Influence of Design Methods a Discrete Model of Separately Excited DC Motor on Parameters Estimation

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Abstract: The authors have developed the methods of preparing the difference schemes required for dynamic identification of the parameters of a DC motor as an object of the electric drives with control system during idling start-up. A system of differential equations describing the separately excited DC motor is reduced to the system of difference equations for constructing the discrete model. The authors have used three methods: direct difference, bilinear transformation method and multipoint approximation for writing the difference equations system. The estimations of the parameters of the discrete model based on the linear algebraic equations system applied this way were obtained by the matrix method. Comparing the estimates obtained the authors detected the influence of the methods of constructing the object discrete model on the error in the parameters' estimations obtained.

Keywords: discrete model; difference equations; bilinear transformation method; multipoint approximation; direct difference; linear algebraic equations system; dynamic identification; parameters estimations

1　Introduction

Dynamic identification of control objects [1] consists in determination of its model parameters based on supplied system input and output information during time limited by transition process continuance. It is necessary to understand that the identification procedure is an ill-posed problem. For transition into

consideration of an ill-posed problem, it is necessary to understand what is the well-posed problem by Jacques Hadamard. Consider the operator equation:

$$\mathbf{A} \cdot x = \mathbf{B} \tag{1}$$

where **A** is an operator acting from a space with infinite dimension x in a space with infinite dimension B. The essence of the problem is to reduce to finding solutions of the equations **x**, the corresponding of the given right side **B**. This problem will be positively posed if its mathematical solution has the following properties:

1.The solution exists.

2. The solution is unique. This condition provided if **A** has one-to-one correspondence [2].

It should be noted that the first and second conditions indicate the existence of the inverse operator $\mathbf{A}^{-1}$, and its domain of definition coincides with the domain of definition of the space **B**.

3. The solution's behavior changes continuously with the initial conditions, i.e. if $\mathbf{B}_n \to \mathbf{B}$, $\mathbf{A} \cdot \mathbf{x}_n = \mathbf{B}_n$, $\mathbf{A} \cdot \mathbf{x} = \mathbf{B}$, then $\mathbf{x}_n \to \mathbf{x}$. This means that the inverse operator $\mathbf{A}^{-1}$ is continuous.

Accordingly, ill-posed problem is considered when at least one of these properties is violated.

Equation (1) is a typical mathematical model for many physical phenomena [3], provided that the operator **A** relates the characteristics of the object **x** with the data **B**. It's necessary to understand that x can't be measured and **B** can't be obtained during the experiment. This problem is called inverse [4] and solution concludes in finding the characteristics of the object **x**. Such an inverse problem often has to be discretized in order to obtain a numerical solution. A functional analysis of inverse problems shows the continuity of the solutions [5], which implies the fulfillment of the third condition. But in connection with the transition to the discrete domain, this condition is violated, which leads to instability of the numerical solutions in calculations with finite accuracy.

Error in the statement of the operator **A** is affected on the fulfillment of the correctness conditions and methods of solving inverse problems. At the same time, it is impossible to completely get rid of the noise component [5-7]. Taking into account the noise of equation (1), it looks as follows:

$$\mathbf{A} \cdot \mathbf{x} + \boldsymbol{\varepsilon} = \mathbf{B}$$

where $\boldsymbol{\varepsilon}$ are the errors arising from the transition from a continuous coordinate system to a discrete one [8]. 1. Presence of natural noise components of signals received by sensors. 2. Vulnerability of digital differentiation to the presence of noise. 3. The value of the sampling period must not contradict the requirements of

the Nyquist–Shannon theorem [9, 10]. Thus, the correct designing of a discrete model of the object and the influence of the quality of digital differentiation is an integral task for solving problems' parameters identification of a separately excited DC motor

2 Possible Schemes of Numerical Differentiation

There are many methods of numerical differentiation [11, 12]:

1.   Forward differences (direct difference) where $x'(t_0) \approx \dfrac{x(t_0 + \Delta t)\text{-}x(t_0)}{\Delta t}$.
The estimation of the numerical differentiation formula error using Taylor's formula with remainder term in integral form is: $\left| \dfrac{x(t_0 + \Delta t) \text{-} x(t_0)}{\Delta t} - x'(t_0) \right| \leq c_1 \int\limits_{t_0}^{t_0 + \Delta t} |x''(\xi)| d\xi$, where c_1 is the constant independent

of $x(t)$, Δt and $x(t) \in C^{(2)}$, so $x(t)$ has continuous derivatives up to the second order and including on the interval $[t_0, t_0 + \Delta t]$.

2.   Backward differences (inverted difference). Estimation of errors of such approximation is: $\left| \dfrac{x(t_0) \text{-} x(t_0 - \Delta t)}{\Delta t} - x'(t_0) \right| \leq c_2 \cdot \int\limits_{t_0 - \Delta t}^{t_0} |x''(\xi)| d\xi$, where we make

the same assumptions as in the previous paragraph relatively to c_2, $x(t)$ and Δt.

