

# Limiting Static and Dynamic Characteristics of an Induction Motor under Frequency Vector Control

Istvan Vajda<sup>1</sup>, Yury N. Dementyev<sup>2</sup>, Kirill N. Negodin<sup>2</sup>, Nikolay V. Kojain<sup>2</sup>, Leonid S. Udut<sup>2</sup>, Irina. A. Chesnokova<sup>2</sup>

<sup>1</sup>Óbuda University, Kandó Kálmán Polytechnic, Bécsi út 96/b, 1034 Budapest, Hungary, e-mail vajda@uni-obuda.hu

<sup>2</sup>Tomsk Polytechnic University, Institute of Power Engineering, av. Lenina 30, 634050 Tomsk, Russian Federation (e-mail: dementev@tpu.ru, knn1@tpu.ru, kojain@tpu.ru, udut@tpu.ru, taksimo@tpu.ru)

---

*Abstract: Static and dynamic characteristics of an induction motor (IM) under frequency vector control are reviewed. Limiting static characteristics enabling to determine the limits of an automatic electric drive, as well as regions of short-term and admissible continuous performance of an induction motor under frequency vector control are presented. Recommendations on the choice of maximum phase voltage of an inverter, DC link voltage of a frequency converter and supply voltage of an induction motor as well as possible ways to reach maximum angular velocity of an induction motor under frequency vector control are suggested.*

*Keywords: induction motor; frequency control; vector control; three-phase inverter; limiting characteristics*

---

## 1 Introduction

A squirrel cage AC induction motor drive is widely applied in adjustable electric drive systems, which are currently used in industry and operate mainly in continuous static modes with a constant or slowly varying load moment. This electric drive consumes more than half of all power generated [1].

The widespread use of a squirrel cage induction motor in the systems of an adjustable electric drive, which are in high demand in industries, can be attributed to its high reliability due to the absence of a brush-collector unit and permanent magnets, a simple design, a small size and a rotor inertia moment, absence of switching constraints on speed and current, etc. [2-4]. The most common law for developing automatic control systems (ACS) of a frequency-controlled induction

motor drive, which implement the assigned static values, was, at an early stage, a simple proportional law of voltage amplitude control of an induction motor stator in its frequency function. However, in some works [3] it is proved that application of this control law makes it impossible to achieve both acceptable mechanical and energy characteristics of an electric drive under a wide range of rotation changes per minute and load changes due to the influence of active resistance and leakage inductance of the stator of an induction motor.

In that regard, a more promising principle of frequency-vector control of an induction motor drive [3-5] was developed. It enables to consider an induction motor as a two-channel object (an analogue of a separately excited DC motor) oriented along the vector of the rotor flux linkage. A vector-frequency control of a squirrel-cage induction motor allows for providing an independent control of the rotor flux linkage vector and electromagnetic moment. Due to that a two-region rotations per minute can be controlled in the vector control system similar to a dc drive [6].

Currently, of particular interest for the research are limiting static characteristics of an induction motor. Corresponding either to the rated motor voltage or maximum output voltage of an inverter of a frequency converter under the assigned voltage of the supply network for various control systems of a three-phase inverter [7]. Thus, enabling to estimate feasibility of reaching a desired speed depending on the load moment in a vector VFD.

The purpose of the article is to analyse the limiting static design characteristics of the motor  $\omega(T_{EM})$  and  $\omega(T_{1ph})$  in the “frequency converter - induction motor” system open at  $q$ -axis coordinate system at the assigned value of the rotor flux linkage and in the closed system of the induction motor drive under frequency-vector control with controlled flux.

## 2 Vector Method of Frequency Control of an Induction Motor

The vector systems of induction motors frequency control are based on a structural scheme of a two-phase motor in rotating coordinates  $d, q$  [8-12]. In the closed loop vector control system, the voltage component  $U_{1d}$  sustains the rotor flux linkage  $\Psi_{2d}=\text{const}$  constant and the voltage component  $U_{1q}$  ensures equality of the motor electromagnetic moment to the static moment on the shaft  $T_{EM}=T_{load}+\Delta T_{Imotor}$  in the steady-state operation mode.

The automatic control system of an induction motor drive with a frequency-vector control is made of two independent but related control systems: a maintenance system of the assigned value of the rotor flux linkage with current  $I_d$  and a maintenance system of the assigned speed with the motor moment (current  $I_q$ ).

The control system of the motor flux linkage is auxiliary and ensures the operation of an induction motor drive control system. The speed control system is the main control system of an induction motor drive and ensures compliance of its characteristics with the requirements. Obviously, static modes of an induction motor drive, both in open and closed systems, can be studied only under the following assumption: a flux control system ensures constancy of the assigned value of the rotor flux linkage [13-16].

If to assume that at a constant voltage supply of an induction motor  $U_{1ph} = \text{const}$  the control system maintains constancy of the flux linkage  $\Psi_{2d}(I_{1d}) = \text{const}$  along the  $d$ -axis, fulfilment of the condition  $T_{EM}(I_{1q}) = T_{EMref}$  will depend not only on the voltage component value  $U_{1q} = \sqrt{(\sqrt{2} \cdot U_{1ph})^2 - U_{1d}^2}$ , but also, primarily, on the angular velocity of the induction motor rotation.

Therefore, if at a constant voltage of an induction motor is to take a flux linkage equal to  $\Psi_{2d} = \Psi_{2dset}$ , then the motor static characteristics  $\omega(T_{EM})$  and  $\omega(I_{1ph})$  in the “frequency converter-induction motor” system open along the  $q$ -axis can then be calculated at the assigned value of the rotor flux linkage.

A block diagram of an induction motor in two-phase rotating coordinates  $d, q$  for the static operation mode of an induction motor drive in “frequency converter-induction motor” system at frequency-vector control is shown in Fig. 1.

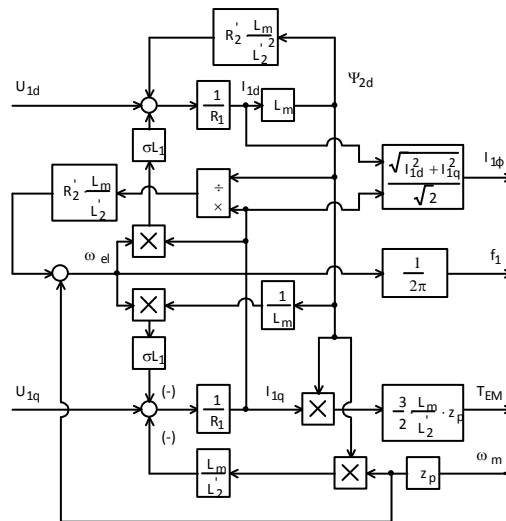


Figure 1

A block diagram of an induction motor in two-phase rotating coordinates  $d, q$  for the static operation mode under vector control

