Nucleation of a Crack under Inner Compression of Cylindrical Bodies

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Abstract: The problem of fracture mechanics of crack nucleation in plunger pair bushing is considered. It is assumed that under the repeated reciprocating motion of a plunger there happens crack nucleation and a failure of materials of pair elements. Crack nucleuses are simulated by a bridged prefracture zone that is considered as areas of weakened interparticle bonds of the material. It is assumed that the inner boundary of the bushing is close to the annular one and has rough surfaces.

Keywords: contact pair; nucleation of a crack; bonds between surfaces; prefracture zone; cohesive forces; rough surfaces

1 Introduction

The bushing-plunger friction pair operates in conditions of a complex stress state. Experience in using a plunger pair shows that the initiation of cracks and the fracture of the materials of the components of the friction pair occur during repeated reciprocating motion. To control the friction and wear processes in the friction pair, the investigation of material fracture and the friction caused by contact interaction and accompanied by the joint action of contact pressure and friction force are necessary. It is therefore necessary in the planning stage of the construction of sliding pairs to take into account the possibility of the occurrence of cracks and to carry out a limit analysis of the components of the contact pair. It should be taken into account that the bushing internal contour and the plunger external contour are nearly circular. As is known, real treated surfaces are never absolutely smooth but always have micro- or macroscopic irregularities (of a technological character) forming the rough surface. Despite the extremely small sizes of such irregularities, they affect the different service properties of tribo-conjugation [1, 2].

2 Formulation of the Problem

The contact deformation of cylindrical bodies of close radii under inner compression is considered. It is assumed that the surfaces of the bodies in the contact area are rough.

We assume that the outer cylinder (bushing) is an unrestricted plate with a hole close to circular, into which is inserted elastic cylinder (shaft). A concentrated force *P* is pressing into the hole's boundary and concentrated pair whose moment is determined from the cylinder's limit equilibrium condition under the action of Coulomb friction forces is applied to the center of the shaft (Fig. 1).

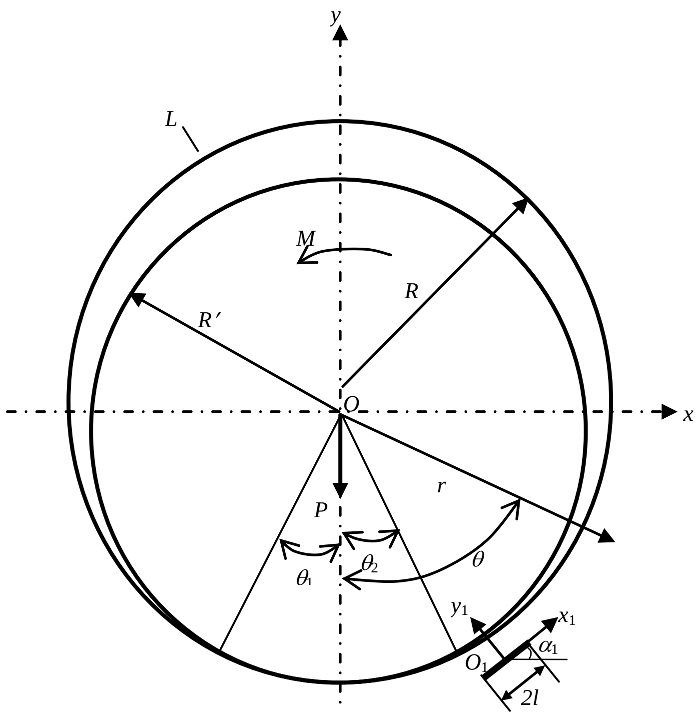

Figure 1
Computational diagram of a problem of the contact fracture mechanics

To determine the contact pressure, it is necessary to consider [3, 4] the contact problem of the pressing of a plunger into the surface of the bushing involving wear.

Let some unknown part shaft with mechanical characteristics G_1 and μ_1 be retained against the internal surface of the bushing with mechanical characteristics G (shear modulus) and μ (Poisson ratio). The condition relating the displacements of the bushing and the plunger is written in the form [3, 4].

$$v_1 + v_2 = \delta(\theta) \qquad \theta_1 \le \theta \le \theta_2 \tag{1}$$

Here $\delta(\theta)$ is sag of the point on the surface of the bushing and the plunger, which is determined by the form of the inner surface of the bushing and the plunger surface, and, also by the magnitude of the pressing force P. $\theta_2 - \theta_1$ is the magnitude of the contact angle area.

In the contact area, in addition to the contact pressure, there is a tangential stress $\tau_{r\theta}$ which is related to the contact pressure $p(\theta,t)$ by the Coulomb law

$$\tau_{r\theta}(\theta,t) = fp(\theta,t) \tag{2}$$

where f is the coefficient of friction of the "bushing-plunger" pair.

