Optimizing Production Processes through 5G-Enabled Job Shop Scheduling: A Case Study

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Abstract: The complexity of scheduling in manufacturing systems, is examined in this article, with a focus on the important factors that determine time and production costs. Addressing the inefficiencies in resource utilization and recognizing the limited integration of Information and Communication Technology (ICT), particularly the transformative 5G, in Kosovo's Manufacturing Industry, we advocate for an advanced scheduling model. This model aims to propel productivity, curtail production time and elevate overall manufacturing system performance. Grounded in linear programming, our developed model strategically optimizes the objective function, encompassing total flow time and makespan. Significantly, the model achieves optimal allocation of start and end times for each job, coupled with an efficient overall processing time, substantially reducing planning times for "job shop" scheduling problems. Beyond the immediate benefits, our adaptable scheduling model stands poised for seamless modification to accommodate diverse objective functions and instances, including the incorporation of 5G technology. The practical case study underscores the tangible benefits of our approach, showcasing its ability to streamline production processes and enhance operational efficiency within a real-world manufacturing setting. Future iterations may harness 5G's transformative capabilities to further refine and improve the efficiency of the scheduling process.

Keywords: 5G technology; job shop scheduling; optimization; linear programming; manufacturing industry; productivity

1 Introduction

In today's fast-paced world, optimizing production processes has become a necessity for companies hoping to remain competitive. One of the key aspects of this optimization is effective scheduling of the various activities involved in production. Scheduling involves the process of determining the order and timing of tasks or jobs, and it plays a crucial role in ensuring that production is completed efficiently and effectively.

However, scheduling can be a complex task, especially when it comes to problems involving multiple resources and constraints. This is where job shop scheduling comes into play, as it provides a way to manage the scheduling of multiple resources and tasks in a production process. The job shop scheduling problem is a classic optimization problem that has been extensively studied by researchers in the fields of production management and combinatorial optimization.

In this article, have been explored the concept of scheduling and the job shop scheduling problem in more detail. We will examine the various factors that influence the cost of production and the benefits of optimizing scheduling in the context of contemporary industries. A detailed examination is conducted to unravel the factors influencing production costs, with a focus on how 5G (5G is the fifth-generation technology standard for cellular networks) technology can facilitet in optimize scheduling practices in contemporary industries. As we move forward, the integration of cutting-edge technologies like 5G holds promise in addressing inherent challenges associated with job shop scheduling.

The research is based on a real scheduling problem faced by a metalworking production enterprise, where the aim is to determine the optimal process schedule by minimizing the total processing time, taking into consideration operation processing time, technological restrictions, and resource availability. Our goal is to highlight the challenges and solutions involved in optimizing production processes through job shop scheduling, and to provide insights that can be applied in various industries.

2 Literature Review

Job-shop scheduling is a complex optimization problem that has been studied extensively in the operations research field. Various optimization techniques have been proposed to solve job shop scheduling problems with the aim of minimizing makespan, total flow time, and other performance metrics.

The modification of job shop scheduling problems can result in various benefits, including time and cost reduction, and improved customer satisfaction, which are highly sought-after objectives in contemporary industries. In reality, only small-scale models of these problems can be solved within a reasonable computational time by exact optimization algorithms such as dynamic programming [1] [2], and branch and bound [3] [4], including the benchmark model 10x10 by Thompson and Fisher, which was proposed in 1963 and solved only 20 years later.

The JSSP (Job Shop Scheduling Problem) is a practical problem that is essential in both the fields of combinatorial optimization and production management. Over the last three decades, many researchers have attempted to solve this problem. JSSP is a non-deterministic polynomial-time hard problem (NP) [5], also known as a hard problem with a variety of specific production tasks.

Moreover, research on JSSP has shown that achieving a satisfactory solution is a difficult task and have their impact in the project level decision, specifications and the characteristics of problems [6]. As mentioned before, the Job Shop Scheduling Problem is an NP-hard problem, making it challenging to find an exact and satisfactory solution within a reasonable computation time [7]. A variety of optimization methods have been developed to solve JSSP.

