Process Network Solution of Extended CPM Problems with Alternatives

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Abstract: In this paper a novel method to extend the problem range of CPM problems is given. First, the CPM problem is transformed into a process network problem. It is shown how the directed bipartite process network has to be generated, and the corresponding mathematical programming model should be formulated. Time optimal, cost optimal, time optimal with additional cost constraints and cost optimal with additional time constraints mathematical programming models are given. Moreover, it is illustrated how alternative cases may appear in the structure. An example illustrates the efficiency of the present work.

Keywords: CPM; process network; alternatives

1   Introduction

The critical path method (CPM) is an algorithmic approach of scheduling a set of activities. CPM is widely used for projects in the field of constructions to software

development. Modeling techniques originate back in the 1950s. The main criteria, in order to use the CPM technique, are the following. First, duration times of the activities have to be known together with the dependencies between the activities. Based on this information the activity network is developed. With the help of the list of activities together with their duration and dependencies on each other as well as on the logical end points, CPM calculates the longest path of the planned activities together with the earliest and latest times that each activity can start or finish without lengthening the project. In this regard, the critical path is the sequence of the activities which add up to the longest overall duration. Please note that there are activity-on-node and activity-on-arc approaches of the CPM from which representations the latter is considered for the present work. For further information please consider the problem definition of Chanas and Zielinski, 2001.

Recent advances consider CPM networks with ant colony optimization methods (Shankar et al. 2011, Li Hui et al. 2013); CPM problems where task durations are uncertain (Li et al. 2015), fuzzy linear programming model (Madhuri et al. 2013). For a special problem CPM with time–cost trade-offs, i.e. process of crashing, was investigated by Sunita et al. 2013, while activity list-based nested partitions algorithm with local adjustments were used by Xiao et al. 2014.

Please note that CPM techniques order the resources to the activities, but this order is not represented in the graph. When another resource is ordered to the same activity then the parameters of the problem have to be reset, in most cases a new graph has to be depicted, and the problem has to be solved again; i.e. a large number of separate problems has to be uniquely considered. Moreover, CPM graph techniques do not handle at all cases where a given subtask can be solved in many different ways.

Friedler et al. 1992a, 1992b, introduced a process network methodology for chemical engineering problems. Based on rigorous mathematical foundations the approach relies both on graph theory as well as combinatorial techniques focusing first on the structure generation of the problem considered. Besides the directed bipartite process network an underlying axiom system is used to derive theorems to generate the potentially feasible structures as well as the so-called maximal structure which includes all feasible solution structures. When the algorithmically and mathematically proven structure generation ends, a mathematical programming model is generated and solved with similar mathematical rigor. One of the main advantages of the developed methodology is that alternative solutions can also be interpreted for the problems considered.

Later on this methodology was used in other fields, for example workflow modelling (Tick 2007, Tick et al. 2013); separation network synthesis problems with multiple feed streams and sharp separators (Kovács et al. 1999 and 2000); generating and analyzing structural alternatives for supply scenarios (Barany et al. 2010; Klemeš et al. 2010, Kalauz et al. 2012); determining the thermodynamically dominant pathways in a metabolic network (Yun et al. 2013), or identifying

feasible pathways of the reaction catalysed by a catalyst with multi-active sites (Fan et al. 2012). Process networks were successfully adopted for solving the routing and scheduling of evacuees, facing a life-threatening situation as well (Garcia-Ojeda et al. 2012).

Please note that there has been no connection between CPM and process network methodology in the literature until today.

1.1   Mapping of CPM to a Process Network

In order to solve CPM problems with the help of process network methodology, the two terminologies have to be mapped. First, the basic elements are considered as described in Table 1. Both the CPM and the process network methodology may have attributes corresponding to their various objects, depending on the application field. For example, in the CPM an activity has a given duration and may have various resources with various costs; while on the other hand in a process network an operating unit may also have a given operation time and various costs.

Table 1
Basic terminology of the CPM and of the process networks

CPM	Process network
Event (node). Activity (arc). Logical connection between the activities (dependencies between the activities). CPM graph: scheduling problem.	Material (node type 1): raw material, intermediate and product. Operating unit (node type 2). Material flow (arc). Process network: network of the operating units producing the products from the raw materials.

