

Torque Ripple Calculation of the Two-phase Permanent Magnet Synchronous Motor Supplied by a Triac Converter

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Abstract: A synchronous motor with ferrite permanent magnet rotor is a good solution for small pump applications. It also has some drawbacks. The most important of them seems to be its inability to start directly on the mains, Permanent magnet motor has to be equipped with an electronic circuit for direct starting, which increases motor price. Another drawback is the torque ripple for the non-harmonic supply. This paper shows analytical calculation of the torque ripple of the small permanent magnet motor, which does a triac converter supply. The converter forms two-phase supply voltage of 25 Hz frequency, from one-phase 50Hz mains.

Keyword: torque ripple, synchronous motor, permanent magnet, triac converter

1 Introduction

Small water pump having rated power about 100W have a large application in automobile industry, central heating and house appliances (washing machines, dishwasher). They are run either by single-phase ac motor, when operated from the mains or by a dc motor in the case in an automobile.

Both, motor and pump are often manufactured as a single piece of equipment. A separated water pump in appliances tends to leak water between the rotating shaft and casing. In an integral motor pump a stainless-steel cylinder has to be

inserted into the air-gap to protect the stator from water, and the whole rotor body is inserted into another stainless steel cylinder, to protect the rotor cage and lamination. The stator and rotor cylinders cause additional losses and significantly deteriorate motor performance.

Water leakage problems and low efficiency of an induction motor are the main reasons why another concept of the water pump was developed, in which the rotor of the motor is immersed into pumping water. In this solution the rotor of the motor is exposed to chemically aggressive water. A squirrel-cage rotor cannot be used, but a ferrite permanent magnet rotor seems to be a good choice. Strontium-ferrite permanent magnets are chemically inert, which make them suitable for applications in aggressive environments. Strontium-ferrite magnets have high specific electric resistance, so they do not experience thermal problems due to eddy-current losses. Their low residual flux density imposes the need for special machine construction when high air-gap flux density is needed.

2 Motor Configuration

Figure 1 shows the sketch of the construction disposition of a two-phase synchronous motor having permanent magnets on the rotor. The stator structure is similar to that of a two-phase salient poles reluctance motor. The stator magnetic circuit is built from the laminations. The rotor form the two-pole cylinder permanent magnet. The Strontium-Ferrite magnet with linear demagnetisation characteristic and remanence 0.5 T was used.

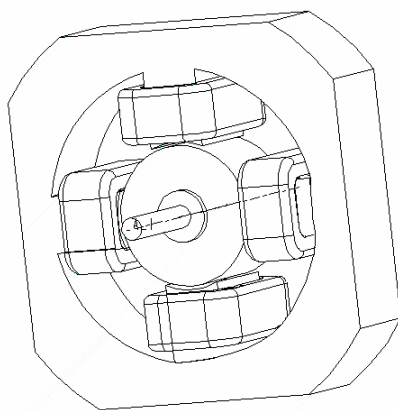


Figure 1
Two-phase permanent magnet motor

The stator windings are of particularly simple form. Two opposite placed winding form one phase. The stator windings can be configured to either a serial or parallel two-phase system. Normally the windings are identical. The windings which forms one phase of the motor are connected like shown on Fig. 1. Two diametrically opposite stator poles are of opposite magnetic polarity.

The electromagnetic torque is mainly developed due to the interaction between stator winding current and rotor permanent magnet flux.

Electromagnetic torque can be calculated with the two corresponding temporal quantities that are phase current and back electromotive force (emf), or with the related spatial quantities that are stator magnetomotive force (mmf) and the rotor induction.

Finite-element analysis was employed for a basic design approach predicting static performance of the motor. The wave of induced emf in stator coil windings was numerically calculated from the wave of the flux. The calculation results that the waveform of the emf can be mathematically substituted by a sinusoidal function.