3.   Central difference $x'(t_0) \approx \dfrac{x(t_0 + \Delta t) \text{-} x(t_0 - \Delta t)}{2\Delta t}$ which approximation error is given by the expression $\left| \dfrac{x(t_0 + \Delta t) \text{-} x(t_0 - \Delta t)}{2\Delta t} - x'(t_0) \right| \leq c_3 \cdot \Delta t \cdot \int\limits_{t_0 - \Delta t}^{t_0 + \Delta t} |x'''(\xi)| d\xi$.

4.   The method of multipoint approximation to approximate the derivative. The approximation order can be improved significantly if we use more points, and namely: $t = t_0 - 2\Delta t, t = t_0 - \Delta t, t = t_0 + \Delta t, t = t_0 + 2\Delta t$ at decomposition of continuous function by the Taylor's formula:

$$x(t) = x(t_0) + (t - t_0)x'(t_0) + \frac{(t - t_0)^2}{2!}x''(t_0) + \frac{(t - t_0)^3}{3!}x'''(t_0) + o((t - t_0)^3) \qquad (2)$$

So we obtain

$$x'(t) \approx \frac{x(t_0 - 2\Delta t) - 8x(t_0 - \Delta t) + 8x(t_0 + \Delta t) - x(t_0 + 2\Delta t)}{12\Delta t} \qquad (3)$$

from the system of linear equations:

$$
\begin{cases}
x(t_0 - 2\Delta t) = x(t_0) - 2\Delta t x'(t_0) + 2\Delta t^2 x''(t_0) - \dfrac{4\Delta t^3}{3} x'''(t_0) + o(\Delta t^3), \\[2mm]
x(t_0 - \Delta t) = x(t_0) - \Delta t x'(t_0) + \dfrac{\Delta t^2}{2} x''(t_0) - \dfrac{\Delta t^3}{6} x'''(t_0) + o(\Delta t^3), \\[2mm]
x(t_0 + \Delta t) = x(t_0) + \Delta t x'(t_0) + \dfrac{\Delta t^2}{2} x''(t_0) + \dfrac{\Delta t^3}{6} x'''(t_0) + o(\Delta t^3) \\[2mm]
x(t_0 + 2\Delta t) = x(t_0) + 2\Delta t x'(t_0) + 2\Delta t^2 x''(t_0) + \dfrac{4\Delta t^3}{3} x'''(t_0) + o(\Delta t^3).
\end{cases}
\tag{4}
$$

The estimation of this approximation error has the form:

$$
\left| \frac{x(t_0 - 2\Delta t) - 8x(t_0 - \Delta t) + 8x(t_0 + \Delta t) - x(t_0 + 2\Delta t)}{12\Delta t} - x'(t_0) \right| \le
$$

$$
\le c_4 \Delta t^3 \left\{ \int_{t_0 - \Delta t}^{t_0 + \Delta t} \left| x'''(\xi) \right| d\xi - 2 \int_{t_0 - 2\Delta t}^{t_0 + 2\Delta t} \left| x''''(\xi) \right| d\xi \right\}, \ x(t) \in C^{(4)}
\tag{5}
$$

3 Methods for Imaging Derivatives

The examined schemes of numerical differentiation allow moving from differential equations to difference ones, which are widely used to describe the stationary discrete systems. z-transform is the convenient tool for solving differential equations, so let us consider separately some methods for imaging derivatives from the Laplace-domain to the z-domain.

Let us consider the method of direct difference as the derivative transformation. The method consists in derivative approximation by the finite difference $\dfrac{dx}{dt} \rightarrow \dfrac{x(n \cdot \Delta t + 1 \cdot \Delta t) - x(n \cdot \Delta t)}{\Delta t}.$

Applying the Laplace transform of the left side and z-transform of the right one, we obtain $xX(s) \rightarrow \dfrac{z - 1}{\Delta t} X(z).$ The explicit form of this transformation is $s \rightarrow \dfrac{z - 1}{\Delta t}$. The inverse transformation $z \leftarrow s \cdot \Delta t + 1$ was obtained from the latter relation. According to [13, 14] in order to construct the filters with desired properties the conversion of the derivative into the finite difference must satisfy the following conditions:

1.   The imaginary axis of s-plane must be displayed into the unit circle of z-plane.

2.   The left half-plane of Laplace-domain $Re(s) \leq 0$ must be displayed into the interior part of a unit circle of z-domain $|z|<1$.