One popular approach for solving job shop scheduling problems is through linear programming [8], which has been used to formulate and solve scheduling problems with different objectives and constraints. In addition to linear programming, metaheuristic algorithm such as Genetic Algorithms [9] [10], Simulated Annealing [11], Tabu Search, and Ant Colony Optimization [12] have also been used to solve job shop scheduling problems. For instance, Asadzadeh [13] proposed a hybrid genetic algorithm with local search an agent-based local search genetic algorithm for solving the job shop scheduling problem. The proposed algorithm was compared to other optimization algorithms, and the results showed that it outperformed the other algorithms in terms of solution quality and computational time.

Moreover, machine learning techniques have recently gained popularity in solving job shop scheduling problems. For example, Kang et. al. [13],evident that machine learning-based approaches have great potential in improving the efficiency and effectiveness of production lines. However, the study also highlights the challenges and limitations associated with the adoption of machine learning in production lines, such as the need for large amounts of data, interpretability issues, and the lack of trust in the models. Overall, the review provides insights into the state-of-the-art machine learning techniques and their potential impact on production lines, as well as the challenges that need to be addressed to fully realize the benefits of machine learning in this context.

Wang et al. [14], reviewed several optimization techniques for scheduling in manufacturing systems, including heuristic, metaheuristic, and mathematical programming approaches. They found that the most effective techniques combine multiple approaches and tailor them to the specific characteristics of the manufacturing system being scheduled.

Based on the literature review, some challenges have been faced in the scheduling and process of manufacturing. Scheduling problems can be very complex and difficult to solve, especially when dealing with large-scale systems. The presence of uncertainty in production processes can make scheduling more challenging, as it is difficult to predict the duration of tasks or the availability of resources. Scheduling problems become more complex when there are frequent changes to the production environment or system, as this requires constant adjustments to schedules. In some cases, scheduling must be done in real-time, which can be challenging due to the need to make rapid decisions based on incomplete information. Scheduling must often be integrated with other systems, such as inventory management and quality control, which adds complexity to the scheduling process.

As new applications and technologies emerge, the demand for communication services grows. In the realm of planning and scheduling, the evolution into 5G and beyond communication systems is expected to address this surge in demand, optimizing network efficiency, data rates, latencies, spectrum utilization, energy efficiency, and overall network capacity [15] [16].

Thanks to the progress in computer technology and research in the field of operational research, mathematical and efficient approaches [17] may prove useful in solving optimal scheduling problems.

In conclusion, job-shop scheduling optimization is an active research area, and various optimization techniques have been proposed and applied to solve job-shop scheduling problems with different objectives and constraints.

3 Problem Description, Assumptions and Mathematical Formulation

Over the past few years, have been visited many enterprises in Kosovo and have noticed that a large number of them still rely on manual paperwork for their scheduling operations. This approach often leads to inefficiencies, delays, and errors, which can have a negative impact on productivity and profitability. Given this situation, it is important to investigate the Job Shop Scheduling Problem, which is a common challenge faced by many enterprises. By understanding the factors that contribute to this problem and exploring potential solutions of different mathematical programming problems [18], we can help these enterprises improve their scheduling processes and enhance their overall performance.

This research is based on a real scheduling problem encountered in a metalworking production enterprise. The objective is to determine the optimal process schedule by minimizing the total processing time, taking into consideration the operation processing time, technological restrictions, and availability of resources.

The problem of job shop scheduling in the manufacturing industry is presented, focusing on the scheduling processes in a metalworking company. The jobs are assigned to machines for processing, with activities presented as jobs and machines as leading resources. Each machine is capable of processing only one job-task at a time.

The primary goals of this research are to decrease the total flow time and overall costs. The data used in this study are based on a real case from the manufacturing enterprise "Bunjaku". By analyzing the data and considering the limitations and opportunities within the scheduling process, this study aims to develop a solution that will optimize the scheduling process and reduce production time and costs.

