10

Enlarged view and steady state error comparison

Conclusions

A high performance adaptive robust controller is proposed for ELSS torque tracking problem using fractional calculus. The proposed control offers advantages including chattering minimization and robustness for tracking errors due to nonzero initial condition. Moreover the uncertain parameters estimation laws offer more degree of freedom to adjust the convergence speed of estimated parameters. Numerical simulations are performed to compare the proposed controller with integer order TSMC and feed forward PID control. From simulation results it is concluded that proposed fractional order controller is superior to integer order TSMC and feed forward PID control.

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