We refer the bushing of the contact pair to the polar system of coordinates $r\theta$; for that we choose an origin of coordinates at the center of circle L of radius R.

We will assume that the inner contour of the bushing and the external contour of the plunger are close to annular one.

Represent the boundary L' of inner contour of the bushing in the form

$$r = \rho(\theta), \quad \rho(\theta) = R + \varepsilon H(\theta)$$

where $\varepsilon = R_{\max}/R$ is a small parameter; $R_{\max}$ is the greatest height of the bulge (cavity) of the unevenness of the friction surface.

The coefficients of the Fourier series for the function $H(\theta)$:

$$H(\theta) = \sum_{k=0}^{n} (a_k^0 \cos k\theta + b_k^0 \sin k\theta)$$

are found by means of a profilogram of the treated surface of the bushing which describes each inner profile of the bushing. In a similar way, the plunger contour may be represented as

$$\rho_1(\theta) = R' + \varepsilon H_1(\theta), \qquad H_1(\theta) = \sum_{k=0}^{n} (a_k^1 \cos k\theta + b_k^1 \sin k\theta)$$

It is assumed that the bushing and plunger wear is of an abrasive character. For displacements of the points of surface of the bushing we have

$$v_1 = v_{1e} + v_{1r} + v_{1w} \tag{3}$$

where v_{1e} are elastic displacements of bushing's contact surface; v_{1r}, v_{1w} are displacements caused by the removal of the micro-bulges and by the bushing surface wear, respectively.

Similarly, for displacements of plunger's contact surface we have

$$v_2 = v_{2e} + v_{2r} + v_{2w} \tag{4}$$

The rate of change of the displacements of the surface in the course of the bushing and the plunger wear will be [4]

$$\frac{dv_{ju}}{dt} = K^{(j)} p(\theta, t) \qquad (j=1,2) \tag{5}$$

where $K^{(j)}$ is the wear coefficient of the bushing and the plunger material (j=1,2), respectively.

Prefracture zones will arise in proportion to the loading of the bushing during the operation of the friction pair with force load, and these zones are modelled as domains in which the interparticle bonds in the material have been weakened. The interaction of the surfaces of these domains is modelled by the introduction of a prefracture zone of the bonds, which have a specified deformation pattern. The physical nature of these and the dimensions of the prefracture zone depend on the form the material. Since the above-mentioned zones (layers) are small compared with the remaining part of the bushing, they can be conceptually eliminated by replacing them with cuts, the surfaces of which interact with one another according to a certain law that the corresponds to the action of the material which has been removed. Taking account of these effects in fracture mechanics is an important but exceedingly difficult problem. In the case being investigated, the occurrence of a crack nucleus involves the transition of a prefracture zone into domain where there are ruptured bonds between the surfaces of the material.

Investigations [5-7] of occurrence domains with a disrupted structured of the material show that, during the initial stage, the prefracture zones have the form of a narrow elongated layer, and then, when the load is increased, a secondary system of zones suddenly appears, and these zones contain material with partially ruptured bonds.

We will assume that the prefracture zone is oriented in the direction of maximal tensile stresses arising in the bushing.

Let us consider the prefracture zone of length $2l$ allocated on the segment $|x_1| \le l, \quad y_1 = 0$. At the center of the prefracture zone, we located an origin of local system of coordinates $x_1O_1y_1$ whose axis x_1 coincides with the line of the zone and makes an angle with the axis x $(\theta = 0)$. The surfaces of the prefracture zone interact in such a way that this interaction (the bonds between the surfaces) restrains the formation of a crack.

For a mathematical description of the interaction of the surfaces prefracture zones, it is assumed that between them for which the law of deformation is known. Under the action of external loads on the bushing, normal $q_{y_1}(x_1)$ and tangential $q_{x_1y_1}(x_1)$ tractions will arise in the bonds joining the surfaces prefracture zones.

Consequently, the normal and tangential stresses numerically equal $q_{y_1}(x_1)$ and $q_{x_1y_1}(x_1)$ respectively, will be applied to the surfaces prefracture zones. The quantities of these stresses are not known beforehand and are to be determined when solving the boundary value problem of fracture mechanics.