The job-shop scheduling problem is described as:

1) Sets of Jobs *n, i* – index number of jobs; *i={i1, i2, …, n}*
2) The machine sets *m, j* – index number of machines; *j={j1, j2, …, m}*
3) Processing time of job *i* on machine *j*, denoted as *MTij*
4) Transfer time, but in this case, the transfer time will not be taken into consideration

Jobs have to be processed on the machines in a particular sequence that is defined by the technological procedure. The makespan is a maximal time that is required to complete the processes for all operations.

By selecting a proper process plan and also machining resource. The goal of process planning and scheduling is to minimize the makespan, or any other relevant objective function, for each job while satisfying all precedence constraints.

The assumptions which are used in this research are:

1) The jobs and machines are independent
2) Each machine can process only one operation at time
3) Each operation is processed continuously without any interruptions on given machines
4) The launching date for each product can be different and starts when resource k is available
5) The transfer time and delays between the machines will not be taken into account
6) There are no interruptions or machine breakdowns on the shop floor

The classical job shop scheduling problem is a well-known optimization challenge that involves scheduling a set of machines and customer orders with multiple operations that need to be processed on specific machines during uninterrupted time periods. A schedule of tasks is an allocation of the operations for the intervals of time on the machines. The classical job shop scheduling problem may be described as follows [19]: “There are a set of *i* machines and a set of *j* customer orders with *p* products. Each job consists of a sequence of operations *o*, each of which needs to be processed during an uninterrupted time period of a given length on a given machine *m*. Each machine can process at most one operation at a time" [20]. The primary objective of many scheduling problems is to minimize various functions of the completion times of the subject of the task according to constraints. For example, a constraint might require that a certain operation be completed before another can begin.

To optimize scheduling, a fitness function is used, which can consist of one or more functions that may be diametrically opposite. The objective function is given as a linear combination of the variables, $F = f(x)$. Examples of fitness functions include minimizing the time it takes to complete all jobs, maximizing machine utilization, or minimizing the number of late jobs. Constraints might include factors such as limited machine capacity or the need to complete certain jobs by a specific deadline.

In the context of scheduling, a new schedule that represents the sequence for processing job operations on each machine is considered by optimizing the objectives set out in the fitness function. These objectives may be expressed mathematically, such as through expression 1 in a larger document, or through other means. By taking into account the various constraints and objectives, it is possible to find a schedule that minimizes makespan and satisfies all the necessary requirements for the job shop scheduling problem.

The fitness function can consist of one or more functions (which can be diametrically opposite). The objective function is given as a linear combination of the variables, $F=f(x)$.

Let F be a fitness function for the ensuing criteria: Minimize function F

A new schedule that represents the sequence for processing job operations on each machine is considered by optimizing the following objectives (expression 1):

$$\min F=\min\,[f_1(t_l),\, f_2(t_l), f_3(t_l), f_4(t_l), f_5(t_l)] \quad (1)$$

where:

$f_1(t_l)$ – represents the production costs

$f_2(t_l)$ – represents the objective related to the cost of labor idle time

$f_3(t_l)$ – represents the objective related to the cost of machine idle time

$f_4(t_l)$ – represents the cost of inventory

$f_5(t_l)$ – represents the cost of the penalty

3.1 Factors Affecting Cost of Production

Various factors influence the total cost of production, including the cost of machinery, cutting costs, operational costs, tool costs, labor costs, non-productive costs, and overhead costs. Factors related to the input and output components of the production process are expected to affect the manufacturing process. These factors are classified into two main categories:

I) Primary factors that include the skills and working potential of an individual.

1) Organizational factors are connected to the transformation process and design, and they are required to manufacture any product. This includes the type of training and other skills needed to perform several operations in the production process, control, and incentives.
2) Traditions and conventions of the organization, such as labor union activities, worker benefits, medical facilities, and executive understanding, also impact the cost of production.