It is worth mentioning that in the CPM networks the work is performed by the resources, while in the process networks work is performed by the operating units. Therefore, these are mapped to each other.
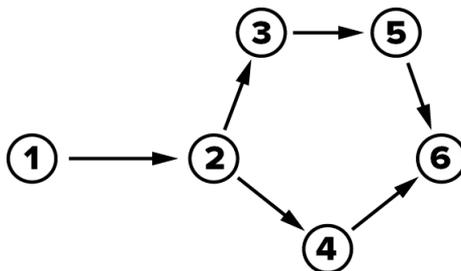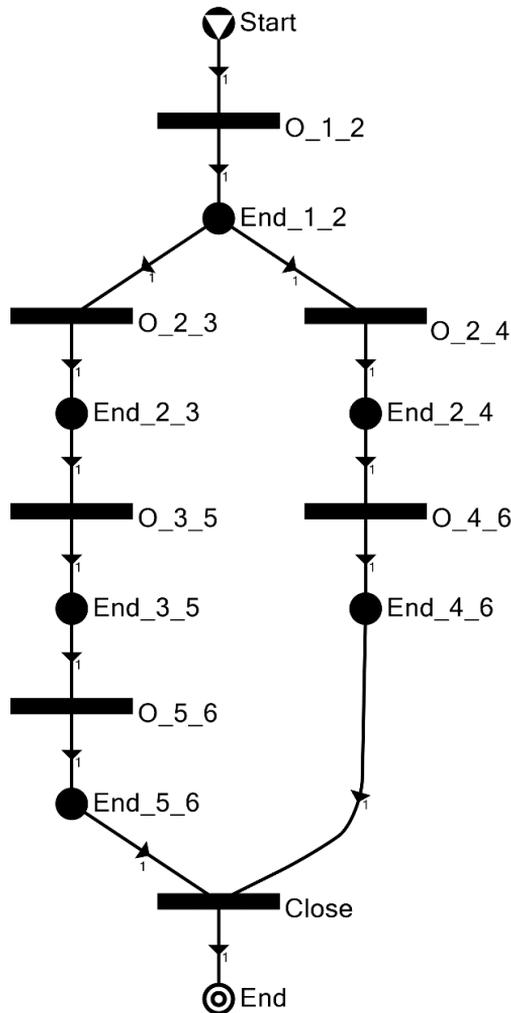
Figure 1a
CPM graph example

Figure 1b

Process network of the CPM graph example A given CPM graph can be mapped into a process network as follows. There is a given start of the CPM problem, see Figure 1a. In the process network it is represented by a raw material, see "Start" in Figure 1b. The activity between event 1 and event 2 of the CPM graph is represented in the process network as operating unit O_1_2. The input material of the operating unit O_1_2 in this case is the "Start." Activity between event 2 and event 3 of the CPM starts when event 2 is accomplished. In the process network it is mapped into the material End_1_2, which is the output material of O_1_2, which means that the operation of O_1_2 is finished. Generally speaking, in the process network such materials will be inputs to an operating unit, which have to be accomplished in the CPM graph before the activity can start. In the process

network of the current example operating unit Close will have two input materials, namely End_4_6 and End_5_6, since in the CPM graph both event 4 and event 5 have to be accomplished before the activity between 6 and 7 can start. Each operating unit of the process network has one output, which represents the operational finish of the operating unit. Since the end point of the CPM graph is event 6 with two different preceding activities, in the process network an addition technical operating unit, called Close, has to be inserted. As a result, the CPM graph of Figure 1a can be mapped into the process network Figure 1b.

When the structural mapping is done according to the above given details, Table 2 illustrates the logical connections between the CPM and process networks.

Table 2
Logical connection between the CPM and process networks

CPM	Process network
Activity.	Operating unit.
Event.	Material (raw material, intermediate and product)
Logical connection between the activities (dependencies between the activities).	Material flow (arcs).
CPM graph: scheduling problem.	Process network: network of the operating units producing the products from the raw materials.

The mapping is done accordingly, then the basic terminology of CPM and process networks can be combined for simplicity.