3.   The conversion should be rational.

The left half-plane of s-plane moves into the left half-domain $Re(s)<1$ of z-domain based on the inverse transform $z \leftarrow s \cdot \Delta t +$, so the second condition is not satisfied. The imaginary axis of s-plane moves into a straight $Re(s)=1$ of z-domain, so the first condition is not satisfied. These conditions can be fulfilled near the point $z=1$ of the complex z-domain, so when $\Delta t \rightarrow 0$ the direct difference method should give a satisfacto-ry result.

The inverse difference method, in which the derivative is approximated by the backward difference: $\dfrac{dx}{dt} \rightarrow \dfrac{x(n \cdot \Delta t) - x(n \cdot \Delta t - 1 \cdot \Delta t)}{\Delta t}$. Applying the Laplace transform of the left part and z-transform of right part, we obtain $s \cdot X(s) \rightarrow \dfrac{1 - z^{-1}}{\Delta t} \cdot X(z)$ whence the explicit form of this transformation $s \rightarrow \dfrac{1 - z^{-1}}{\Delta t}$ was obtained. The inverse transformation is as follows $z \leftarrow \dfrac{1}{1 - s \cdot \Delta t}$.   Since $Re\left(\dfrac{1}{1 - j \cdot \omega \cdot \Delta t}\right) = \dfrac{1}{1 + \omega^2 \cdot \Delta t^2}$ and

$Im\left(\dfrac{1}{1 - j \cdot \omega \cdot \Delta t}\right) = \dfrac{\omega \cdot \Delta t}{1 + \omega^2 \cdot \Delta t^2}$,   then

$$\left(Re\left(\dfrac{1}{1 - j \cdot \omega \cdot \Delta t}\right) - \dfrac{1}{2}\right)^2 + \left(Im\left(\dfrac{1}{1 - j \cdot \omega \cdot \Delta t}\right)\right)^2 = \left(\dfrac{1}{1 + \omega^2 \cdot \Delta t^2} - \dfrac{1}{2}\right)^2 +$$

$+ \dfrac{\omega^2 \cdot \Delta t^2}{\left(1 + \omega^2 \cdot \Delta t^2\right)^2} = \dfrac{1}{4}$, so the points of the imaginary axis $s=j \cdot \omega$ the backward difference method transforms into the points of circle on z-plane with center in $z = \dfrac{1}{2}$ and radius equal to $\dfrac{1}{2}$. The point $s=0$ is imaged into the point $z=1$, so if $s \rightarrow +\infty$ then $z \rightarrow -j \cdot 0$, and if $s \rightarrow -\infty$ then $z \rightarrow +j \cdot 0$. Under this imaging the left half-plane of Laplace-domain is displayed inside the circle.

Let us consider the bilinear transformation $s \to \dfrac{2}{\Delta t} \cdot \dfrac{1 - z^{-1}}{1 + z^{-1}}$ that allows the

inverse transformation $z \leftarrow \dfrac{\dfrac{2}{\Delta t} + s}{\dfrac{2}{\Delta t} - s}$. The last expression shows that the points of

the imaginary axis $s = j \cdot \omega$ are transformed into the point of circle $|z| = 1$ by bilinear

transformation. As $\dfrac{\dfrac{2}{\Delta t} + j\omega}{\dfrac{2}{\Delta t} - j\omega} = \dfrac{\dfrac{4}{\Delta t^2} + \dfrac{4}{\Delta t}\omega j - \omega^2}{\dfrac{4}{\Delta t^2} + \omega^2}$, so $\mathrm{Re}\left(\dfrac{\dfrac{2}{\Delta t} + j\omega}{\dfrac{2}{\Delta t} - j\omega} \right) = \dfrac{\dfrac{4}{\Delta t^2} - \omega^2}{\dfrac{4}{\Delta t^2} + \omega^2}$

and $\mathrm{Im}\left(\dfrac{\dfrac{2}{\Delta t} + j\omega}{\dfrac{2}{\Delta t} - j\omega} \right) = \dfrac{\dfrac{4}{\Delta t}\omega}{\dfrac{4}{\Delta t^2} + \omega^2}$, therefore $\mathrm{Re}^2\left(\dfrac{\dfrac{2}{\Delta t} + j\omega}{\dfrac{2}{\Delta t} - j\omega} \right) + \mathrm{Im}^2\left(\dfrac{\dfrac{2}{\Delta t} + j\omega}{\dfrac{2}{\Delta t} - j\omega} \right) = 1$.

Points of the left half-plane $\mathrm{Re}(s) < 0$ are transformed into the region bounded by a circle $|z| < 1$. The transformation must be linear fractional and is fulfilled as well. It is the third condition.

Finally, applying the Laplace transform of the left part and z-transform of the right

part of the expression (1), we obtain $s \cdot X(s) \to \dfrac{z^{-2} - 8z^{-1} + 8z - 8z^2}{12\Delta t} X(z)$ and its

ex-plicit form $s \to \dfrac{z^{-2} - 8z^{-1} + 8z - 8z^2}{12\Delta t}$. The authors did not manage to find the

inverse transformation in the last expression, so the analysis of conversion when displaying the derivative found by the formula (3) is not given in the paper.