The system of equations describing the block diagram in fig. 1 can be presented as follows:

$$\begin{aligned} T_{EM} &= \frac{3}{2} \cdot \frac{L_m}{L_2'} \cdot z_p \cdot \Psi_{2d} \cdot I_{1q} ; & \Psi_{2d} &= I_{1d} \cdot L_m ; \\ I_{1q} &= \left( U_{1q} - \frac{L_m}{L_2'} \cdot z_p \cdot \Psi_{2d} \cdot \omega_{motor} - \frac{\sigma \cdot L_1}{L_m} \cdot \Psi_{2d} \cdot \omega_{electric1} \right) \cdot \frac{1}{R_1} ; \\ I_{1d} &= \left( U_{1d} + R_2' \cdot \frac{L_m}{L_2'^2} \cdot \Psi_{2d} + \sigma \cdot L_1 \cdot I_{1q} \cdot \omega_{electric1} \right) \cdot \frac{1}{R_1} ; \\ \omega_{electric1} &= z_p \cdot \omega_{motor} + R_2' \cdot \frac{L_m}{L_2'} \cdot \frac{I_{1q}}{\Psi_{2d}} . \end{aligned}$$

It is known that under vector control, both a module and a spatial position of the stator current vector change [10-13, 16, 17]. The current vector changes so that the projection of the stator current vector  $\vec{I}_1$  of the induction motor on  $d$ -axis, oriented along the vector of the rotor flux linkage  $\vec{\Psi}_2$  remains unchanged and it can be determined for the first control area of the induction motor speed ( $f_1 \leq f_{1n}$ ) under the flux linkage  $\Psi_{2d} = \Psi_{2n} = \text{const}$  in the following way

$$I_{1d} = \frac{\Psi_{2n}}{L_m} = \text{const} ; \quad (1)$$

Component  $I_{1q}$  of the stator current vector  $\vec{I}_1$ , the value of which determines the motor moment, can be calculated, in the steady state mode, with the following equation:

$$I_{1q} = \frac{T_{EM}}{\frac{3}{2} \cdot \frac{L_m}{L_2'} \cdot z_p \cdot \Psi_{2n}} . \quad (2)$$

To meet the conditions (1) and (2) voltage values  $U_{1d}$  and  $U_{1q}$  must be maintained in accordance with the next equations:

$$\begin{aligned} U_{1d} &= R_1 \cdot I_{1d} - R_2' \cdot \frac{L_m}{L_2'^2} \cdot \Psi_{2n} - \\ &\quad - \frac{\sigma \cdot L_1 \cdot R_2' \cdot L_m}{L_2' \cdot \Psi_{2n}} \cdot I_{1q}^2 - \sigma \cdot L_1 \cdot z_p \cdot I_{1q} \cdot \omega_{motor} \end{aligned} ; \quad (3)$$

$$U_{1q} = \left( R_1 + \frac{\sigma \cdot L_1}{L_2'} \cdot R_2' \right) \cdot I_{1q} + \left( \frac{L_m}{L_2'} + \frac{\sigma \cdot L_1}{L_m} \right) \cdot z_p \cdot \Psi_{2n} \cdot \omega_{motor} . \quad (4)$$

The active value of the motor phase voltage  $U_{1ph}$  and voltage vector components  $\bar{U}_{dq}$  in a two-phase rotating coordinate system  $d, q$  are connected by the following relation:

$$\left(\sqrt{2} \cdot U_{1ph}\right)^2 = U_{1d}^2 + U_{1q}^2. \quad (5)$$

The given equations (1) - (5) allow calculating static mechanical  $\omega_{motor}(T_{EM})$  and electromechanical  $\omega_{motor}(I_{1ph})$  characteristics, as well as dependence of an angular velocity on frequency  $\omega_{motor}(f_1)$  for the induction motor in the “frequency converter-induction motor” system open along the  $q$ -axis at a constant voltage supply of the motor  $U_{1ph} = \text{const}$ .

Besides, equations (1) – (5) enable to determine the required maximum voltage  $U_{1ph,n}$ , which ensures the motor operation at the assigned values of the maximum speed of the electrical drive and the maximum moment of the static load.

Of practical importance is calculation of limiting characteristics of the motor, corresponding either to the maximum allowed value of the motor voltage  $U_{1ph,n}$  or the maximum output voltage of the converter  $U_{i,ph,m}$  at the assigned value of the supply voltage. In the first case, it is assumed that the supply voltage can be selected in accordance with the allowed value  $U_{1ph,m}$ . In the second case, the supply voltage is assigned and determines the maximum output voltage of the converter  $U_{i,ph,m}$ .

### 3 Control Systems of Three-Phase Frequency Converter Inverters

Currently, control systems of three-phase inverters of frequency converters are implemented with a simple sinusoidal PWM, a sine PWM and a third harmonic in control signals and with a vector PWM [7, 11]. Systems with a vector PWM are controlled by sinusoidal signals and have characteristics similar to the sinusoidal PWM with a superposition of a third harmonic [7].

The control system of three-phase inverter with sinusoidal PWM has common for all three phases of inverter reference sawtooth configuration signal with singular amplitude and  $f_{PWM}$  frequency. Three sinusoidal control signals with  $u_{1m} \leq 1$  amplitude are buckled to input PWM block.

$$u_{1a} = u_{1m} \cdot \cos(2\pi \cdot f_1 \cdot t);$$

$$u_{1b} = u_{1m} \cdot \cos\left(2\pi \cdot f_1 \cdot t - \frac{2\pi}{3}\right);$$

$$u_{1c} = u_{1m} \cdot \cos\left(2\pi \cdot f_1 \cdot t - \frac{4\pi}{3}\right),$$

Common for all controls third harmonic signal is buckled to control signals in systems with sinusoidal PWM and third harmonic:

$$u_{3f} = \frac{1}{6} \cdot u_{1m} \cdot \cos(2\pi \cdot 3 \cdot f_1 \cdot t).$$

Realization principle of control system of three-phase inverter with sinusoidal PWM and putting in three-phase system of control influences of third harmonic signal is shown on Fig. 2.

The third harmonic putting in system of control signals of inverter leads to shape change and amplitude of resulting control on input PWM block reducing to  $\sqrt{3}/2 = 0.866$  times. It allows to improve amplitude of resulting control influences  $u_{1a}^*(t)$ ,  $u_{1b}^*(t)$ ,  $u_{1c}^*(t)$  to  $k = 2/\sqrt{3} = 1.1547$  times to amplitude of sawtooth reference voltage and to increase amplitude of the first harmonic of output inverter voltage to the same times.