3 Mathematical Model

The analytical description of two-phase synchronous motor with permanent magnet rotor is simpler than that of three-phase, due to the fact that stator windings of such two-phase motors are physically orthogonal and thus magnetically decoupled. The mathematical description bears strong resemblance to its single-phase counterpart.

Figure 2 shows the per-phase equivalent circuit of the machine.

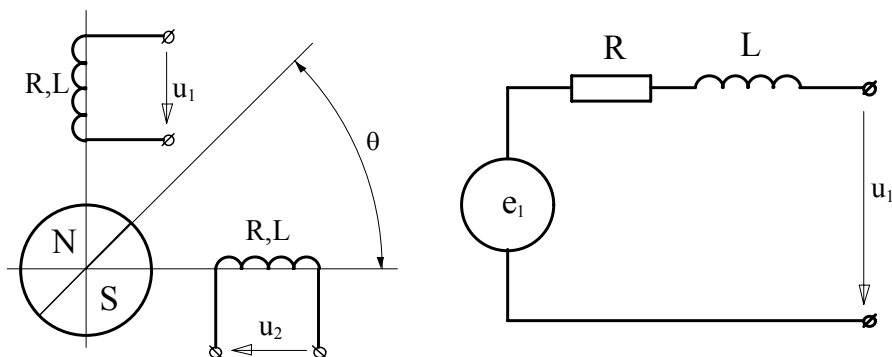


Figure 2
Equivalent circuit

Let us assume that the reluctance torque is negligible. This one depends on the air-gap between the poles. In accordance with the stator cross section shown in the Figure 1, the reluctance torque would exhibit four-maximums and four-minimums per complete rotation.

Assuming that all windings are identical and the magnetic circuit is symmetrical.

Instantaneous value of electrical input power is determined as [1],[2],[3]:

$$p = u_1 i_1 + u_2 i_2 \quad (1)$$

It consists of three parts:

$$p = p_j + p_m + p_e \quad (2)$$

$$p_j = R(i_1^2 + i_2^2) \quad \text{represents the copper losses in the stator coil;}$$

$$p_m = L \left(i_1 \frac{di_1}{dt} + i_2 \frac{di_2}{dt} \right) \quad \text{represents the magnetic reactive power;}$$

$$p_e = e_1 i_1 + e_2 i_2 \quad \text{represents the electrical output-power.}$$

Where: R - is the armature resistance; L - is the synchronous inductance; e_1, e_2 - phase electromotive forces;

The product of the torque and the speed gives the electrical output power of the machine:

$$p_e = m \cdot \omega \quad (3)$$

Where: m - instantaneous value of the torque, ω - speed of the machine.

To determine the waveform of the torque of the machine, it's necessary to determine the waveform of the phase currents. These one can be calculated from the voltage equations [4]

$$\begin{aligned} u_1 &= R i_1 + L \frac{di_1}{dt} + e_1 \\ u_2 &= R i_2 + L \frac{di_2}{dt} + e_2 \end{aligned} \quad (4)$$

Assume that machine speed is constant at steady state, solution will be simplified by replacing the time by the angle.

$$\begin{aligned} u_1 &= R i_1 + L \omega \frac{di_1}{d\theta} + e_1 \\ u_2 &= R i_2 + L \omega \frac{di_2}{d\theta} + e_2 \end{aligned} \quad (5)$$

Where: θ is angle of position of the rotor.

The phase-torque is proportional to a product of the phase current and phase emf, and disproportional to the rotor speed.

$$\begin{aligned} m_1 &= \frac{i_1 e_1}{\omega} \\ m_2 &= \frac{i_2 e_2}{\omega} \end{aligned} \quad (6)$$

Total motor torque is given by sum of the phase-torque.

$$m = m_1 + m_2 \quad (7)$$

The electromotive force induced in the stator coils can be expressed as a sinusoidal or co-sinusoidal function, which is lagged by the feeding voltage by angle γ . Angle γ depends from the torque of the machine.

$$\begin{aligned} e_1 &= E \cdot \sin(\theta - \gamma) \\ e_2 &= E \cdot \cos(\theta - \gamma) \end{aligned} \quad (8)$$

Where: E - is the maximal value of the induced electromotive force.