4 The Difference Scheme to Identify the Separate Excitation DC Motor Parameters Applying the Direct Difference Method

The separately excited DC motor model at idle is described by a differential equations system [15]:

$$\begin{cases} U(t) = R \cdot i(t) + L \cdot \dfrac{di(t)}{dt} + c \cdot \omega(t) \\ J \cdot \dfrac{d\omega(t)}{dt} = c \cdot i(t) \end{cases} \tag{6}$$

where $U(t)$ is the voltage (V), applied to the anchor at time t; R is the resistance of the armature circuit (Ohm); $i(t)$ is the armature current (A); L is the anchor circuit inductance (H); c is the special coefficient (V•sec/rad); J is the equivalent inertia moment, cast to the motor shaft; $\omega(t)$ is the shaft rotation frequency of DC motor.

Taking into account the sampling time interval Δt of the measurement system let us move from the differential equation $U(t) = R \cdot i(t) + L \cdot \dfrac{di(t)}{dt} + c \cdot \omega(t)$ to the differential equations system for the current $n \cdot \Delta t$ and previous time $n \cdot \Delta t - 1 \cdot \Delta t$ at a constant assessment of resistance and inductance:

$$\begin{cases} i(n \cdot \Delta t) \cdot \hat{R} + D^{(1)}\big[i(n \cdot \Delta t)\big] \cdot \hat{L} = U(n \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t), \\ i(n \cdot \Delta t - 1 \cdot \Delta t) \cdot \hat{R} + D^{(1)}\big[i(n \cdot \Delta t - 1 \cdot \Delta t)\big] \cdot \hat{L} = U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t - 1 \cdot \Delta t), \end{cases} \tag{7}$$

where $i(n \cdot \Delta t)$, $i(n \cdot \Delta t - 1 \cdot \Delta t)$ are the currents; $D^{(1)}[i(n \cdot \Delta t)]$, $D^{(1)}[i(n \cdot \Delta t - 1 \cdot \Delta t)]$ are the derived currents; $U(n \cdot \Delta t)$, $U(n \cdot \Delta t - 1 \cdot \Delta t)$ are the voltages; $\omega(n \cdot \Delta t)$, $\omega(n \cdot \Delta t - 1 \cdot \Delta t)$ are the shaft rotation frequencies of DC motor on the current and previous steps, respectively. The last system can be written in matrix form:

$$\begin{bmatrix} i(n \cdot \Delta t) & D^{(1)}\big[i(n \cdot \Delta t)\big] \\ i(n \cdot \Delta t - 1 \cdot \Delta t) & D^{(1)}\big[i(n \cdot \Delta t - 1 \cdot \Delta t)\big] \end{bmatrix} \cdot \begin{bmatrix} \hat{R} \\ \hat{L} \end{bmatrix} = \begin{bmatrix} U(n \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t) \\ U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t - 1 \cdot \Delta t) \end{bmatrix} \tag{8}$$

The solution of this system explicitly by the matrix method at the current step of time sampling has the form:

$$\begin{bmatrix} \hat{R}(n \cdot \Delta t) \\ \hat{L}(n \cdot \Delta t) \end{bmatrix} = \frac{1}{i(n \cdot \Delta t) \cdot D^{(1)}\big[i(n \cdot \Delta t - 1 \cdot \Delta t)\big] - i_{j-1} \cdot D^{(1)}\big[i(n \cdot \Delta t)\big]} \cdot$$

$$\cdot \begin{bmatrix} \big(U(n \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t)\big) \cdot D^{(1)}\big[i(n \cdot \Delta t - 1 \cdot \Delta t)\big] - \\ \big(U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t - 1 \cdot \Delta t)\big) \cdot i(n \cdot \Delta t) - \\ - \big(U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t - 1 \cdot \Delta t)\big) \cdot D^{(1)}\big[i(n \cdot \Delta t)\big] \\ - \big(U(n \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t)\big) \cdot i(n \cdot \Delta t - 1 \cdot \Delta t) \end{bmatrix} \tag{9}$$