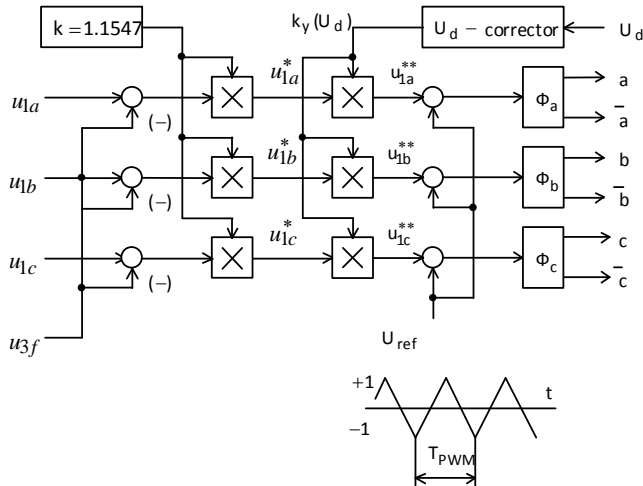


Figure 2

Realization principle of sinusoidal PWM with third harmonic putting and inverter control correction

The comparative evaluation of a simple sinusoidal PWM system and a system with an additional third-harmonic signal and an amplification gain of the modulated signal  $k=1.1547$  are given in [7].

The automatic control system of an induction motor drive with frequency-vector control, primarily, creates and maintains the assigned (nominal, in the first region)

induction motor flux, and then creates the desired moment [1, 3, 5]. When an inverter is in an under-voltage status, the desired flux values of an induction motor and, mainly, the moment values will be achieved by changing rotational emf of an induction motor, i.e. by reducing the angular velocity of an induction motor. It can be achieved in the “frequency converter - induction motor” system by decreasing frequency of the inverter output voltage:

$$f_1 = \frac{\omega_{\text{electric1}}}{2 \cdot \pi} = \frac{1}{2 \cdot \pi} \cdot \left( z_p \cdot \omega_{\text{motor}} + R'_2 \cdot \frac{L_m}{L'_2} \cdot \frac{I_{1q}}{\Psi_{2d}} \right)$$

Thus, under vector control the inverter output voltage is the main factor when creating flux and the desired moment of an induction motor. Its value can be determined by its regulating characteristic (fig. 3) which is limited at the level  $U_{\text{iph}} = U_{\text{i.ph.m}}$ .

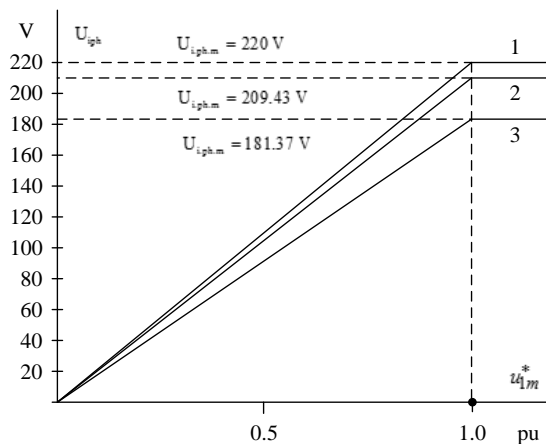


Figure 3

Regulating characteristics of a voltage inverter: 1 and 2 - sinusoidal PWM with a third harmonic at  $U_{\text{epn}} = 400\text{V}$  and  $U_{\text{epn}} = 380\text{V}$ ; 3 - a simple sinusoidal PWM and  $U_{\text{epn}} = 380\text{V}$

As can be seen from Fig. 2, the inverter voltage limitation affects only in the upper part of the speed control range and almost does not affect the control system operation of the frequency converter in its lower part. Induction motor AB250S6 natural mechanical characteristic 1 ( $f_1 = 50\text{ Hz}$  and  $U_1 = U_{\text{i.ph.n}} = 220\text{ V}$ ) and limiting mechanical characteristics 2, 3 and 4 of the open “frequency converter-induction motor” system are shown in Fig. 3 for the next implementations of a three-phase inverter control system, respectively:

-  $U_{\text{epn}} = 400\text{ V}$ , a sinusoidal PWM with a third harmonic superposition and  $k=1.1547$  ( $U_{\text{i.ph.m}} = 209.43\text{ V}$ );

-  $U_{epn} = 380 \text{ B}$ , a sinusoidal PWM with a third harmonic superposition and  $k=1.1547$  ( $U_{i,ph,m} = 209.43 \text{ V}$ );

-  $U_{epn} = 380 \text{ B}$ , a simple sinusoidal PWM ( $U_{i,ph,m} = 181.37 \text{ V}$ ).

Characteristic 5 is a static mechanical characteristic of the motor calculated at  $U_{1ph} = 181.37 \text{ V}$  and  $f_1 = 50 \cdot \frac{181.37}{220} = 41.22 \text{ Hz}$ . It must be noted that a voltage

drop in the inverter circuit is neglected and is taken to be equal to  $U_{1ph,m} = U_{i,ph,m}$  in the given calculations of the inverter voltage.

The analysis of characteristics given in Fig. 4 shows that when a frequency converter is powered from the mains with the rated voltage, limitation of the inverter output voltage causes a substantial reduction of the induction motor speed control range in the upper part of the control region at a rated load moment up to the speed value:

$$\omega_{ED,max} < \omega_{motor,n} \cdot \frac{U_{i,ph,m}}{U_{1ph,n}}, \text{ Rad / s,}$$

and reduction of drive overload at high speeds.

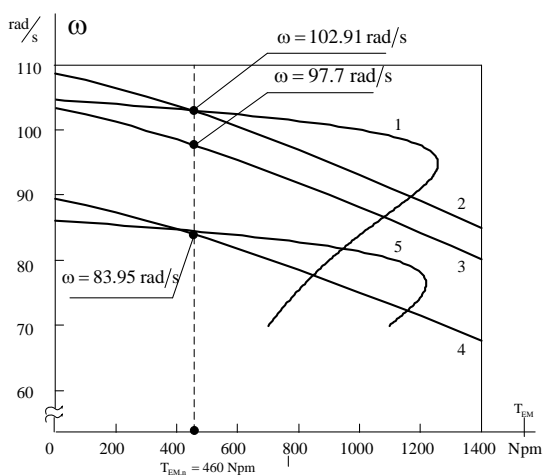


Figure 4

Static mechanical characteristics of induction motor AB250S6

Characteristics explanation of Fig. 4 is follow: 1 – natural characteristic  $f_{1n} = 50 \text{ Hz}$  and  $U_{1ph,n} = 220 \text{ V}$ ; 2, 3 and 4 limiting characteristics of the open-loop “frequency converter-induction motor” system under vector control  $U_{1ph,m} = 220 \text{ V}$ ,  $U_{1ph,max} = 209.43 \text{ V}$  and  $U_{1ph,max} = 181.37 \text{ V}$  respectively; 5 –



forced characteristic at  $U_{1ph} = 181.37 \text{ V}$  and  $f_1 = 41.22 \text{ Hz}$ .

Fig. 5 shows static mechanical characteristics of the closed-loop system of an electric motor drive with frequency-vector control of AB250S6 induction motor obtained under the following conditions:  $U_{1ph,max} = 209.43 \text{ V}$  current carrying rating  $I_{ED,max} = 170 \text{ A}$  corresponding to the maximum electromagnetic moment of the motor  $T_{ED,max} = 945 \text{ Npm}$ .