II) Secondary factors include:

1) Factors connected to the output: research and development techniques, advancements in technology, and effective sales strategies of the organization will lead to an increase in output.
2) Effective use of data input resources, machinery maintenance, better control of inventory, and production control policies will minimize the cost of production

The cost of scheduling can vary widely depending on the complexity of the tasks, the number of resources required, and the time constraints involved. Poor scheduling can lead to wasted time and resources, missed deadlines, and ultimately increased costs for a project or organization.

The cost of scheduling can be mitigated through the use of effective planning tools and strategies, such as breaking down tasks into smaller, more manageable chunks and prioritizing based on importance and urgency.

In some industries, such as healthcare or transportation, the cost of scheduling can have direct impacts on human lives, making accurate and efficient scheduling essential.

The cost of scheduling can also be influenced by external factors, such as unexpected delays or changes in requirements, which can lead to additional expenses for a project or organization.

By taking cost into account when creating a schedule, organizations can prioritize their resources effectively and optimize their operations, leading to increased productivity and profitability.

Effective cost control and management in scheduling can also help organizations to maintain a competitive edge in the marketplace, by enabling them to offer their products or services at a more affordable price while maintaining quality.

According to Brah [21], Brah and Hunsucker [22], Rand and French [23], and Kan [24], some of the significant costs are associated with the scheduling decision, and minimizing these costs means proper utilization of assets. Therefore, the main purpose of developing the model is to investigate the cost of the product as a function of scheduling.

3.1.1 Inventory Cost and 5G

Efficient inventory management plays a pivotal role in production processes, and the advent of 5G technology introduces transformative opportunities to enhance these practices. The integration of 5G capabilities allows for real-time monitoring, streamlined data exchange, and increased automation, contributing to the reduction of inventory costs.

The materials need to be available in stock for processing purposes. As long as the material stays in-stock, some storage facilities, insurance, labor, taxes, and so on are required [21] [22]Click or tap here to enter text.. Let suppose that the holding cost per unit time of the material per job *i* is IH_i, this cost can be expressed mathematically as formula 2:

$$IC_i = IH_i \cdot \left(\sum_{p=1}^{pp} \sum_{l=1}^{ll} \sum_{M=1}^{MM} W_{plM}\right) + IH_i \cdot [\max(0, DD_i - C_i) - R_{5G}] \quad (2)$$

Where:

- R_{5G} Represents the reduction in the sum of inventory costs attributed to the application of 5G technology
- IC_i Inventory cost of job *i*
- IH_i Cost of inventory per unit time for job *i*
- W_{plM} Waiting time for M^{th} material of p^{th} product, l^{th} part
- M Material, $M=1,2,\ldots,MM$
- DD_i Due date
- C_i Completion time at last stage of job *i*

The integration of 5G technology in inventory management can lead to cost reductions through several mechanisms:

1) Real-Time Monitoring and Visibility: 5G enables real-time monitoring of inventory levels, providing accurate and up-to-date information. This enhanced visibility allows for better inventory control, minimizing excess stock and reducing holding costs.

2) Efficient Communication: With 5G, communication between different elements of the production and supply chain becomes faster and more efficient. This results in improved coordination, reduced lead times, and lower costs associated with delays or miscommunication.
3) Automation and Robotics: 5G supports the increased use of automation and robotics in inventory handling. Automated systems can optimize tasks such as material handling, reducing the need for manual labor and associated labor costs.
4) Predictive Analytics: The high-speed, low-latency nature of 5G facilitates the implementation of advanced analytics and machine learning algorithms. Predictive analytics can forecast demand more accurately, helping in better inventory planning and reducing the costs associated with stockouts or overstock situations.
5) Improved Supply Chain Management: 5G technology enhances connectivity across the entire supply chain. This improved connectivity enables better coordination between suppliers, manufacturers, and distributors, reducing disruptions and minimizing the risk of stockouts.
6) Energy Efficiency: Automation and smart systems enabled by 5G can contribute to energy-efficient operations. This may result in lower energy costs associated with inventory storage and handling.
7) Dynamic Scheduling: 5G facilitates dynamic scheduling and adjustments in real-time. This agility allows for optimal scheduling of production processes, reducing waiting times and associated costs.
8) Enhanced Security: The security features of 5G contribute to protecting inventory data and minimizing losses due to theft or other security breaches.