Substituting $D^{(1)}\big[i(n \cdot \Delta t)\big]$ as the estimation of the derivatives in the form of a direct difference $\dfrac{i(n \cdot \Delta t) - i(n \cdot \Delta t - 1 \cdot \Delta t)}{\Delta t}$, we finally obtain:

$$\begin{bmatrix} \hat{R}(n \cdot \Delta t) \\ \hat{L}(n \cdot \Delta t) \end{bmatrix} = \frac{1}{[i(n \cdot \Delta t - 1 \cdot \Delta t)]^2 - i(n \cdot \Delta t)i(n \cdot \Delta t - 1 \cdot \Delta t)} \cdot$$

$$\cdot \begin{bmatrix} \left(U(n \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t)\right) \cdot \dfrac{i(n \cdot \Delta t - 1 \cdot \Delta t) - i(n \cdot \Delta t - 2 \cdot \Delta t)}{\Delta t} - \\ \left(U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t - 1 \cdot \Delta t)\right) \cdot i(n \cdot \Delta t) - \end{bmatrix} \tag{10}$$

$$-\left(U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t - 1 \cdot \Delta t)\right) \cdot \dfrac{i(n \cdot \Delta t) - i_{j-1}}{\Delta t}$$
$$-\left(U(n \cdot \Delta t) - c \cdot \omega(n \cdot \Delta t)\right) \cdot i(n \cdot \Delta t - 1 \cdot \Delta t)$$

Based on the second equation of the system (2) let us find the estimation of the inertia moment transforming the equation to the form

$$\frac{d\omega(t)}{dt} \cdot \hat{J} = c \cdot i(t) \tag{11}$$

Taking into account the comments given in the derivation of the previous system of difference equations, let us form according to the equation (11) the proper difference equation $\dfrac{\omega(n \cdot \Delta t) - \omega(n \cdot \Delta t - 1 \cdot \Delta t)}{\Delta t} \cdot \hat{J}(n \cdot \Delta t) = c \cdot i(n \cdot \Delta t)$ from which we directly obtain $\hat{J}(n \cdot \Delta t)$.

5 The Difference Scheme to Identify the Separate Excitation DC Motor Parameters Applying the Bilinear Transformation Method

Applying the Laplace transform to the system (6) we obtain:

$$\begin{cases} L \cdot s \cdot i(s) + R \cdot i(s) = U(s) - c \cdot \omega(s), \\ J \cdot s \cdot \omega(s) = c \cdot i(s). \end{cases} \tag{12}$$

Then using the bilinear transformation [16] $s \to \dfrac{2}{\Delta t} \cdot \dfrac{1 - z^{-1}}{1 + z^{-1}}$ we obtain:

$$\begin{cases} \dfrac{2}{\Delta t} \cdot \left(1 - z^{-1}\right) \cdot i(z) \cdot L + \left(1 + z^{-1}\right) \cdot i(z) \cdot R = \left(U(z) - c \cdot \omega(z)\right) \cdot \left(1 + z^{-1}\right), \\ \dfrac{2}{\Delta t} \cdot \left(1 - z^{-1}\right) \cdot J \cdot \omega(z) = c \cdot i(z). \end{cases} \tag{13}$$

Let us write down the first equation of the system (13) relative to the current $n \cdot \Delta t$ and the previous time $n \cdot \Delta t \Delta t$ considering the fact that according to z-transform the image multiplication by z^{-1} corresponds to a delay per one sample:

$$\begin{cases} \dfrac{2}{\Delta t} \cdot \left(i(n \cdot \Delta t) - i(n \cdot \Delta t - 1 \cdot \Delta t) \right) \cdot L + \left(i(n \cdot \Delta t) + i(n \cdot \Delta t - 1 \cdot \Delta t) \right) \cdot R = \\ = U(n \cdot \Delta t) + U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \left(\omega(n \cdot \Delta t) + \omega(n \cdot \Delta t - 1 \cdot \Delta t) \right) \\ \dfrac{2}{\Delta t} \cdot \left(i(n \cdot \Delta t - 1 \cdot \Delta t) - i(n \cdot \Delta t - 2 \cdot \Delta t) \right) \cdot L + \left(i(n \cdot \Delta t - 1 \cdot \Delta t) + i(n \cdot \Delta t - 2 \cdot \Delta t) \right) \cdot R = \\ = U(n \cdot \Delta t - 1 \cdot \Delta t) + U(n \cdot \Delta t - 2 \cdot \Delta t) - c \cdot \left(\omega(n \cdot \Delta t - 1 \cdot \Delta t) + \omega(n \cdot \Delta t - 2 \cdot \Delta t) \right) \end{cases} \tag{14}$$