For determining the displacements v_{1e} and v_{1r} it is necessary to solve the following problem of elasticity theory for a bushing

$$\sigma_n = -p(\theta)\,; \quad \tau_{nt} = -fp(\theta) \quad \text{for} \quad r = \rho \quad \text{in the contact area} \tag{6}$$

$$\sigma_n = 0\,; \quad \tau_{nt} = 0 \quad \text{for} \quad r = \rho \quad \text{out of the contact area}$$

on surfaces prefracture zone

$$\sigma_{y_1} = q_{y_1}(x_1)\,; \quad \tau_{x_1y_1} = q_{x_1y_1}(x_1) \quad \text{for} \quad |x_1| \le l\,, \tag{7}$$

n, *t* are natural coordinates; σ_n, σ_t and τ_{nt} are stress tensor components.

In a similar way, we state the problem of elasticity theory for determining displacements v_{2e} and v_{2r} of the contact surface of the plunger

$$\sigma_n = -p(\theta)\,; \quad \tau_{nt} = -fp(\theta) \quad \text{for} \quad r = \rho \quad \text{in the contact area} \tag{8}$$

$$\sigma_n = 0\,; \quad \tau_{nt} = 0 \quad \text{for} \quad r = \rho \quad \text{out of the contact area}$$

The magnitudes of θ_1 and θ_2, that is, of the ends of the segment over which the plunger and the bushing are in contact, are unknown. In order to determine them, we will use a condition which expresses the continuous fall of the pressure $p(\theta)$ to zero when then point θ falls outside the segment where the surface touch

$$p(\theta_1) = 0\,, \qquad p(\theta_2) = 0$$

The equations and conditions (1)-(8) have to be supplemented with a relation between the expansion of the prefracture zone and bond tractions. Without loss of generality, we will represent this relation in the form

$$(v^+ - v^-) - i(u^+ - u^-) = C(x_1, \sigma_1)\left[q_{y_1}(x_1) - iq_{x_1y_1}(x_1)\right], \qquad \sigma_1 = \sqrt{q_{y_1}^2 + q_{x_1y_1}^2} \tag{9}$$

Here the function $C(x_1, \sigma_1)$ may be considered as an effective compliance of the bonds, which depends on their tension; σ_1 is the modulus of the vector of the bond tractions; $(u^+ - u^-)$ is the tangential, $(v^+ - v^-)$ is the normal component of the expansion of the prefracture zone.

In order to determine the value of the external load (the contact pressure) at which a crack is initiated, it is necessary to supplement the formulation of the problem with a condition (criterion) for the appearance of a crack (the rupture of the interparticle bonds in the material). As such a condition, we will adopt the criterion for the critical expansion of the prefracture zone

$$\left|(v^{+}-v^{-})-i(u^{+}-u^{-})\right|=\delta_{cr}$$

where δ_{cr} is a characteristic of the fracture toughness of the bushing material.

The additional condition enables us to determine the parameters of the contact pair for which a crack appears in the bushing.

3 The Method of the Boundary-Value Problem Solution

Using the perturbation method, we find the boundary conditions at each approximation:

for zero approximation of the problem

$$\sigma_r^{(0)}=-p^{(0)}(\theta)\,;\quad \tau_{n\theta}^{(0)}=-fp^{(0)}(\theta)\quad \text{for } r{=}R \text{ in the contact area} \tag{10}$$

$$\sigma_r^{(0)}=0\,;\quad \tau_{n\theta}^{(0)}=0\quad \text{for } r{=}R \text{ out of the contact area}$$

on surfaces prefracture zone

$$\sigma_{y_1}^{(0)}=q_{y_1}^{(0)}(x_1)\,;\quad \tau_{x_1y_1}^{(0)}=q_{x_1y_1}^{(0)}(x_1)\quad \text{for } |x_1|\le l\,, \tag{11}$$

for the first approximation of the problem

$$\sigma_r^{(1)}=N-p^{(1)}(\theta)\,;\quad \tau_{n\theta}^{(1)}=T-fp^{(1)}(\theta)\quad \text{for } r{=}R \text{ in the contact area} \tag{12}$$

$$\sigma_r^{(1)}=N\,;\quad \tau_{n\theta}^{(1)}=T\quad \text{for } r{=}R \text{ out of the contact area}$$

on surfaces prefracture zone

$$\sigma_{y_1}^{(1)}=q_{y_1}^{(1)}(x_1)\,;\quad \tau_{x_1y_1}^{(1)}=q_{x_1y_1}^{(1)}(x_1)\quad \text{for } |x_1|\le l\,, \tag{13}$$

Here $$N=-H(\theta)\frac{\partial\sigma_r^{(0)}}{\partial r}+2\tau_{r\theta}^{(0)}\frac{1}{R}\frac{dH}{d\theta}\,;\quad \text{for } r{=}R$$

$$T=(\sigma_\theta^{(0)}-\sigma_r^{(0)})\frac{1}{R}\frac{dH}{d\theta}-H(\theta)\frac{\partial\sigma_r^{(0)}}{\partial r}$$