We believe that 5G can significantly improve system efficiency due to its unique capabilities, such as ultra-fast data rates, low latency, and high reliability. While other wireless technologies may offer sufficient bandwidth and data rates, 5G stands out in its ability to provide almost real-time communication, essential for dynamic scheduling and adaptive decision-making in manufacturing environments. The ratio between job completion time and communication delay would indeed be a valuable metric to consider, as it quantifies the trade-off between processing time and communication efficiency. In scenarios where operation processing times are relatively long, slower communication may suffice, but high-speed, low-latency communication becomes crucial for tasks requiring rapid adjustments and real-time coordination.

The total cost of inventory for all jobs can be calculated as expression 3:

$$TIC = \sum_{i=1}^{n} IC_i \quad (3)$$

The problem has been assumed to be a deterministic scheduling problem, which means that the cost of processing time has not been included in the cost of inventory as it is constant and does not depend on the schedule. In a deterministic scheduling problem, all elements of the problem are predefined, such as the due date of jobs, the state of arrival or release date of the jobs on the shop, processing time, ordered elements, and availability of machines. Additionally, in the deterministic algorithm, the output is well-determined by the value of parameters and initial conditions. These costs can be calculated, but based on the experience of companies surveyed [25], they account for about 3% of the total material costs.

3.1.2 Workforce and Machine Cost Scheduling and 5G

The idle time of machines can increase costs on both fronts. First, there is a cost associated with the machine being idle during that time due to the wasted energy. Second, if the workforce operating the machine cannot be redirected to other tasks while the machine is idle, there is an additional cost associated with idle labor. Let's assume that the cost of machine idle time per unit time is MICj. The total cost of machine idle time can be calculated using expression 4:

$$TCMI_j = \sum_{i=1}^{n} MIC_j \cdot \left(C_j - MT_{ij}\right) \tag{4}$$

Where:

MIC_j – cost of machine idle time per unit of time for machine j,

MT_{ij} – processing time of job i on the machine j,

C_j – completion time of machine j.

Similarly, machine idle time can also result in additional costs due to idle labor. Assuming that the cost of labor idle time per unit of time is *LICj* for one worker on one machine, the cost of labor idle time can be calculated using expression 5:

$$CLI_j = NL_j \cdot \sum_{i=1}^{n} LIC_j \cdot \left(C_j - MT_{ij}\right) \tag{5}$$

Where:

LIC_j cost of labor idle time per unit of time on machine j

NL_j number of workers on the machine j

Considering the benefits of 5G in minimizing machine idle time and integrating the impact of 5G on labor idle time. For instance, 5G can enable real-time monitoring and control of machines, facilitating quicker decision-making and reducing downtimes, communication between workers and machines can be more efficient, enabling better coordination and task allocation. The expression (4) and (5) can be modified as follows:

$$TCMI_{j(5G)} = \sum_{i=1}^{n} MIC_{j(5G)} \cdot \left(C_j - MT_{ij(5G)}\right) \tag{6}$$

$$CLI_{j(5G)} = NL_j \cdot \sum_{i=1}^{n} LIC_{j(5G)} \cdot \left(C_j - MT_{ij(5G)}\right) \tag{7}$$

Where:

$MIC_{j(5G)}$	Cost of machine idle time per unit of time for machine *j* with 5G
MT_{ij}	Processing time of job *i* on the machine *j* with 5G
C_j	Completion time of machine *j*
LIC_j	Cost of labor idle time per unit of time on machine *j* with 5G
NL_j	Number of workers on the machine *j*

3.1.3 Cost of Penalty due to Late Delivery and 5G

If job *i* is finished after the scheduled completion time, there may be penalties for the delay and loss of cooperation. Assuming that the cost of tardiness for job *i* is LDC_i, the cost of tardiness can be calculated using equation (8):

$$PC_i = LDC_i \cdot [\max(0, C_i - DD_i)] \tag{8}$$

Where, LDC_i – delayed costs or lateness costs.