1.2   Illustrative Example of the Mapping

Let us consider the example illustration published by Chanas and Zielinsky (2001). The CPM graph of the example is given in Figure 2a; while Figure 2b illustrates the process network representation of the example after the mapping. Please note that in Figure 2a, the activity between event 4 and event 7 starts only when both activities between event 2 and event 4 and between event 3 and event 4 are finished. Similarly, in Figure 2b operating unit O_4_7 starts its operation only when both operating units O_2_4 and O_3_4 finished their operations, i.e. materials End_2_4 and End_3_4 are available respectively, serving as input materials of O_4_7. Please also note that since the operating units under consideration are different, their end points are also different.
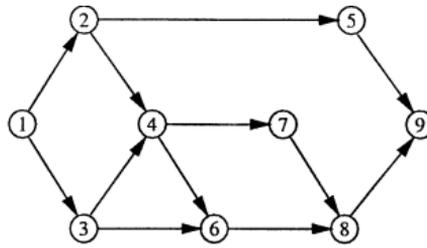
Figure 2a
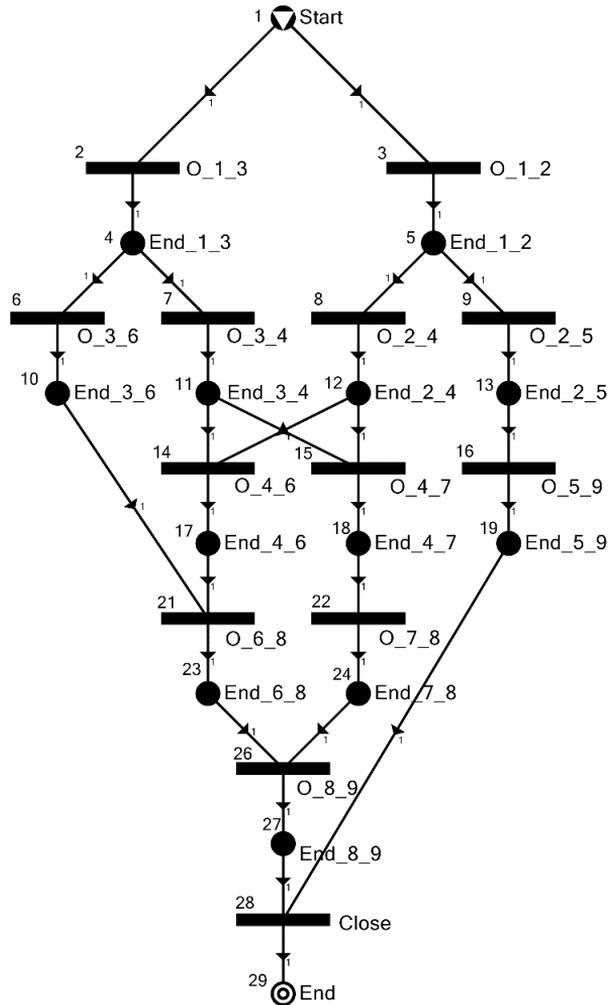CPM graph of the illustrative example

Figure 2b
Process network of the illustrative example

2 Alternatives

It is important to note that real case examples raise the question of alternatives. In case a given problem can be solved by performing more than one activity or more than one series of activities, then it is called a problem of alternatives. This situation is not handled by CPM, moreover, crucial decisions in this regards have to be made prior to the depiction of the CPM graph. Nevertheless, these decisions may fundamentally influence the overall duration of the final result of the CPM solution. Obviously, it would be of high importance not to exclude any possibilities at the beginning, but to have these decisions as a result of a solution process as well. The transformation presented in this paper gives the possibility of making these decisions later on. Alternatives can be added into the process network and thus CPM problems extended with alternatives can be considered within this framework. In the process network representation of the problem these are considered as alternative arcs. This can be easily illustrated in the process network by adding a parallel operating unit for the alternative group, see Figure 3.
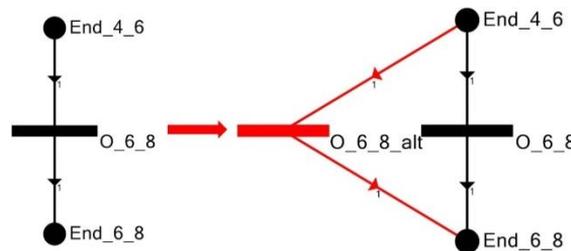
Figure 3
Illustration of adding an alternative

Based on the mapping of the basic elements and logical connections of the CPM and process network representations described earlier in this paper, it is obvious that the problem of alternative arcs and alternative paths can be described as alternative operating units of the process network. As a result, project processes can be illustrated in more details in the process network representation than in CPM graphs. A given activity is performed by one operating unit or another, or one phase of the work is performed by one operating unit and another phase by another operating unit. This can be imagined for example when during a production process a decision maker controls the alternative operating units (arcs), namely which operating unit should operate during the work process.