As can be seen from Fig. 5, the peak characteristic 2 of the open-loop system limits the maximum speed of an induction motor drive in the first control region depending on the load moment. For example, the angular velocity at point 2 is limited by the value  $\omega_2 = 97.7 \text{ rad/s}$ , and at the maximum motor moment  $T_{EM,n} = 460 \text{ Npm}$  it is limited by the speed value  $\omega_2 = 89 \text{ rad/s}$  at point 1.

Characteristics of transient processes and a dynamic characteristic of an induction motor drive with closed-loop vector control at a constant nominal rotor flux linkage and an idling torque  $T_{EM} = 92 \text{ Npm}$ , when performing the speed assignment, corresponding to fig. 5  $\omega_{set} = 97.7 \text{ rad/s}$ , are shown in Figs. 6, 7, respectively. The transient curves of a frequency-controlled induction motor drive, shown in Figs. 6, 7 were obtained at a constant value of the rotor flux linkage  $\Psi_{2dset} = \Psi_{2n}$  and next parameters of the control system of an induction motor drive:

- supply voltage  $U_{epn} = 400 \text{ V}$ ;
- PWM inverter frequency  $f_{PWM} = 5 \text{ kHz}$ ;
- AD capacity of a current transducer  $n_{ADCcs} = 10$ ;
- interval for calculating in the loop current is  $0.0002 \text{ sec.}$ ;
- number of speed sensor pulses (quadrupled) per shaft speed is 4000;
- interval for calculating in the speed loop is  $0.002 \text{ sec.}$

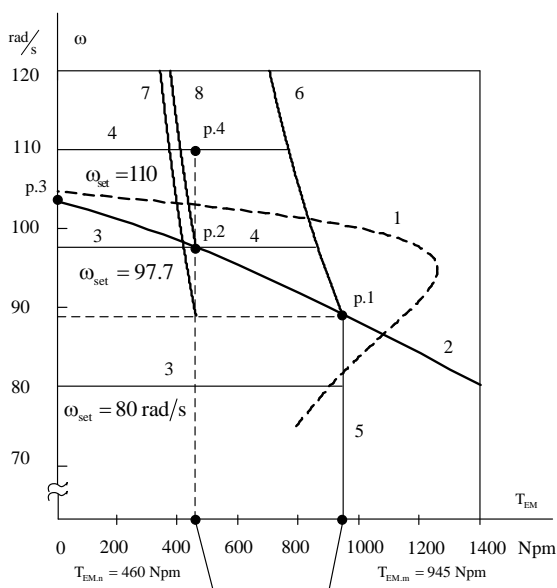


Figure 5

Static mechanical characteristics of an induction motor drive under frequency-vector control

Characteristics explanation of Fig. 5 are as follows: 1 - natural characteristic of AB250S6 induction motor; 2 - limiting characteristic of the open loop system at  $U_{i,ph,m} = 209.43 \text{ V}$ ; 3, 4 - limiting characteristics of the closed-loop system in the first and second region; 5, 6 - characteristics of admissible short-term duty; 7, 8 - characteristics of admissible long-term duty.

If we know parameters of an induction motor and assign a load moment and an angular motor speed, e.g.  $T = T_{EM,n}$  and  $\omega = \omega_{motor,n}$ , the desired voltage values  $U_{id}$  and  $U_{iq}$  can be calculated with equations (1) - (5) for an induction motor at a given point. Then, inverter voltage, DC link voltage and mains supply can be calculated with the following equations:

$$U_{i,ph,m} = 1.05 \cdot \frac{\sqrt{U_{id}^2 + U_{iq}^2}}{\sqrt{2}}; U_d = \sqrt{3} \cdot \sqrt{2} \cdot U_{i,ph,m}; U_{epn} = \frac{U_d}{1.35},$$

where, the coefficient 1.05 takes into account a voltage drop in the inverter circuit.

For AB250S6 induction motor the above given load and speed values can be achieved with a sinusoidal PWM inverter, a superposition of a third harmonic and  $k=1.1547$  only when the inverter voltage is  $U_{i,ph,m} > U_{i,ph,n} = 220 \text{ V}$ . It is possible when a frequency converter is supplied from the mains with  $U_{epn} > 420 \text{ V}$ .

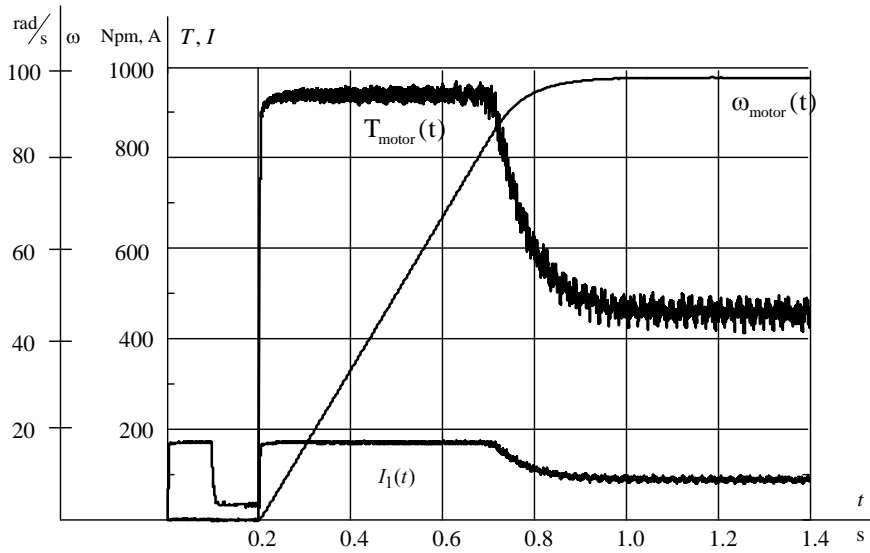


Figure 6

Transient processes  $I_{1ph}(t)$ ,  $T_{EM}(t)$  and  $\omega_{motor}(t)$  when an induction motor drive performs the assignment  $\omega_{set} = 97,7 \text{ rad/s}$