Similarly we can write the boundary conditions at each approximation for the plunger. Additional relation (9) accepts the following form:

at the zero approximation

$$(v^{(0)+}(x_1,0)-v^{(0)-}(x_1,0))-i(u^{(0)+}(x_1,0)-u^{(0)-}(x_1,0))= \tag{14}$$
$$=C(x_1,\sigma_1^{(0)})\left[q_{y_1}^{(0)}(x_1)-iq_{x_1y_1}^{(0)}(x_1)\right]$$

at the first approximation

$$(v^{(1)+}(x_1,0)-v^{(1)-}(x_1,0))-i(u^{(1)+}(x_1,0)-u^{(1)-}(x_1,0))= \tag{15}$$
$$=C(x_1,\sigma_1^{(1)})\left[q_{y_1}^{(1)}(x_1)-iq_{x_1y_1}^{(1)}(x_1)\right]$$

By means of the Kolosov-Muskheleshvili formulas [8], we write the boundary conditions of the problem at zero approximation (10)-(11) for complex potentials $\Phi^{(0)}(z)$ and $\Psi^{(0)}(z)$. On annular boundaries of the bushing they will be of the form

$$\Phi^{(0)}(z)+\overline{\Phi^{(0)}(z)}-e^{2i\theta}\left[\bar{z}\Phi^{(0)\prime}(z)+\Psi^{(0)}(z)\right]=X^{(0)}(\theta) \tag{16}$$

$$z=Re^{i\theta};\quad X^{(0)}(\theta)=\begin{cases}-(1-if)\,p^{(0)}(\theta) & \text{on the contact area}\\ 0 & \text{out the contact area}\end{cases}$$

Boundary conditions on the surfaces prefracture zone will be written as:

$$\Phi^{(0)}(z)+\overline{\Phi^{(0)}(z)}+\bar{t}\Phi^{(0)\prime}(t)+\Psi^{(0)}(t)=q_{y_1}^{(0)}+iq_{x_1y_1}^{(0)} \tag{17}$$

where *t* is affix of points of the surfaces prefracture zone.

We look for the potentials $\Phi^{(0)}(z)$, $\Psi^{(0)}(z)$, $\Phi_1^{(0)}(z)$, $\Psi_1^{(0)}(z)$, $\Phi_2^{(0)}(z)$, $\Psi_2^{(0)}(z)$ and in the form

$$\Phi^{(0)}(z)=\sum_{k=0}^{2}\Phi_k^{(0)}(z),\qquad \Psi^{(0)}(z)=\sum_{k=0}^{2}\Psi_k^{(0)}(z) \tag{18}$$

$$\Phi_1^{(0)}(z)=\frac{1}{2\pi}\int_{-l}^{l}\frac{g_k^0(t)dt}{t-z_1},\qquad \Psi_1^{(0)}(z)=\frac{1}{2\pi}e^{-2i\alpha}\int_{-l}^{l}\left[\frac{\overline{g_k^0(t)}}{t-z_1}-\frac{\overline{T}_k e^{i\alpha}}{(t-z_1)^2}g^0(t)\right]dt \tag{19}$$

$$T_1=te^{i\alpha}+z_1^0;\qquad z_1=e^{-i\alpha}(z-z_1^0)$$

$$\Phi_2^{(0)}(z)=\frac{1}{2\pi}\int_{-l}^{l}\left[\left(-\frac{1}{z}-\frac{\overline{T}_1}{z-\overline{T}_1}\right)e^{i\alpha}g^0(t)+\frac{1-T_1\overline{T}_1}{\overline{T}_1(1-z\overline{T}_1)^2}e^{-i\alpha}\overline{g^0(t)}\right]dt, \tag{20}$$

$$\Psi_2^{(0)}(z)=\frac{1}{2\pi z}\int_{-l}^{l}\left[\left(\frac{1}{zT_1}-\frac{2}{z^2}-\frac{\overline{T}_1}{z(1-z\overline{T}_1)}-\frac{\overline{T}_1^2}{(1-z\overline{T}_1)^2}\right)e^{i\alpha}g^0(t)+\right.$$
$$\left.+\left(-\frac{1}{1-zT_1}+\frac{1-T_1\overline{T}_1}{z\overline{T}_1(1-z\overline{T}_1)^2}-\frac{2(1-T_1\overline{T}_1)}{(1-z\overline{T}_1)^3}\right)e^{-i\alpha}\overline{g^0(t)}\right]dt$$

Here $g^0(t)$ is the required function, which characterizes the expansion of the prefracture zone.