In the process network representation, this means information additional to the structural representation. Until this point structural construction was performed, in other words from the raw materials and set of operating units the production process was generated to produce the desired products with the help of additional intermediate materials. Here, a decision maker may set up priorities and may decide between alternatives. Normally, this information is represented in the cost function of the operating units.

All in all, based on the mapping described previously, the CPM graph was transformed into a process network and then alternative solutions were also added into the process network representation, see alternative operating units O_6_8_alt and O_8_9_alt in Figure 4. Please note that the input and output materials of the added alternative operating units are identical to that of the original operating units.
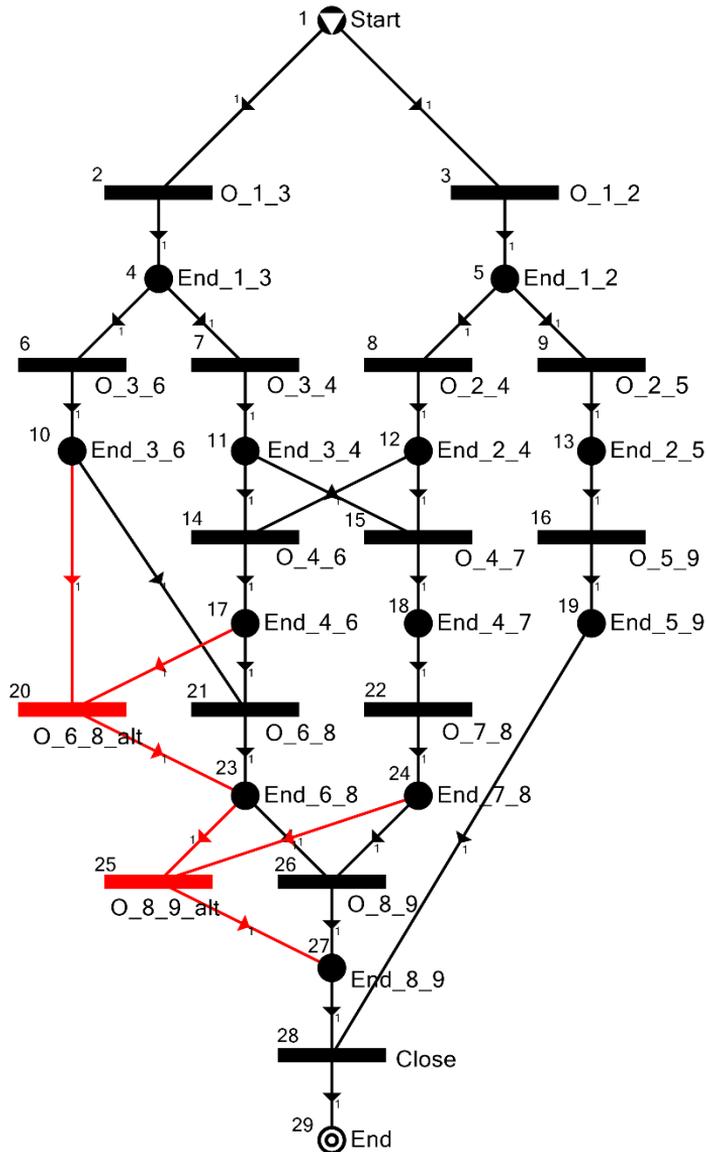

Figure 4

Illustrative example with alternatives

The axioms of process networks (see Friedler et al. 1992a) determine a solution structure within the process network. According to the terminology of the CPM, the axioms of process networks have to be extended with the following: each event in any solution structure, which is represented by a material in the process network, has one and only one input arc, except the Start event, which has zero indegree. In other words, it is also important to lock out from the solution point of view the parallel alternatives. Therefore, it has to be stated that from every operating unit there exists one and only one path to the final product (which is the end of the project in case of the CPM model).