In matrix form this system has the form:

$$\begin{bmatrix} \dfrac{2}{\Delta t} \cdot \left(i(n \cdot \Delta t) - i(n \cdot \Delta t - 1 \cdot \Delta t) \right) & i(n \cdot \Delta t) + i(n \cdot \Delta t - 1 \cdot \Delta t) \\ \dfrac{2}{\Delta t} \cdot \left(i(n \cdot \Delta t - 1 \cdot \Delta t) - i(n \cdot \Delta t - 2 \cdot \Delta t) \right) & i(n \cdot \Delta t - 1 \cdot \Delta t) + i(n \cdot \Delta t - 2 \cdot \Delta t) \end{bmatrix} \cdot \begin{bmatrix} \hat{L}(n \cdot \Delta t) \\ \hat{R}(n \cdot \Delta t) \end{bmatrix} = \tag{15}$$

$$= \begin{bmatrix} U(n \cdot \Delta t) + U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \left(\omega(n \cdot \Delta t) + \omega(n \cdot \Delta t - 1 \cdot \Delta t) \right) \\ U(n \cdot \Delta t - 1 \cdot \Delta t) + U(n \cdot \Delta t - 2 \cdot \Delta t) - c \cdot \left(\omega(n \cdot \Delta t - 1 \cdot \Delta t) + \omega(n \cdot \Delta t - 2 \cdot \Delta t) \right) \end{bmatrix}$$

We obtain the solution of the latter system explicitly:

$$\hat{L}(n \cdot \Delta t) = \frac{\Delta t}{4 \cdot \left(i(n \cdot \Delta t) \cdot i(n \cdot \Delta t - 1 \cdot \Delta t) - \left[i(n \cdot \Delta t - 1 \cdot \Delta t) \right]^2 \right)} \cdot$$
$$\cdot \Big\{ \left(i(n \cdot \Delta t - 1 \cdot \Delta t) + i(n \cdot \Delta t - 2 \cdot \Delta t) \right) \cdot$$
$$\cdot \left[U(n \cdot \Delta t) + U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \left(\omega(n \cdot \Delta t) + \omega(n \cdot \Delta t - 1 \cdot \Delta t) \right) \right] - \tag{16}$$
$$- \left(i(n \cdot \Delta t) + i(n \cdot \Delta t - 1 \cdot \Delta t) \right) \cdot$$
$$\cdot \left[U(n \cdot \Delta t - 1 \cdot \Delta t) + U(n \cdot \Delta t - 2 \cdot \Delta t) - c \cdot \left(\omega(n \cdot \Delta t - 1 \cdot \Delta t) + \omega_{j-2} \right) \right] \Big\},$$

$$\hat{R}(n \cdot \Delta t) = \frac{1}{2 \cdot \left(i(n \cdot \Delta t) \cdot i(n \cdot \Delta t - 1 \cdot \Delta t) - \left[i(n \cdot \Delta t - 1 \cdot \Delta t) \right]^2 \right)} \cdot$$
$$\Big\{ \left(i(n \cdot \Delta t) - i(n \cdot \Delta t - 1 \cdot \Delta t) \right) \cdot$$
$$\cdot \left[U(n \cdot \Delta t - 1 \cdot \Delta t) + U(n \cdot \Delta t - 2 \cdot \Delta t) - c \cdot \left(\omega(n \cdot \Delta t - 1 \cdot \Delta t) + \omega(n \cdot \Delta t - 2 \cdot \Delta t) \right) \right] - \tag{17}$$
$$- \left(i(n \cdot \Delta t - 1 \cdot \Delta t) - i(n \cdot \Delta t - 2 \cdot \Delta t) \right) \cdot$$
$$\cdot \left[U(n \cdot \Delta t) + U(n \cdot \Delta t - 1 \cdot \Delta t) - c \cdot \left(\omega(n \cdot \Delta t) + \omega(n \cdot \Delta t - 1 \cdot \Delta t) \right) \right] \Big\}.$$

From the second equation of the system (13) obtain the estimation of the equivalent inertia moment J in a similar way

$$\hat{J}(n \cdot \Delta t) = \frac{\Delta t}{2} \cdot c \cdot \frac{i(n \cdot \Delta t) + i(n \cdot \Delta t - 1 \cdot \Delta t)}{\omega(n \cdot \Delta t) - \omega(n \cdot \Delta t - 1 \cdot \Delta t)} \tag{18}$$

Estimations of the parameters $\hat{L}$, $\hat{R}$, $\hat{J}$ are delivered by the expressions (9) and (11), in which the derivatives are approximated by the formula (3).