For defining the potentials $\Phi_0^{(0)}(z)$ and $\Psi_0^{(0)}(z)$ we use the N. I. Muskheleshvili method [8]

$$\Phi_0^{(0)}(z) = -\frac{1}{2\pi z}\int_L \frac{X^{(0)}(\sigma)d\sigma}{\sigma - z}, \qquad \sigma = e^{i\theta} \tag{21}$$

$$\Psi_0^{(0)}(z) = \frac{1}{z^2}\Phi_0^{(0)}(z) + \frac{1}{z^2}\overline{\Phi}_0^{(0)}(z) - \frac{1}{z}\Phi_0^{(0)\prime}(z)$$

Satisfying the boundary condition on the surfaces prefracture zone by the functions (18)-(20), we find singular integral equation with respect to the function $g^0(x_1)$:

$$\int_{-l}^{l}\left[R(t,x_1)g^0(x_1) + S(t,x_1)\overline{g^0(x_1)}\right]dt = \pi\left[q_{y_1}^{(0)} - iq_{x_1y_1}^{(0)} + f^0(x_1)\right] \qquad |x_1| \le l \tag{22}$$

$$f^{(0)}(x_1) = -\left[\Phi_0^{(0)}(x_1) + \overline{\Phi_0^{(0)}(x_1)} + x_1\overline{\Phi_0^{(0)\prime}(x_1)} + \overline{\Psi_0^{(0)}(x_1)}\right]$$

To the singular integral equation for the inner prefracture zone at zero approximation, we should add equality

$$\int_{-l}^{l} g^{(0)}(t)dt = 0 \tag{23}$$

Using the procedure for converting to an algebraic form [10, 11], the singular integral equation (22) with condition (23) reduced to the system of M complex algebraic equations for determining M unknowns $g^{(0)}(t_m) = v^0(t_m) - iu^0(t_m)$ (m=1,2,…,M)

$$\frac{1}{M}\sum_{m=1}^{M} l\left[g^{(0)}(t_m)R(lt_m, lx_r) + \overline{g^{(0)}(t_m)}S(lt_m, lx_r)\right] = \tag{24}$$

$$= q_{y_1}^{(0)}(x_r) - iq_{x_1y_1}^{(0)}(x_r) + f^{(0)}(x_r)$$

$$\sum_{m=1}^{M} g^{(0)}(t_m) = 0, \qquad r=1,2,\dots,M-1$$

where $t_m = \cos\frac{2m-1}{2M}\pi$; $x_r = \cos\frac{\pi r}{M}$.

If in (24) we go over to complexly conjugated values, we get M algebraic equations more. The right hand side of (24) contains unknown values of the forces $q_{y_1}^{(0)}(x_r)$ and $q_{x_1y_1}^{(0)}(x_r)$ in bonds.

The additional relation (14) at zero approximation is the condition determining forces in the bonds arising on the surfaces prefracture zone

$$g^{(0)}(x_1)=\frac{2G}{i(i+k_b)}\frac{d}{dx_1}\left[C(x_1,\sigma_1^{(0)})(q_{y_1}^{(0)}(x_1)-iq_{x_1y_1}^{(0)}(x_1)\right] \tag{25}$$

where $k_b=3-4\mu$ for plane strain, $k_b=(3-\mu)/(1+\mu)$ for plane stress state.

For constructing the missing algebraic equations for finding the approximate values of the forces $q_{y_1}^{(0)}(x_r)$ and $q_{x_1y_1}^{(0)}(x_r)$ at the nodal points, we require the conditions (25) to be fulfilled at the nodal points. For that, we use the finite differences method.

We need two complex equations determining the dimensions of the prefracture zone for closeness of the obtained system. Writing the stress finiteness conditions, we find two missing equations more in the following form:

$$\sum_{m=1}^{M}(-1)^m g^{(0)}(t_m)\cot\frac{2m-1}{4M}\pi=0 \tag{26}$$

$$\sum_{m=1}^{M}(-1)^{M+m} g^{(0)}(t_m)\tan\frac{2m-1}{4M}\pi=0$$

By means of complex potentials (18)-(20) and the Kolosov-Muskheleshvili formulae [8] and integration of the kinetic equation (5) wear of bushing's material at zero approximation, we find the displacements $v_1^{(0)}$ of the bushing's contact surface. In a similar way, we find the solution of the elasticity theory problem for the shaft in the first approximation. Using the solution and kinetic equation of shaft's material wear at zero approximation, we find the displacements $v_2^{(0)}$ of the shaft's contact surface.