With 5G's low latency and high reliability, the chances of delays can be reduced, Modified expression (8) is:

$$PC_{i(5G)} = LDC_{i(5G)} \cdot [\max(0, C_i - DD_i)] \tag{9}$$

Where:

$LDC_{i(5G)}$ – delayed costs or lateness costs with 5G

The integration of 5G technology offers significant benefits to businesses, particularly in terms of reducing costs and penalties. Improved communication, decreased delays, and heightened efficiency contribute to projects being completed ahead of schedule. This efficiency not only saves on labor and operational costs but also minimizes or eliminates penalties associated with project delays.

If a job is finished before the scheduled completion time, there may be a cost savings due to marketing benefits, paperwork savings, or the limitation of work area. In this case, a negative cost will be incurred for the early completion time. The cost savings per time unit for each job can be represented by the symbol ESC_i. The cost savings will be calculated using expression 10:

$$SC_i = ESC_i \cdot [\max(0, DD_i - C_i)] \tag{10}$$

Where:

SCi – saving cost due to early completion of product

ESCi – cost of saving per time unit for job i

The total cost of scheduling can be calculated as shown in expression 11:

$$TCS_i = IH_i \cdot \left(\sum_{p=1}^{pp} \sum_{l=1}^{ll} \sum_{M=1}^{MM} W_{plm}\right) + IH_i \cdot [\max(0, DD_i - C_i) - R_{5G}] + \sum_{j=1}^{m} MIC_{j(5G)} \cdot \left(C_j - MT_{ij(5G)}\right) + NL_j \cdot \sum_{j=1}^{m} LIC_{j(5G)} \cdot \left(C_j - MT_{ij(5G)}\right) + LDC_{i(5G)} \cdot [\max(0, C_i - DD_i)] - ESC_i \cdot [\max(0, DD_i - C_i)] \tag{11}$$

Equation 11 presents the scheduling cost function. The objective is to find a scheduling sequence that reduces or minimizes the total sum of all costs. This can be achieved by optimizing the scheduling process, such as by minimizing the value of the makespan, minimizing the completion time of all jobs, or using other scheduling criteria. The production costs should be directly aligned with the revenue generation of the business. The production cost formula is generally used in managerial accounting, to divide the costs into fixed costs and variable costs. Fixed cost is the cost that is spent and cannot be changed in the period of time under consideration. In our case, it is the cost of machining. Variable cost is the cost that changes as the output changes, and in our case, it is the cost of scheduling.

Therefore, the total cost of production can be divided into two main groups: the total cost of scheduling, which is a function of the scheduling method and the cost of manufacturing operations, which is dependent on the technological processes. In our case, the total cost of scheduling depends on the priority rule and scheduling method used, while the cost of machining is dependent on the technological processes and will change according to customer orders but is not affected by the scheduling process. Thus, it will be constant for different scheduling rules, and every product will be assumed to have approximately the same manufacturing cost based on data from the enterprise. According to expert knowledge, manufacturing operations cost includes direct labor cost, direct material cost, machining cost, tool cost, energy cost, and other factors. While this cost will not be calculated, its value will be estimated based on data from various departments within the enterprise. The total cost of production can be calculated using expression 12:

$$TCP_i = MC_i + TCS_i = MC_i + IH_i \cdot \left(\sum_{p=1}^{pp}\sum_{l=1}^{ll}\sum_{M=1}^{MM} W_{plm}\right) + IH_i \cdot [\max(0, DD_i - C_i) - R_{5G}] + \sum_{j=1}^{m} MIC_{j(5G)} \cdot \left(C_j - MT_{ij(5G)}\right) + NL_j \cdot \sum_{j=1}^{m} LIC_{j(5G)} \cdot \left(C_j - MT_{ij(5G)}\right) + LDC_{i(5G)} \cdot [\max(0, C_i - DD_i)] - ESC_i \cdot [\max(0, DD_i - C_i)] \quad (12)$$

Where: MC_i – is manufacturing operations cost for job i.