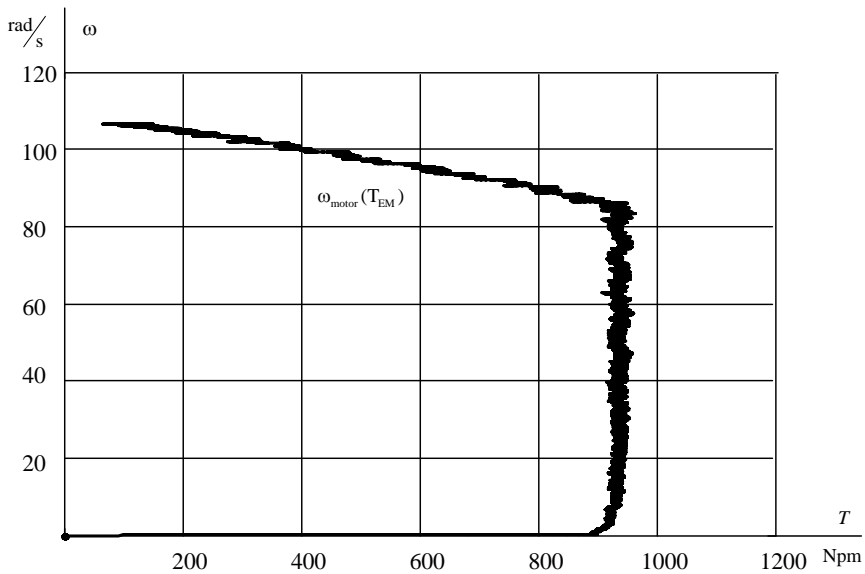


Figure 7

Dynamic characteristic  $\omega_{motor}(T_{EM})$  when an electric drive performs  $\omega_{set} = 97,7 \text{ rad/s}$

The desired maximum angular velocity of an induction motor under frequency-vector control in an induction motor drive can be achieved if:

- frequency converter is supplied from  $U_{epn} \geq 420$  V network, the maximum inverter voltage is limited at  $U_{i,ph,m} = 1.05 \cdot U_{i,ph,n}$ , maintaining the coefficient  $k$  in accordance with the expression:

$$k = \frac{\sqrt{3} \cdot \sqrt{2} \cdot 1.05 \cdot U_{i,ph,n}}{U_d} \cdot 1.1547 \leq 1.1547 \quad (6)$$

- to increase the amplitude of control signals  $u_{im}^* > 1$  at  $U_{epn} < 420$  V network, allowing 10% increase of the maximum value of the inverter output voltage. However, it can cause a substantial increase of higher odd (higher than a third one) harmonics (the 5<sup>th</sup> – up to 20%, the 7<sup>th</sup> – up to 14.3%) in the inverter output signal (similar to the inverter with  $\pi$ -switching);
- at  $U_{epn} < 420$  V network to select a motor with excess power and run it with a constantly weakened flux, or, more reasonably, to implement a two-region speed control with flux weakening only in the second region, at the speed rate higher than the base speed according to the equation:

$$\Psi_{2dset} = \Psi_{2n} \cdot \frac{\omega_{start}}{\omega}, \text{ where } \omega \geq \omega_{start},$$

where  $\omega_{start}$  is the selected value of the initial speed of field weakening. Here we understand base speed as speed values corresponding to the limiting characteristic of the open-loop system under true values of a load moment.

From the condition of maximum field weakening speed we must start at the initial speed at point 1  $\omega_{start} = \omega_1$  (Fig. 6). However, in this case the induction motor will have a much weaker excitation flux at a steady-state operating mode. For example, when the induction motor operates at point 2 with torque  $T_{EM} = 460$  Npm and at angular speed  $\omega_2 = 97.7$  rad/s the final value of flux linkage will be equal to

$$\Psi_{2d} = \frac{\omega_1}{\omega_2} \cdot \Psi_{2n} = 0.9 \cdot \Psi_{2n}$$

at the desired value

$$\Psi_{2d} = \frac{\omega_2}{\omega_2} \cdot \Psi_{2n} = \Psi_{2n}.$$

Similarly, when an induction motor operates at point 4 at angular velocity  $\omega_4 = 110$  rad/s the final value of flux linkage will be equal to

$$\Psi_{2d} = \frac{\omega_1}{\omega_4} \cdot \Psi_{2n} = 0.73 \cdot \Psi_{2n}$$

at the desired value

$$\Psi_{2d} = \frac{\omega_2}{\omega_4} \cdot \Psi_{2n} = 0.89 \cdot \Psi_{2n}$$

From the foregoing, it follows that the value of the initial field weakening speed must be chosen in accordance with the final value of the motor electromagnetic moment and changed in accordance with the moment changes. Thus, the initial field weakening speed of an induction motor is the function of electromagnetic moment  $\omega_{\text{start}} = f(T_{\text{EM}}, U_{\text{1ph.m}})$  and represents a limiting characteristic under the final voltage value  $U_{\text{1ph.m}}$ , e.g., characteristic 2 at  $U_{\text{1ph.m}} = 220 \text{ V}$  in Fig. 4.

When the supply voltage and the induction motor load (i.e. motor supply current) change, the initial field weakening value of the induction motor flux will also depend on the actual voltage on DC link and is determined with the following dependence:

$$\omega_{\text{start}} = f(T_{\text{EM}}, U_{\text{i.ph.m}} = 1.05 \cdot U_{\text{1ph.n}}) \cdot \frac{U_d}{\sqrt{3} \cdot \sqrt{2} \cdot 1.05 \cdot U_{\text{1ph.n}}}$$

where,  $U_d \leq 420 \text{ V}$ .

The block diagram of formation of the assignment at the input control loop of the rotor flux linkage in the frequency-vector control system of the double-region induction motor drive is shown in Fig. 8. The function converter forms limiting characteristic of the open-loop system of an induction motor drive at maximum voltage of the inverter  $U_{\text{i.ph.m}} = 1.05 \cdot U_{\text{1ph.n}}$

$$\omega_{\text{start}}^* = f(T_{\text{EM}}, U_{\text{i.ph.m}} = 1.05 \cdot U_{\text{1ph.n}}) \quad (7)$$

To ensure efficiency of the flux linkage control device the following conditions must be met:

- when assigning functional converter characteristics (7) the next condition must be met:  $\omega_{\text{start}}^* < \omega_{\text{start}}$  at a common moment value;
- an inertial filter (F) must be in the flux linkage control channel at the time constant  $T_F$ .

The desired nature of transient processes in the second region of speed control can be achieved by selecting the time constant of the filter  $T_F$ . Selecting smaller values of the initial field weakening speed  $\omega_{\text{start}}^*$  on the functional converter characteristic in the low moment area we can make an induction motor operate in the low

moment area with weakening of the flux and the lowest current consumption.

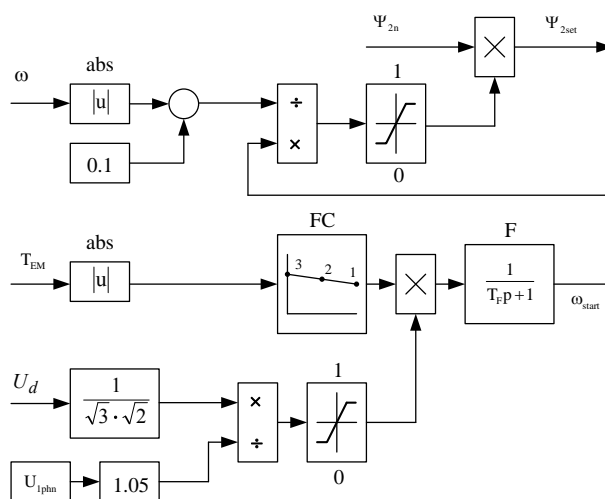


Figure 8

Block diagram of the rotor flux linkage formation at the input control loop

Fig. 9 shows dependence of the current consumed by induction motor AB250S6 when operating at the assigned speed  $\omega_{\text{set}} = 0.9 \cdot \omega_n$  and the following values of the rotor flux linkage: characteristic 1 -  $\Psi_{2\text{set}} = \Psi_{2n}$ , characteristic 2 -  $\Psi_{2\text{set}} = 0.8 \cdot \Psi_{2n}$  and characteristic 3 -  $\Psi_{2\text{set}} = 0.6 \cdot \Psi_{2n}$ . To reduce current consumption of an induction motor and exclude its thermal overheating under heavy loads the nominal flux linkage value must be assigned. On the contrary, under light load it is advisable to reduce the flux linkage value.