6   The Results of Numerical Modeling of Dynamic Identification of Separate Excitation DC Motor Parameters

For solution of the identification problem [17] the authors found the analytical solution of the system (3), which is shown in Figure 1. The identification processes for the parameters $\hat{L}$, $\hat{R}$, $\hat{J}$ were plotted for the direct difference method, bilinear transformation, multipoint approximation (Figure 2–4) and compared with the actual values. The lack of transients in Figure 2–4 can be explained by the fact that the noise component was not taken into account at simulation.
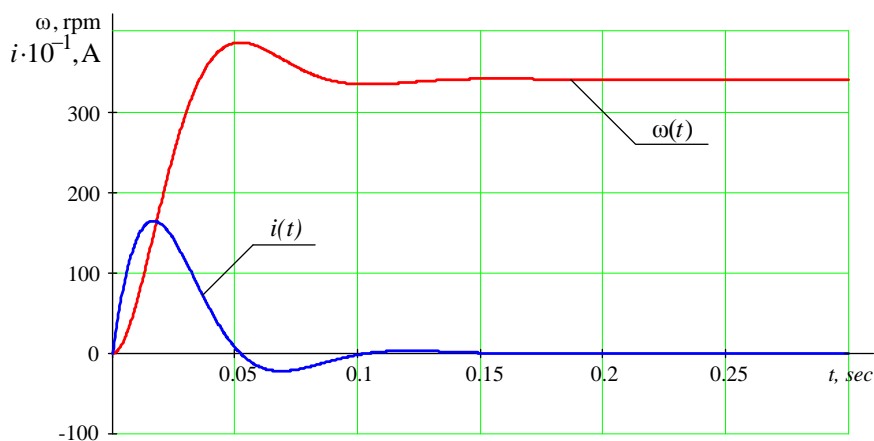

Figure 1

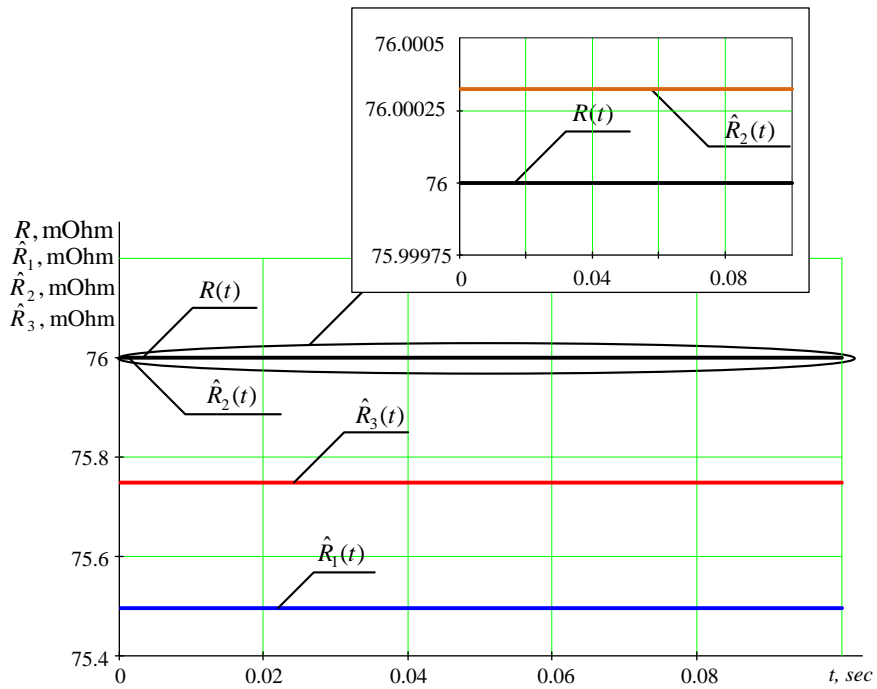Transition processes of current i(t) and rotor speed ω(t)

Figure 2

The dynamic of DC motor resistance estimation. R is the real resistance value; $\hat{R}_1$ − the resistance estimation by direct difference method; $\hat{R}_2$ is the resistance estimation by bilinear transformation method; $\hat{R}_3$ is the resistance estimation by multipoint approximation
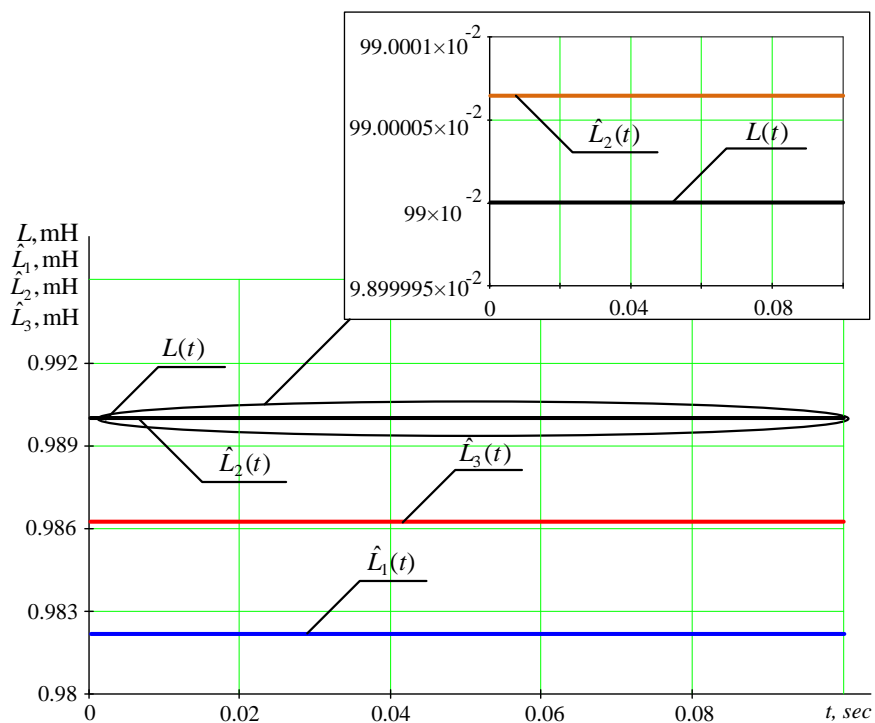
Figure 3