We substitute the found quantities $v_1^{(0)}$ and $v_2^{(0)}$ into the basic contact equation (1) at zero approximation

$$p^{(0)}(\theta,t)=p_0^0(\theta)+tp_1^0(\theta)+\ldots; \tag{27}$$

$$p_0^0(\theta)=\sum_{k=0}^{\infty}(\alpha_k^0\cos k\theta+\beta_k^0\sin k\theta),$$

$$p_1^0(\theta)=\sum_{k=0}^{\infty}(\alpha_k^1\cos k\theta+\beta_k^1\sin k\theta)$$

For the algebraization of the basic contact equation, the unknown functions of the contact pressure at zero approximation are found in the form of expansions. Substituting the relation in the basic contact equation at zero approximation, we get the functional equations for the sequential determination of $p_0^0(\theta)$, $p_1^0(\theta)$, etc. For constructing the algebraic system for finding α_k, β_k, we equate the coefficients for the same trigonometric functions in the left and right hand sides of the functional of the contact problem. We get an infinite algebraic system with respect to α_k^0 (k=0,1,2,…), β_k^0 (k=1,2,…) and α_k^1, β_k^1, etc.

On account of the unknown quantities θ_1, θ_2 and l_1, the joint system of equations is nonlinear even in the case of linear elastic bonds. To determine the quantities θ_1 and θ_2 ($\theta_1 = \theta_1^0 + \varepsilon\theta_1^1 + \ldots; \theta_2 = \theta_2^0 + \varepsilon\theta_2^1 + \ldots$), we have the condition:

for the zero approximation $p^{(0)}(\theta_1^0) = 0$; $p^{(0)}(\theta_2^0) = 0$;

for the first approximation $p^{(1)}(\theta_1^1) = 0$; $p^{(1)}(\theta_2^1) = 0$.

The right hand sides of infinite algebraic systems with respect to α_k, β_k contain integrals of unknown function $q^{(0)}(x_1)$. Thus, the infinite algebraic systems with respect to α_k, β_k and finite systems with respect to $q^{(0)}(x_1)$, $q_{y_1}^{(0)}(x_r)$, $q_{x_1y_1}^{(0)}(x_r)$ and l are connected between themselves and they must be solved jointly. The combined system equations even for linear-elastic bonds become nonlinear because of unknown quantities θ_1, θ_2, l. For its solution at zero approximation, the reduction and successive approximations methods were used [10].

In the case of the nonlinear law of deformation of bonds for determining forces in bonds, we also use the iteration algorithm similar to the method of elastic solutions [11]. Nonlinear part of the bonds deformation curve is represented in the form of bilinear dependence, whose outgoing section corresponds to the elastic deformation of bonds ($0 < \mathrm{V}(x_1) < \mathrm{V}_*$) with maximal tension of bonds. For $\mathrm{V}(x_1) < \mathrm{V}_*$, the deformation law was described by a nonlinear dependence determined by two points (V_*, σ) and $(\delta_{cr}, \sigma_{cr})$; moreover, for $\sigma_{cr} \geq \sigma_*$ we have increasing linear dependence (linear hardening corresponds to the elastoplastic deformation of the bonds).

After defining the quantities of the desired zero approximation, we can construct the solution of the problem at the first approximation N and T determined on the base of obtained solution for r=R. The boundary conditions (12), (13) may be written in the form of a boundary value problem for finding complex potentials $\Phi^{(1)}(z)$ and $\Psi^{(1)}(z)$ that we seek in the form of (18), with obvious changes. The further course of the solution is at the zero approximation. The obtained complex integral equation with respect to $g^{(1)}(t)$, $\overline{g^{(1)}(t)}$ under additional condition of type (23) by means of the algebraization system is reduced to the system of M algebraic equations for determining $N_0 \times M$ unknowns $g^{(1)}(t)$ (m=1,2,..M).

The desired expansion coefficients of the contact pressure $p^{(1)}(\theta)$ and the unknown values of forces in bonds $q_{y_1}^{(1)}(x_1)$ and $q_{x_1y_1}^{(1)}(x_1)$ are contained in the right hand side of this system.

The construction of the missing equations for determining the unknown forces at the nodal points and prefracture zone sizes are realized as in the zero approximation. The problem of the theory of elasticity for a shaft at the first approximation is solved in some way. The algebraization of solving the equation of the contact problem at the first approximation is carried out similar to the zero approximation. For that, the desired functions of the contact pressure are represented in the form:

$$p^{(1)}(\theta.t) = p_0^1(\theta) + tp_1^1(\theta) + \ldots; \tag{28}$$

$$p_0^1(\theta) = \alpha_{0.0}^1 + \sum_{k=0}^{\infty}(\alpha_{k,0}^1 \cos k\theta + \beta_{k,0}^1 \sin k\theta);$$

$$p_1^1(\theta) = \alpha_{0.1}^1 + \sum_{k=0}^{\infty}(\alpha_{k,1}^1 \cos k\theta + \beta_{k,1}^1 \sin k\theta);$$

As the result we get infinite linear algebraic systems with respect to $\alpha_{0,0}^1$, $\alpha_{k,0}^1$, $\beta_{k,0}^1$ and $\alpha_{0,1}^1$, $\alpha_{k,1}^1$, $\beta_{k,1}^1$ (k=1,2,…), etc.