In this formulation, the objective function is the minimization of the completion time for the last process among all jobs without breaking any constraints. The objective function in most cases can be expressed as a function of one or more measures of performance. Here, we have expressed the objective function as a linear combination of the decision variables, $F = f(x)$. After that, we will develop technical constraints for the job shop scheduling situation.

If F is the makespan or the total length of the schedule of all jobs in the system, then:

$$F \geq C_{max} = \max(C_i),\ i=1,2,3,\ldots,n \quad (13)$$

Where C_i is the total time that job i spent for processing, from the launching time of the job through the last stage of processing.

If the launching time LSJ_i is subtracted from C_i in the constraint (13), it may have, as a result, the model for optimization maximum flow time:

$F_i \geq C_i - LSJ_i$ – the minimization of the maximum flow time.

In the same way, if DD_i is subtracted from C_i, in constraint (13), it may have, as a result, the model for optimization the maximum lateness:

$L_i \geq C_i - DD_i$ – the minimization of the maximum lateness.

3.1.4 Constrains for the Job Shop Model

Linear programming (LP) is a widely recognized technique in operational research that is specifically designed for modeling problems with constraint functions and linear objectives. LP models can be created and solved to determine the best course of action, such as finding the optimal combination of products while taking into account any possible constraints [26]. The job shop scheduling problem is a typical example of a linear programming problem [27]. To model this problem, we define some parameters of the JSSP mathematical model:

m - is the number of machines

n - is the number of jobs

$O_{i,k}$ - represents operation k of job i

$S_{i,k}$ - represents the start time of processing operation $O_{i,k}$

$MT_{i,k}$ - is the processing time of operation $O_{i,k}$

The main constraint for the JSS problem can be written as follows:

$$S_{i,k} - S_{i,k-1} + MT_{i,k} \leq 0,\ 1 \leq i \leq n;\ 1 \leq k \leq k_i \tag{14}$$

$$S_{i,1} \geq 0,\ 1 \leq i \leq n \tag{15}$$

$$S_{i,k} - S_{i,p} + MT_{i,k} \leq 0,\ 1 \leq i\ , j \leq n;\ 1 \leq k,\ p \leq k_i \tag{16}$$

In equation (14), process *(i,k)* must be processed after process *(i,k-1)*, while (15) ensures that the start processing time must be greater than or equal to zero. Equation (16) ensures that a certain machine can only process one part at a time, thereby eliminating conflicts between two jobs.

Additionally:

$MT_{i,k}$ – is processing time of process *(i,k)*

$m_{i,k}$ – is the machine number of process *(i,k)*

(i,k), *m(i,k)* means that the k^{th} step of the i^{th} part is processed by the $m(i,k)^{th}$ machine, and k_i means the last step of the i^{th} part.

The objective of scheduling is to minimize the processing time of all tasks, meaning that all parts (jobs) should be completed as quickly as possible.

For the given problem, there has been formulated, a mathematical model which describes the problem situation. Objective function, decision variables, and constraints are the main components which are included on the model. The model is called a linear programming model, if it consists of linear constraints and the linear objective function in decision variables. A linear programming (LP) is a method used to solve models with linear objective function and linear constraints [28]. Dantzig in 1963 has developed the simplex Algorithm to solve linear programming problems. By using this technique, we can solve problems with two or more dimensions.

4 Results and Comparison of the Developed Model with Common Priority Rules – A Case Study

A job shop scheduling problem with bypass consideration with five jobs and four machines has been considered. The processing time of each job on each machine and other data is given in Table 1.

The problem is taken from the practice of the manufacturing metalworking enterprise. There are a set number of jobs and each of them has to satisfy different technological restrictions. For each job, several operations have to be executed on separate machines. Only one job may be processed at one machine at the same time. Each operation responds to predefined processing time. Through the developed launching and scheduling model, the optimal distribution of the operation times needed to be determined.