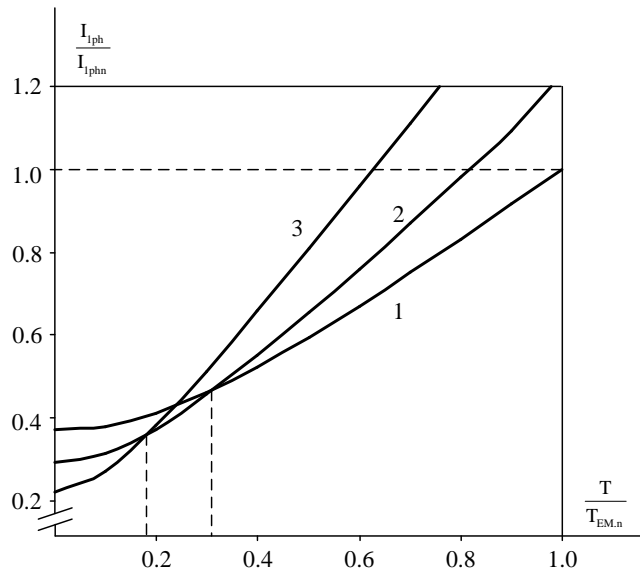


Figure 9

Dependence of current consumed by an induction motor on the load moment at: 1 -  $\Psi_2 = \Psi_{2n}$ ; 2 -

$$\Psi_2 = 0.8 \cdot \Psi_{2n}; 3 - \Psi_2 = 0.6 \cdot \Psi_{2n}$$

Characteristics of transient processes of an induction motor drive under two-region speed control when achieving the assigned speed:  $\omega_{\text{set}} = 97.7$  rad/s and  $\omega_{\text{set}} = 110$  rad/s at the load moment  $T_{\text{EM},n} = 460$  Npm are shown in Figs. 10-13. Characteristics of transient processes shown in Fig. 10 prove greater efficiency of the induction motor drive with controlled flux of an induction motor as compared to the characteristics of the induction motor drive with constant flux of an induction motor (Fig. 6). Characteristics in Fig. 10 and characteristic 2 in Fig. 13 correspond to the adjustment with the constant value  $\omega_{\text{start}} = \omega_1$ , while the diagram in Fig. 12 and characteristic 2 in Fig. 13 correspond to the adjustment with the variable value of the initial speed of weakening of the induction motor flux.

The induction motor drive systems with the constant speed value of field weakening start exhibit a slightly higher speed in the second control region, though the motor flux in the steady-state mode is too weak (characteristic 2 in Fig. 13). The main advantage of the systems with selection of speed of the field weakening start in accordance with the motor electromagnetic moment is to provide optimum value of the flux linkage at a steady-state mode of the induction motor drive (characteristic 1 in Fig. 13). It reduces the motor current consumed from the inverter and allows increasing the moment at the rated motor current in the steady state mode (characteristic 8 in Fig. 5).

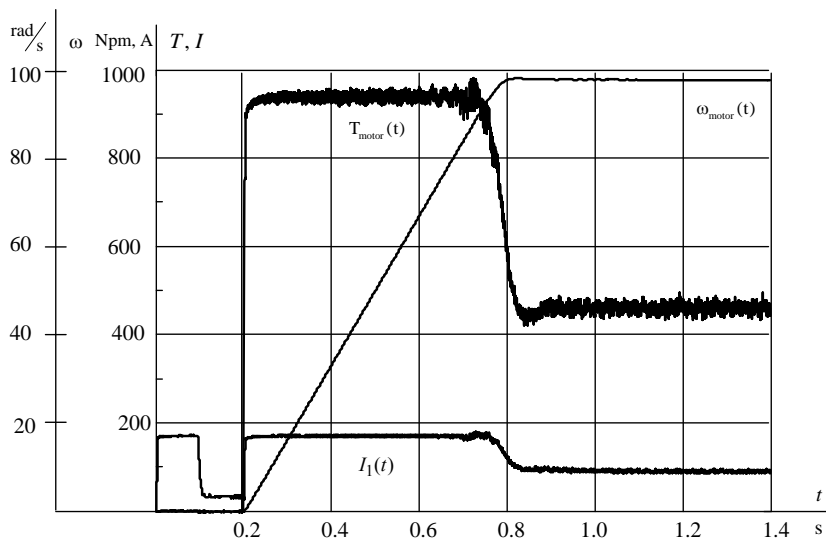


Figure 10

Transient processes  $I_{1ph}(t)$ ,  $T_{EM}(t)$  and  $\omega_{motor}(t)$  when performing  $\omega_{set} = 97.7 \text{ rad/s}$  in a double-region electric drive

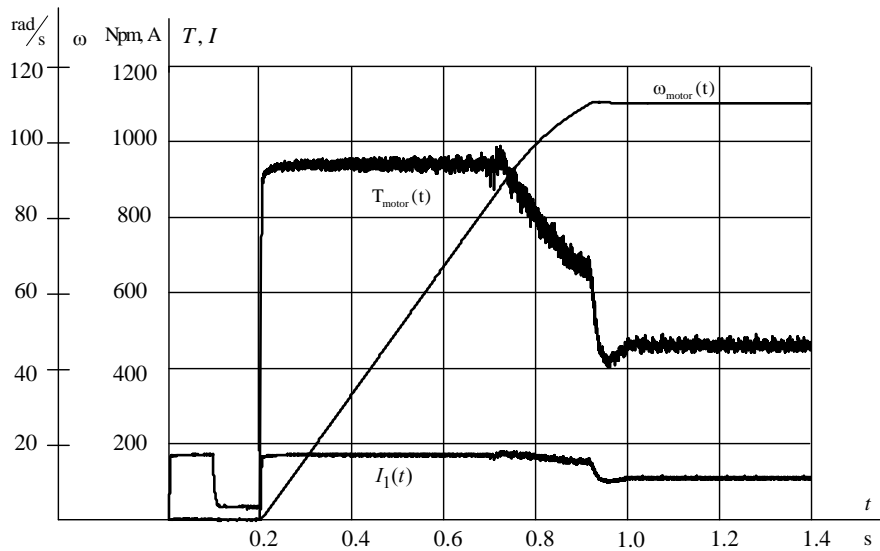


Figure 11

Transient processes  $I_{1ph}(t)$ ,  $T_{EM}(t)$  and  $\omega_{motor}(t)$  when performing  $\omega_{set} = 110 \text{ rad/s}$  and  $\omega_{start} = \text{const}$  in a double-region electric drive