In this regards, the solution structures of the process network now correspond to the CPM graph. Therefore, adding the alternatives within the process network, multiple CPM graphs are described for the original problem. As a result, the optimal solution with a given set of constraints of the original problem is generated from the mathematical programming model of the process network with alternatives. The proposed solution method considering the alternatives is also given in Figure 5.
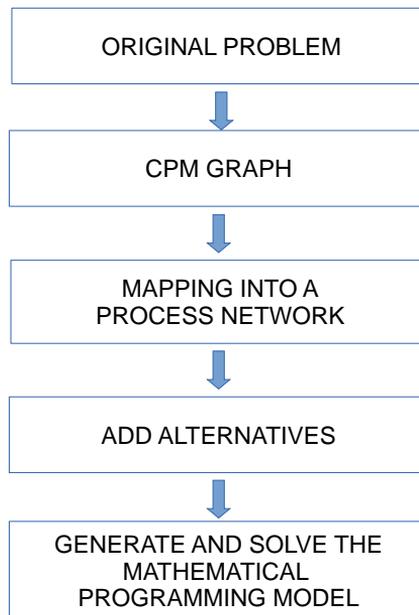

Figure 5
Proposed solution method

3   Mathematical Programming Model

Let A, E, V be finite sets, where A denotes the set of Activities, E denotes the set of Events and V denotes the set of vertices. Let G be the bipartite process network as follows.

$$G(A,E,V);\; A \cap E = \varnothing,\; V \subseteq \left(A \times E\right) \cup \left(E \times A\right) \; bipartite\; graph$$

$$A = \left\{ i \in N \right\} \quad activities$$

$$E = \left\{ j \in N \right\} \quad events$$

As an illustration, the above formula means in Figure 4 the followings:

$$A = \{2,3,6,7,8,9,14,15,16,20,21,22,25,26,28\}$$

$$E = \{1,4,5,10,11,12,13,17,18,19,23,24,27,29\}$$

$$V = \{(1,2),(1,3),(2,4),(3,5),\ldots\}$$

Let us denote x_i the i-th activity in the CPM graph and operating unit in the process network, where

$$x_i = \begin{cases} 0 & \text{the } i\text{-th activity is not performed} \\ 1 & \text{the } i\text{-th activity is performed} \end{cases}$$

t_i = the time from start-up to the i-th event occurs.

T_i = the duration of the i-th activity.

T = planned upper time limit for the total project.

C_i = cost of the i-th activity.

C = planned upper budget for the total project.

$x_{Close} = 1$	$Close \in A$	(1)
$\displaystyle\sum_{\{i:\, i \in A \text{ and } (i,j) \in V\}} x_i = 1$	$\forall j \in E \text{ and } j \neq Start$	(2)

Line (1) refers to the fact that the project has to be finished; line (2) refers to that only one alternative can be considered. Let $\left\{ b_1, b_2, \ldots, b_n \right\}$ be the solution of the above equations and let $S = \left\{ i : i \in A \text{ and } b_i = 1 \right\}$ be a subset of A. The S set of nodes and the corresponding edges designates a part of $G(A, E, V)$ process graph, which exactly specify a CPM graph.

As an illustration, all CPM graphs of the illustrative example depicted in Figure 4 are as follows:

CPM graph 1: $b_i = \begin{cases} 0 & if\ i = 20, and\ i = 25 \\ 1 & otherwise \end{cases}$ where $i \in A$;

CPM graph 2: $b_i = \begin{cases} 0 & if\ i = 21, and\ i = 26 \\ 1 & otherwise \end{cases}$ where $i \in A$;

CPM graph 3: $b_i = \begin{cases} 0 & if\ i = 20, and\ i = 26 \\ 1 & otherwise \end{cases}$ where $i \in A$;

CPM graph 4: $b_i = \begin{cases} 0 & if\ i = 21, and\ i = 25 \\ 1 & otherwise \end{cases}$ where $i \in A$.