The dynamic of DC motor inductance estimation. L is the real inductance value; $\hat{L}_1$ is the inductance estimation by the direct difference method; $\hat{L}_2$ is the inductance estimation by the bilinear transformation method; $\hat{L}_3$ is the inductance estimation by the multipoint approximation
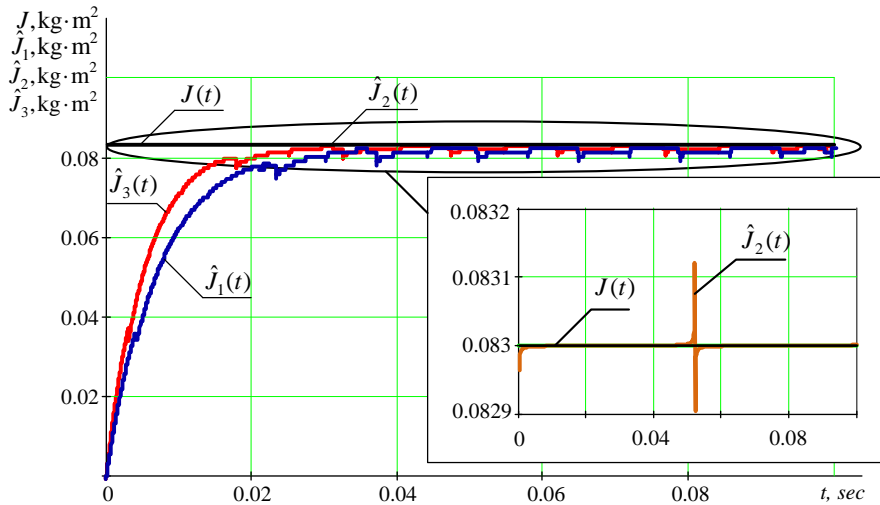
Figure 4

The dynamics of estimation of DC motor equivalent inertia moment. J is the real value of the equivalent inertia moment; $\hat{J}_1$ is the equivalent inertia moment estimation by the direct difference method; $\hat{J}_2$ is the equivalent inertia moment estimation by the bilinear transformation method; $\hat{J}_3$ is the equivalent inertia moment estimation by multipoint approximation

To analyze the quality of the evaluation process of the parameter x (respectively) in Figure 2–4 the authors considered the integral mean-square error of the parameter estimation during the load-on

$$\delta_x = \sqrt{\frac{1}{j_{fin} - j_{st}} \cdot \sum_{k=j_{fin}}^{j_{st}} \left(\frac{x - \hat{x}_k}{x}\right)^2} \cdot 100\%$$ corresponding to the numbers $j_{st} = 2$

and $j_{fin} = 1002$. All errors are presented in Table 1.

Estimations of the equivalent inertia moment by the direct difference method $\hat{J}_1$ and by the multipoint approximation $\hat{J}_3$ were subjected to nonlinear predictive filter. The filtration is required due to the presence of a large dispersed component of the estimates which value is many times higher than the actual value of the inertia moment. This leads to inability to obtain adequate estimates.

Table 1

Indicators of quality estimates

	Estimate	Real value	Estimated value	δ_x, %
Resistance, mOhm	$\hat{R}_1$	76	75.49	0.66
	$\hat{R}_2$		76.032	$43 \cdot 10^{-5}$
	$\hat{R}_3$		75.66	0.437
Equivalent moment of inertia estimates, kg·m²	$\hat{J}_1$	0.083	0.08	2.61
	$\hat{J}_2$		0.083	$78 \cdot 10^{-4}$
	$\hat{J}_3$		0.081	1.48
Inductance, mH	$\hat{L}_1$	99	98.24	0.76
	$\hat{L}_2$		99	$65 \cdot 10^{-6}$
	$\hat{L}_3$		98.6238	0.38

Thus the estimation calculation errors are considerably lower (Table 1) in the equations obtained by the bilinear transformation than in the dif-ference equations, obtained by the methods of direct difference and multipoint difference approximation, so using the bilinear transformation for identifying the estimates of the separate excitation DC motor parameters is preferred as the filtration is not required in this case.

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