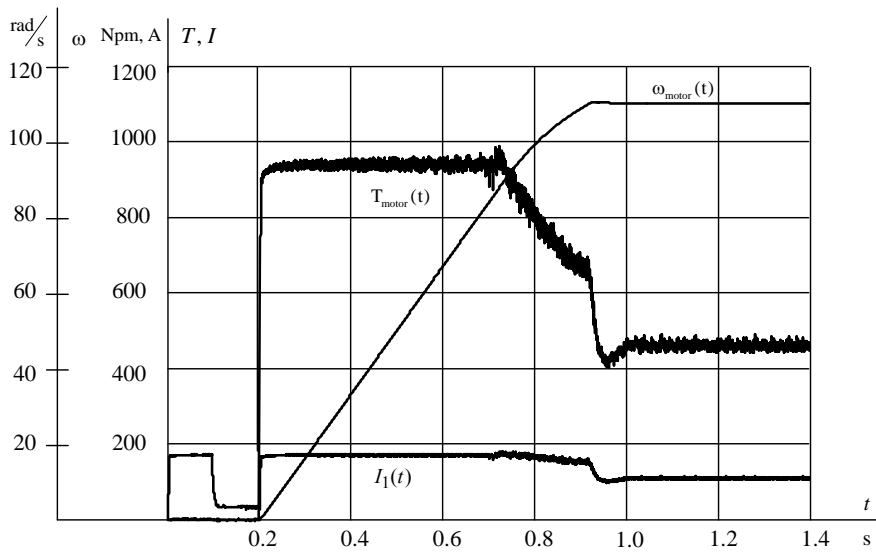


Figure 12

Transient processes  $I_{1ph}(t)$ ,  $T_{EM}(t)$  and  $\omega_{motor}(t)$  when performing  $\omega_{set} = 110 \text{ rad/s}$  and  $\omega_{start} = f(T_{EM})$  in a double-region electric drive

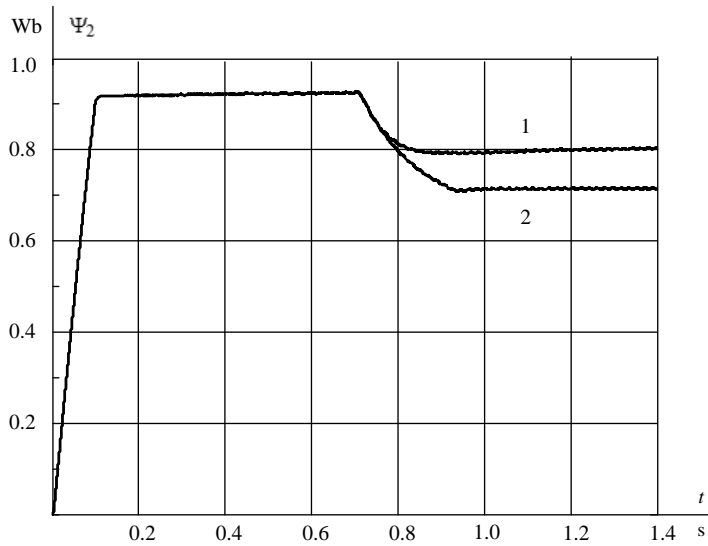


Figure 13

Flux linkage changes  $\Psi_2(t)$  when performing  $\omega_{set} = 110 \text{ rad/s}$  in a double-region electric drive: 1 –  $\omega_{start} = f(T_{EM})$ ; 2 –  $\omega_{start} = \text{const}$

Dynamic characteristics of an induction motor drive with a frequency-vector control given in Figs. 9-12 prove greater efficiency of an induction motor drive with controlled flux of the induction motor. Moreover, an induction motor drive with the constant speed value of initial weakening of the induction motor flux  $\omega_{\text{start}} = \omega_1$  exhibits a slightly higher speed in the second control region.

#### 4 Peculiarities of Frequency Inverter Voltage Selection under Vector Control Considering PWM of the Inverter Output Voltage

If a frequency converter has a sinusoidal PWM of the inverter, with the introduction of a third harmonic and control signals gain  $k = 1.1547$ , the sequence of choice of frequency inverter voltages under frequency vector control of an induction motor is as follows:

1) If a supply voltage is selected, then, using equations (3) - (7) and at the assigned values of maximum speed of an induction motor drive  $\omega_{\text{ED.max}} \leq \omega_{\text{motor.n}}$ , static load moment  $M_c$  and known parameters of an induction motor, the desired value of maximum voltage of the motor  $U_{\text{1ph.n}}$  can be accurately calculated, and, further, if maintenance conditions permit it, the maximum output voltage of an inverter can be chosen equal to:

$$U_{\text{i.ph.m}} \approx 1.05 \cdot U_{\text{1ph.n}} \cdot \frac{\omega_{\text{ED.max}}}{\omega_{\text{motor.n}}}, \text{ V.}$$

$$U_{\text{d}} \geq \sqrt{3} \cdot \sqrt{2} \cdot U_{\text{i.ph.m}}, \text{ V;}$$

$$U_{\text{epn}} \geq \frac{U_{\text{d}}}{1.35}, \text{ V,}$$

In this case, the maximum value of an inverter amplification gain is taken equal to

$$k_{\text{i}} = k_{\text{i.max}} = \sqrt{2} \cdot U_{\text{i.ph.m}}.$$

In a double-region electric motor drive at the assigned value of the maximum angular velocity of an electric motor drive  $\omega_{\text{ED.m}} > \omega_{\text{motor.n}}$ , the maximum value of phase output voltage of an inverter can be determined as

$$U_{\text{i.ph.m}} = 1.05 \cdot U_{\text{1ph.n}}$$

while DC link voltage of a frequency converter and the supply voltage can be calculated and chosen according to the next formulas and conditions:

$$U_d \geq \sqrt{3} \cdot \sqrt{2} \cdot 1.05 \cdot U_{1ph,n}$$

$$U_{epn} \geq \frac{U_d}{1.35}, V,$$

Then, the maximum value of an inverter amplification gain must be chosen and taken equal to:

$$k_i = k_{i,max} = 1.05 \cdot \sqrt{2} \cdot U_{1ph,n}.$$

2) If supply voltage  $U_{epn}$  is given, then DC link voltage and the maximum value of inverter output voltage can be calculated with the next equations:

$$U_d = 1.35 \cdot U_{epn}, V;$$

$$U_{i,ph,m} = \frac{U_d}{\sqrt{3} \cdot \sqrt{2}} \cdot V.$$

In this case, the inverter amplification gain is equal to

$$k_i = \sqrt{2} \cdot U_{i,ph,m} = \frac{U_d}{\sqrt{3}}$$

and the maximal speed of an electric drive when operating with a nominal magnetic flux will be limited by the value:

$$\omega_{ED,m} \approx \omega_{motor,n} \cdot \frac{1}{1.05} \cdot \frac{U_{i,ph,m}}{U_{1ph,n}}, \text{Rad / s.}$$