4 Analysis of the Simulation Results

The system of equations becomes nonlinear because of the unknown quantities θ_1^1 and θ_2^1. The constructed combined system of equations is closed and under the given functions $H(\theta)$ and $H_1(\theta)$ allows us to find the contact pressure, forces in the bonds, the prefracture zone sizes, the stress-strain state, and the bushing and contact pair wear by numerical calculations. The functions $H(\theta)$ and $H_1(\theta)$, describing the roughness of the internal surface of the bushing and the plunger, were considered as the determined totality of unevenness of contours profile and also stationary random function with zero mean value and known variance.

As a rule, the greatest values of contact pressure depend on the angle of contact and the friction coefficient. The presence of friction forces in the contact zone leads to displacements of the graph contact pressure distribution to the contrary action of the moment.

The numerical calculations were carried out for the bushing of a U8-6MA2 double-stoke slush pump for a velocity of the plunger of 0,2 *m/sec*. As constants, we used the following values of the parameters: $2R = 57mm$, $2R' = 56.7mm$, f=0.2, $E = 1.8 \cdot 10^5 MPa$, $\mu = 0.25$, $V_* = 10^{-6} m$, $\sigma_* = 75MPa$, $\sigma_{cr}/\sigma_* = 2$, $\delta_{cr} = 2.5 \cdot 10^{-6} m,$ $K^{(1)} = 2 \cdot 10^{-8}$, $K^{(2)} = 2 \cdot 10^{-9}$, $C_b = 2 \cdot 10^{-7}\, m/MPa$ (C_b is the effective compliance of the bonds).

Using the solution of the problem to calculate displacements on surfaces prefracture zone:

$$-\frac{1+k_b}{2G}\int_{-l}^{l} g(x_1)dx_1 = v(x_1,0) - iu(x_1,0)$$

Assuming $x_1 = x_0$ applying change of variable, changing the integral by the sum, we find displacement vector on the surfaces prefracture zone for $x_1 = x_0$

$$V_0 = \sqrt{u^2 + v^2} = \frac{1+k_b}{2G}\frac{\pi l}{M}\sqrt{A^2 + B^2} \quad (29)$$

$$A = \sum_{m=1}^{M}\left(v^0(t_m) + \varepsilon v^1(t_m)\right), \qquad B = \sum_{m=1}^{M_1}\left(u^0(t_m) + \varepsilon u^1(t_m)\right)$$

Here M_1 is the number of nodal points contained in the interval $(-l, x_0)$.

In the place of crack nucleation condition, we accept the criterion of critical opening of surfaces prefracture zone. Considering relation (9) we can write the limit condition in the form

$$C(x_0, \sigma(x_0))\sigma(x_0) = \delta_{cr} \quad (30)$$

The joint solution of the combined algerbraic system and conditions (30) makes it possible to determine the ultimate size of the external load (contact pressure), the size of surfaces prefracture zone for the limiting equilibrium state under which a crack arises under the given characteristics of the crack resistance of the material.

The graphs of the length of the prefracture zone $\lambda = l/R$ for the bushing borehole pump against the dimensionless values of the contact pressure p_0/σ_* are shown in Fig. 2 (Curve 1 refers to the smooth surface, curve 2 refers to the rough surface).

The distributions of the normal force q_{y_1}/p_0 in the bonds between the surfaces prefracture zone as a function of the dimensionless coordinate x_1/l are shown in Fig. 3. Curve 1 corresponds to the linear bond and curve 2 to the bilinear bond. The dependence of the critical load p_{cr}/σ_* on the dimensionless length of the prefracture zone is shown in Fig. 4.