3.1 Mathematical Programming Model of the Time Optimal Project Plan

The mathematical programming model for the usual CPM problem, extended with alternatives is given below.

$$x_{Close} = 1 \qquad\qquad Close \in A \qquad\qquad (3)$$

$$\sum_{\{i:\,i\in A\ and\ (i,j)\in V\}} x_i = 1 \qquad \forall j \in E\ and\ j \neq Start \qquad (4)$$

$$t_{Start} = 0 \qquad\qquad Start \in E \qquad\qquad (5)$$

$$t_k + \sum_{\{i:\,i\in A\ and\ (i,j)\in V\}} x_i T_i \le t_j \quad \forall k, j \in E \setminus Start\ where\ \exists i\ (k,i)\ and\ (i,j) \in V \quad (6)$$

$$\overline{t_{End} \to \min} \qquad\qquad\qquad (7)$$

Please note that lines (3) and (4) are equal to lines (1) and (2) above; while line (4) refers to that only one alternative can be considered, line (5) refers to the fact that the project started at time zero, line (6) indicates that the activities have been completed at the earliest time. Line (7) refers to the aim, which is to minimize the overall duration of the project.

Please also note that the solution of this mathematical programming model equals to the solution in which the shortest duration time is chosen among the alternatives.

3.2   Mathematical Programming Model of the Cost Optimal Project Plan

The mathematical programming model for the resource allocation problem of the given CPM graph, extended with alternatives is given below:

$$x_{Close} = 1 \qquad\qquad Close \in E \tag{8}$$

$$\sum_{\{i:\, i \in A \ and \ (i,j) \in V\}} x_i = 1 \qquad \forall j \in E \ and \ j \neq Start \tag{9}$$

$$\overline{\sum_{\{i:\, i \in E\}} x_i C_i} \rightarrow \min \tag{10}$$

Please note that lines (8) and (9) are equal to lines (1) and (2) above. Line (10) refers to the aim, which is to minimize the overall cost of the project.

3.3   Mathematical Programming Model of the Time Optimal Project Plan with Additional Cost Constraint

In real case examples, not only the time constraints are important for the project planning but also the different costs of the alternatives are also of importance. Thus the previously described mathematical programming model can be reformulated as follows:

$$x_{Close} = 1 \qquad\qquad Close \in E \tag{11}$$

$$\sum_{\{i:\, i \in A \ and \ (i,j) \in V\}} x_i = 1 \quad \forall j \in E \ and \ j \neq Start \tag{12}$$

$$t_{Start} = 0 \qquad\qquad Start \in E \tag{13}$$

$$t_j \geq x_i T_i + t_k \qquad\qquad \forall j \in E \setminus Start \ and \ \forall i : (i,j) \in V \ and \ \forall k : (k,i) \in V \tag{14}$$

$$\sum_{\{i:\, i \in A\}} x_i C_i \leq C \tag{15}$$

$$\overline{t_{End}} \rightarrow \min \tag{16}$$

Please note that lines (11) and (12) are equal to lines (1) and (2) above; while line (13) refers to the fact that the project started at time zero, line (14) indicates that the activities have been completed at the earliest time, while line (15) refers to the fact that the total project cost should not exceed the given upper budget limit. Line (16) refers to the aim, which is to minimize the overall duration of the project.

3.4 Mathematical Programming Model of the Cost Optimal Project Plan with Time Constraint

Finally, the mathematical programming model of the cost optimal project plan with time constraint can be given as follows:

$$x_{Close} = 1 \qquad\qquad Close \in A \tag{17}$$

$$\sum_{\{i:\, i \in A \; and \; (i,j) \in V\}} x_i = 1 \qquad \forall j \in E \; and \; j \neq Start \tag{18}$$

$$t_{Start} = 0 \qquad\qquad Start \in E \tag{19}$$

$$x_i T_i + t_k \leq t_j \qquad \forall j \in E \setminus Start \; and \; \forall i : (i,j) \in V \; and \; \forall k : (k,i) \in V \tag{20}$$

$$t_{End} \leq T \qquad\qquad End \in E \tag{21}$$

$$\overline{\sum_{\{i:\, i \in A\}} x_i C_i} \rightarrow \min \tag{22}$$

Please note that lines (17) and (18) are equal to lines (1) and (2) above; while line (19) refers to the fact that the project started at time zero, line (20) indicates that the activities have been completed at the earliest time, while line (21) refers to the fact that the total project duration should not exceed the given upper time limit. Line (22) refers to the aim, which is to minimize the overall cost of the project.