4 Converter Configuration

To form two-phase voltage supply system, was used the triac converter. The converter consists of the double secondary winding supply transformer set by two triacs on each of the phases. The triacs are controlled by a micro-controller like that way to constitute two phase voltage on the input motor terminal. The micro-controller must be on the mains voltage synchronised. The output waveform for each of the phases is given on the Figure 4.

The converter output voltage can be mathematically expressed by Fourier series as follows:

$$\begin{aligned} u_1 &= \frac{8U_m}{\pi} \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{4 - (2k+1)^2} \sin[(2k+1)\omega t] \\ u_2 &= \frac{8U_m}{\pi} \sum_{k=1}^{\infty} \frac{1}{4 - (2k-1)^2} \cos[(2k+1)\omega t] \end{aligned} \quad (9)$$

Where: U_m is a maximal value of the mains voltage; k is a positive integer with $k = 1, 2, 3, \dots$

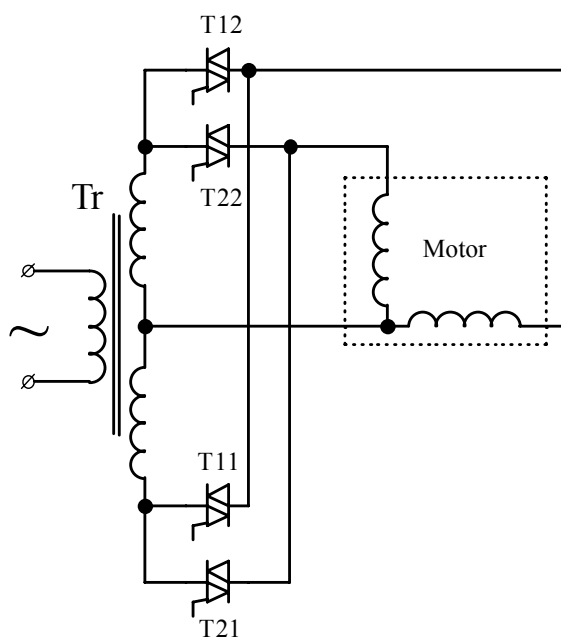


Figure 3
Triac converter circuit

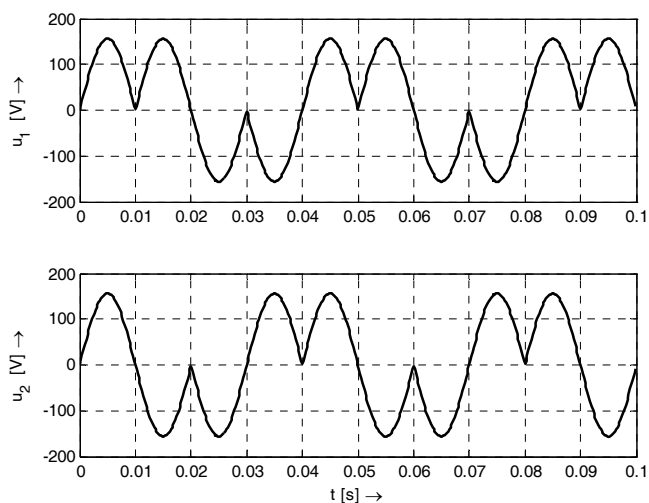


Figure 4
The converter output voltages waveform

The angle of rotor position θ is proportional to time t by expression:

$$\theta = \omega t \quad (10)$$

5 Analytical Solution of the Voltage Equation

To predict the phase currents waveform, it's necessary to solve the voltage differential equations (5). Substituting (8) and (9) into (5) we obtain:

$$\begin{aligned} \frac{8U_m}{\pi} \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{4 - (2k+1)^2} \sin[(2k+1)\theta] &= Ri_1 + L\omega \frac{di_1}{d\theta} + E \sin(\theta - \gamma) \\ \frac{8U_m}{\pi} \sum_{k=1}^{\infty} \frac{1}{4 - (2k-1)^2} \cos[(2k+1)\theta] &= Ri_2 + L\omega \frac{di_2}{d\theta} + E \sin(\theta - \gamma) \end{aligned} \quad (11)$$