It should be noted that final values of the maximum output voltage of an inverter and the inverter amplification gain essentially depend on the supply voltage and the motor load:

$$U_{i,ph,m,fact} = (1.41 \div 1.35) \cdot \frac{(0.85 \div 1.1) \cdot U_{epn}}{\sqrt{3} \cdot \sqrt{2}}, V;$$

$$k_{i,fact} = (1.41 \div 1.35) \cdot \frac{(0.85 \div 1.1) \cdot U_{epn}}{\sqrt{3}}.$$

If  $U_{i,ph,m} > 1.05 \cdot U_{1ph,n}$ , then it must be limited at the level  $U_{i,ph,m} = 1.05 \cdot U_{1ph,n}$  while reducing the amplitude of inverter control signals in the function of DC link voltage  $U_d$  in accordance with the equation (6).

Therefore, when calculating settings of an electric motor drive control system, the maximum value of an inverter amplification gain must be considered:

$$k_i = k_{i,max} = 1.05 \cdot \sqrt{2} \cdot U_{1ph,n}$$

## Conclusions

- 1) It is found that in an induction motor drive with frequency-vector control in case of output under-voltage of an inverter of the frequency converter the desired values of the induction motor flux and moment are achieved due to decreasing its angular velocity resulting from decreasing output frequency  $f_1$ .
- 2) In case of the mains under-voltage, a two-region control of the induction motor speed is reasonable for the “frequency converter-induction motor” system to meet the condition  $\omega_{ED,m} \geq \omega_{motor,n}$ . The initial speed of weakening of an induction motor flux is to be selected in accordance with the final value of the induction motor moment, using a limiting static characteristic of the “frequency converter - induction motor” system open at speed.
- 3) It was proved that the main advantage of control systems which have an option to select the speed of initial weakening of an induction motor flux in accordance with an electromagnetic moment of the motor  $\omega_{start} = f(T_{EM})$  is a possibility to maintain the optimal value of a flux linkage in steady state modes of an induction motor drive. It ensures a bigger moment at rated current of an induction motor in steady state mode.
- 4) To reduce current consumption of an induction motor and exclude its overheating, the rated flux linkage value must be assigned at high load, while at light loads the flux linkage value must be reduced.

## Acknowledgments

The research is funded from Tomsk Polytechnic University Competitiveness Enhancement Program grant, Project Number TPU CEP\_IPE\_97\2017.

## References

- [1] Udut L. S., Maltseva O. P., Kojain N. V. Design and Study of Automatic Electrical Drives. Part 8. Induction motor drive with frequency control. - Tomsk Polytechnic University. – 2d revised and corrected edition. – Tomsk: TPU Publ., 2014, 648 p.
- [2] Sandler A. S., Sarbatov R. S. Automatic Frequency Control of Induction Motors. Moscow, Energy Publ., 1974, 328 p.
- [3] Pankratov V. V. Vector Control of Induction Motor Electric Drives. Novosibirsk, NGTU Publ., 1999, 66 p.
- [4] H. K. Lam, F. H. F. Leung, P. K. S. Tam Stable and Robust Fuzzy Control for Uncertain Nonlinear Systems, IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans, Vol. 30, No. 6, pp. 825-840, 2000
- [5] Rudakov V. V., Stolyarov I. I., Dartau V. A. Induction Motor Electric Drive with Vector Control. Leningrad. Energoatomizdat Publ., 1987, 136 p.

- [6] Blaschke F. Das Prinzip der feldorientierung die Grundlage für die Transvektor – Regelung von Drehfeldmaschinen.//Siemens Zeitschrift, 1971/Bd.45, – H.10. – S. 757-760
- [7] Dementyev Yu. N. , Bragin A. D. , Koyain N. V., Udut L. S. Control System with Sinusoidal PWM Three-Phase Inverter with a Frequency Scalar Control of Induction Motor // 2015 International Siberian Conference on Control and Communications (SIBCON): proceedings, Omsk, May 21-23, 2015, IEEE Russia Siberia Section, 2015, pp. 1-6
- [8] Chernyshov A. Yu., Dementyev Yu. N., Chernyshov I. A. Electrical AC Drive. – Tomsk Polytechnic University. – 2d edition. – Tomsk: TPU Publ., 2015, 210 p.
- [9] Shrejner R. T. Mathematical Modeling of AC Drives with Solid-State Frequency Converters. Ekaterinburg. URO RAN Publ., 2000, 654 p.
- [10] Teryokhin V. B., Dementyev Yu. N. Computer Modelling of AC and DC Drives Systems. – Tomsk Polytechnic University. – Tomsk: TPU Publ., 2015, 307 p.
- [11] Dementyev Y. N., Umurzakova A. D. The Engine Mechanical Coordinates Measuring in the Asynchronous Motor // (Article number 01017) // MATEC Web of Conferences, 2014, Vol. 19, pp. 1-5
- [12] Odnokopylov I. G., Dementyev Y. N., Usachyov I. V., Lyapunov D. Y., Petrusyov A. S. Load Balancing of Two-Motor Asynchronous Electric Drive // 2015 International Siberian Conference on Control and Communications (SIBCON): proceedings, Omsk, May 21-23, 2015, IEEE Russia Siberia Section, 2015, pp. 1-4
- [13] M. Malinowski, M. P. Kazmierkowski, S. Hansen, F. Blaabjerg and G. D. Marques, Virtual-Flux-based Direct Power Control of Three-Phase PWM Rectifiers, IEEE Trans. on Industry Applications, Vol. 37, No. 4, 2001, pp. 1019-1026
- [14] Tishihiko Noguchi, Hiroaki Tomiki, Seiji Kondo and Isao Takahashi, “Direct Power Control of PWM Converters without Power-Source Voltage Sensors”, IEEE Trans. on Industry Applications, Vol. 34, No. 3, 1998, pp. 473-479
- [15] R. E. Precup, S. Preitl PI-Fuzzy Controllers for Integral Plants to Ensure Robust Stability, Information Sciences, Vol. 177, pp. 4410-4429, 2007
- [16] A. Gharbi, M. Benrejeb, P. Borne Study of the Stabilization of Uncertain Nonlinear Systems Controlled by State Feedback, Acta Polytechnica Hungarica, Vol. 13, No. 4, pp. 21-38, 2016
- [17] Glazachev A. V., Dementyev Y. N., Negodin K. N., Umurzakova A. -. Mathematical Description of an Asynchronous Motor with the Indirect Control of the Output Mechanical Variables // EPJ Web of Conferences, 2016, Vol. 110, Article number 01044, pp. 1-6