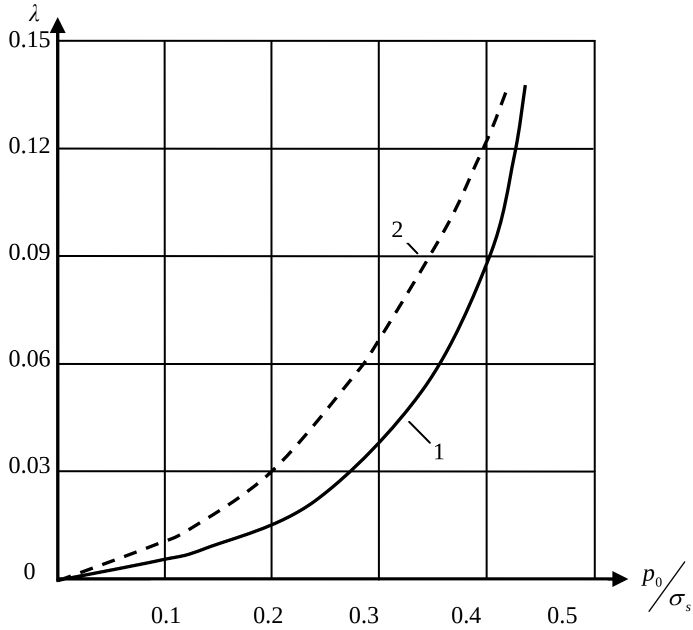

Figure 2
Dependence of length of the prefracture zone $\lambda = l/R$ for the bushing borehole pump on dimensionless contact pressure p_0/σ_*

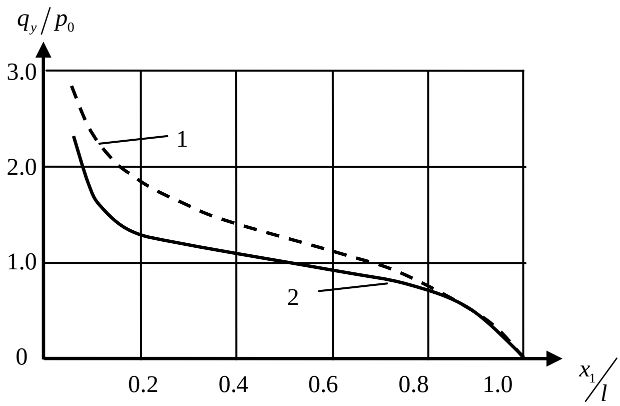

Figure 3
The distributions of the normal force q_{y_1}/p_0 in the bonds between the surfaces prefracture zone as function of the dimensionless coordinate x_1/l

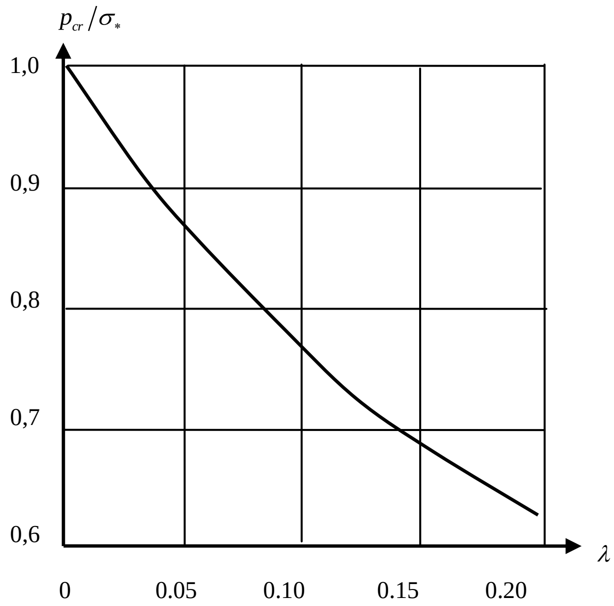

Figure 4
The dependence of the critical load p_{cr}/σ_* on the dimensionless length of the prefracture zone

Conclusions

An analysis of the critical equilibrium state of the bushing contact pair at which the crack appears reduces to a parametric study of the resolving algebraic system (24), (25), (26), etc. and the criterion of crack emergence (30) with different laws of bond deformation, physical and elastic constants of material, and geometric characteristics of the latter. The forces in the bonds and the opening of the prefracture zone are found directly by solving the resultant algebraic systems in each approximation.

An effective algorithm for solving contact fracture mechanics problems on crack nucleation in a bushing friction pair is proposed. This algorithm allows the solution to be constructed in a single manner in each approximation by the method of perturbations.

The model of the prefracture zone with bonds between its faces makes it possible to do the following: to study the basic features of the distribution of forces in the bonds with different deformation laws; to analyze the ultimate equilibrium state of the prefracture zone with allowance for the determination condition of fracture; to estimate the critical external load and crack resistance of the material; and to determine the conditions of equilibrium and growth of the prefracture zone size, as well as conditions of crack nucleation based on the analysis of the ultimate equilibrium state with allowance for mechanical parameters of the bonds. This model allows us to account for not only each specific realization of the roughness profile (deterministic approach), but also to carry out the statistic description of the roughness of bushing and plunger surfaces by realization of stationary random function. The results of the present work allow us to choose the class of roughness of friction pairs, providing the loading ability of conjugation, optimal in strength and stiffness.

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