4 Illustrative Example

The illustrative example from Chanas S. and P. Zielinski (2001) is revisited below, for details please consider the process network given in Figure 4. Please note that the given fuzzy times of the example were first transformed to triangular fuzzy times and then they were defuzzificated to the crisp times below. Thus, the triangular fuzzy time and the defuzzificated crisp times are indicated in Table 3.

Table 3
Duration of the i-th activity

Triangular fuzzy times (CPM)	Defuzzificated crisp times (process network)
T_{12}=(1,25;1;1)	T_3=1,25
T_{13}=(2,5;1;1)	T_2=2,5
T_{24}=(0;0;0)	T_8=0
T_{25}=(2,5;1;1)	T_9=2,5
T_{34}=(0;0;0)	T_7=0
T_{36}=(6,5;1;1)	T_6=6,5

T_{46}=(5;1;1)	T_{14}=5
T_{47}=(9;1;1)	T_{15}=9
T_{59}=(8,5;2;2)	T_{16}=8,5
T_{68}=(4;2;2)	T_{21}=4
T_{78}=(3,5;1;1)	T_{22}=3,5
T_{89}=(7,5;2;2)	T_{26}=7,5

The mathematical programming model of the time optimal project plan with additional cost constraint given above in line (11) – (16) was formulated. Let us consider the following costs of the activities given as follows:

$$\left\{\begin{array}{l} C_2 = 5, C_3 = 5, C_6 = 5, C_7 = 5, C_8 = 5, \\ C_9 = 5, C_{14} = 5, C_{15} = 5, C_{16} = 5, \\ C_{21} = 10, C_{22} = 5, C_{26} = 10, C_{28} = 5 \end{array}\right\} \;;$$

with the duration of the alternative activities: T_{20}=8; T_{25}=9,5; and the costs of the alternative activities: C_{20}=5; C_{25}=5. Let us consider the planned upper budget for the total project $C = 80$. Thus, the solution of the mathematical programming model results in the overall project duration is 22,5; and in this optimal solution, nodes 21 and 26 are selected, and the alternatives are excluded.

Should the planned upper budget for the total project be $C = 70$; then the solution of the mathematical programming model results in the overall project duration 24,5; while in this optimal solution, alternative node 20 is selected instead of the original 21, and the alternative node 25 is excluded.

Conclusions

The critical path method (CPM) gives the longest path of the planned activities together with its overall duration. In each case only time is considered and only one solution is found at a time. Resources together with their costs do not appear in CPM graphs.

In the present paper a novel method to extend the problem range of CPM problems is given. First, it was illustrated how a CPM problem should be transformed into a process network problem. This mapping has no literature antecedents since this situation is not handled in the CPM graph, moreover there is no CPM and process network connection yet. This transformation serves as the key to commonly handle resources together with their costs already within the considered process network. Since real case examples raise the question of alternatives, namely when more than one activity or more than one series of activities are considered for a subtask, this can also be represented in the process network.

CPM methods order the resources to the activities only without any influence to the CPM graph. When another resource is considered to be used to the same activity then in most cases all the parameters have to be reset and a new graph has

to be depicted, and thus the problem has to be solved again. This means a large number of problems to be solved separately, and the parameters considered for each separate problem are independent from the other separate problems' parameters. However, these parameters are dependent on each other in real case problems. Moreover, CPM graph techniques do not handle at all such cases where a given problem can be solved by performing more than one activities, i.e. alternatives. In other words crucial decisions regarding alternatives have to be made before the CPM graph is depicted. In the present work as a novel solution alternatives can be added and are handled within the given model. Moreover, when alternatives are added to the process network all resources together with their parameters depending on each other are considered and handled within the model, as in the case of real case problems.

After describing the extended problem with alternatives, four different mathematical programming models are given in the present work. These mathematical programming models cover a wider range of problems than that of the CPM methodology. These mathematical programming models can be generated and solved algorithmically. The solution of the mathematical programming model is the optimal solution of the original CPM problem, i.e. the exact project definition, solution. Another novelty of the present work is that all solutions of the original CPM problem can be generated and ranked in order according to the cost function.

Acknowledgement

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