Presented linear differential equations have an analytical solution of the form:

$$\begin{aligned} i_1 &= \frac{8U_m}{\pi\omega L} \sum_{k=1}^{\infty} \left\{ \frac{(-1)^{k+1}}{4 - (2k-1)^2} \cdot \frac{(\omega L)^2}{R^2 + [\omega L(2k-1)]^2} \left[\frac{R}{\omega L} \sin[(2k-1)\theta] - (2k-1) \cos[(2k-1)\theta] \right] \right\} + \\ &\quad \frac{ER}{R^2 + (\omega L)^2} \cos(\theta - \gamma) + \frac{E\omega L}{R^2 + (\omega L)^2} \sin(\theta - \gamma) + C.e^{-\frac{R}{\omega L}\theta} \\ i_2 &= \frac{8U_m}{\pi\omega L} \sum_{k=1}^{\infty} \left\{ \frac{1}{4 - (2k-1)^2} \cdot \frac{(\omega L)^2}{R^2 + [\omega L(2k-1)]^2} \left[\frac{R}{\omega L} \cos[(2k-1)\theta] + (2k-1) \sin[(2k-1)\theta] \right] \right\} - \\ &\quad \frac{ER}{R^2 + (\omega L)^2} \sin(\theta - \gamma) + \frac{E\omega L}{R^2 + (\omega L)^2} \cos(\theta - \gamma) + C.e^{-\frac{R}{\omega L}\theta} \end{aligned} \quad (12)$$

For steady state: $\theta \rightarrow \infty$, $C.e^{-\frac{R}{\omega L}\theta} \rightarrow 0$;

6 Current and Torque Calculation

To calculate the currents and torque waveform, the following parameters were used:

- terminal supply voltage $U = 110 \text{ V}$;
- resistance of the phase coil: $R = 17.1 \Omega$;
- inductance of the phase coil: $L = 0,536 \text{ H}$

- stator induced voltage: $e = 45\text{ V}$ for the speed of $\omega = 157\text{ rad/s}$;

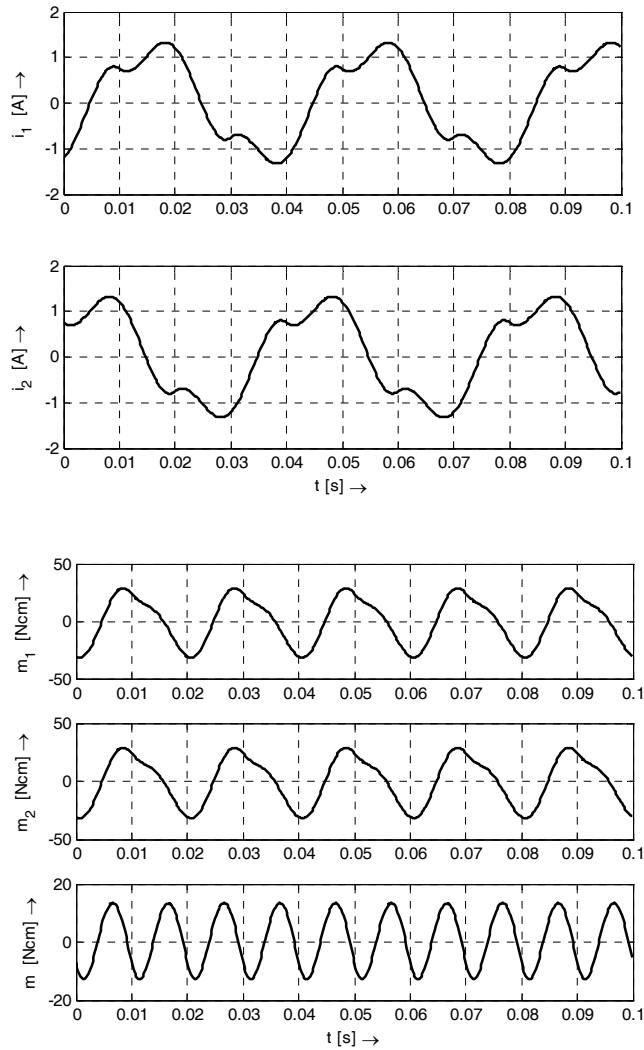


Figure 5

Plot of the currents and torque for no loaded machine

Figure 5 shows instantaneous values of the phase currents of the machine for steady state and no loaded machine. The speed of the machine is 157 rad/s .

There is shown of per-phase and total torque waveforms too. The torque ripple with amplitude about 15 *Ncm* is present in plot of total torque.

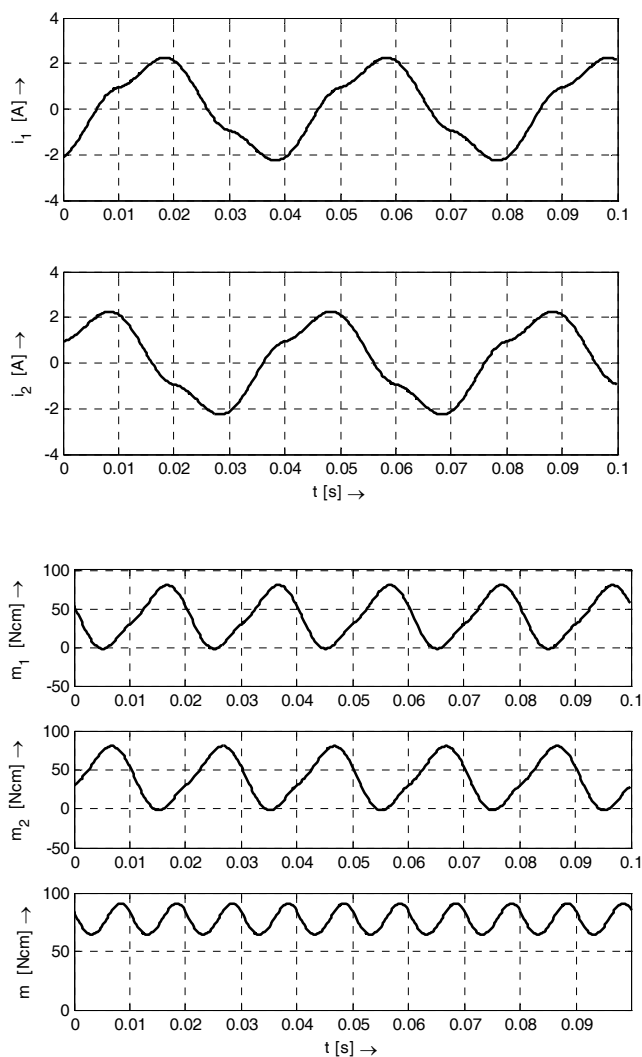


Figure 6

Plot of the currents and torque for loaded machine

Figure 6 shows instantaneous values of the phase currents of the machine for steady state and loaded machine. The machine is loaded by 80 *Ncm* torque. The second waveform of the Fig. 6 shows per-phase and total torque waveforms.

Conclusions

Steady-state performance of the permanent magnet synchronous motor, supplied by triac converter is shown. Equations, which enable to predict steady-state characteristics were developed. It was shown that the constant torque ripple in waveform of electromagnetic torque of the machine is present.

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