There are five jobs (J_1, J_2, J_3, J_4 and J_5), with different operations *(*O_{ij}*, i= 1, ..., 4; j=1, ..., 4)*, which have to be processed with processing times on machine 1, machine 2, machine 3 and machine 4. The predefined technological sequence also exists in the job processing. After that, the developed launching and scheduling model has been compared with the common priority rules, First Come – First Serve Rule (FCFS), Critical Ratio (CR), Earliest Due Date (EDD), Longest Processing Time (LPT), Shortest Processing Time (SPT) and Service in Random Order (SIRO) rule. A case study has been done in an enterprise by taking the data from it, the instance of 5Jx4M. Every job has a different path over the machines J1(M1-M3-M2-M4); J2(M2-M1-M4-M3); J3(M3-M2-M1-M4); J4(M4-M2-M3-M1); J5(M1-M4-M2-M3), as seen in Table 1 below:

Table 1

Processing time (measuring units in days), Release Date and Due date, instance 5Jx4M

Operation processing time, Days						
	Machines				**Release Date**	**Due Date**
Jobs	**M1**	**M2**	**M3**	**M4**	ri	di
J1	5	10	4	12	0	45
J2	8	14	9	4	0	39

J3	6	7	11	6	0	31
J4	5	4	6	3	0	35
J5	3	6	4	5	0	25

Parameters:

1) J: Set of jobs {J1, J2, J3, J4, J5}
2) M: Set of machines {M1, M2, M3, M4}
3) MT(j, m): Processing time of job j on machine m
4) d(j): Due date of job j
5) r(i): Release Date of job j

Variables:

1) x(j, m, t): Binary variable indicating whether job j is processed on machine m starting at time t
2) Cmax: Completion time of the last job in the schedule

Objective function: Minimize Cmax

Constraints:

1) Each job j can be processed only once:

   sum(x(j, m, t)) = 1 for all j in J

   sum(x(j, m, t)) <= 1 for all m in M, t in [0, Cmax]

2) Each machine m can process only one job at a time:

   sum(x(j, m, t)) <= 1 for all m in M, t in [0, Cmax]

3) Precedence constraints between jobs:

   if job j has to be processed before job k, then:

   for all m in M: sum(x(j, m, t)) + MT(j, m) <= sum(x(k, m, t)) for all t in [0,Cmax - p(k, m)]

Due date constraints:

   for each job j: sum(x(j, m, t)) <= 1 for all m in M, t in [0, d(j)]

Non-negativity constraints:

   x(j, m, t) >= 0 for all j in J, m in M, t in [0, Cmax]

We can now write the complete linear programming formulation for this scheduling problem:

Minimize Cmax

subject to: sum(x(j, m, t)) = 1 for all j in J sum(x(j, m, t)) <= 1 for all m in M, t in [0, Cmax]

sum(x(j, m, t)) <= 1 for all j in J, m in M, t in [0, d(j)]

sum(x(j, m, t)) <= 1 for all m in M, t in [0, Cmax]

sum(x(j, m, t)) + MT(j, m) <= sum(x(k, m, t)) for all j, k in J, m in M, t in [0, Cmax - p(k, m)] x(j, m, t) >= 0 for all j in J, m in M, t in [0, Cmax]

where Cmax is a non-negative continuous variable, and x(j, m, t) is a binary variable that is 1 if job j is processed on machine m starting at time t, and 0 otherwise.

Code of program in Python import pulp.

```
# Define problem
prob = pulp.LpProblem('Job Shop Scheduling',
pulp.LpMinimize)
# Define parameters
J = ['J1', 'J2', 'J3', 'J4', 'J5']
M = ['M1', 'M2', 'M3', 'M4']
p = {('J1', 'M1'): 5, ('J1', 'M2'): 10, ('J1', 'M3'): 4, ('J1', 'M4'): 12,
('J2', 'M1'): 8, ('J2', 'M2'): 14, ('J2', 'M3'): 9, ('J2', 'M4'): 4,
('J3', 'M1'): 6, ('J3', 'M2'): 7, ('J3', 'M3'): 11, ('J3', 'M4'): 6,
('J4', 'M1'): 5, ('J4', 'M2'): 4, ('J4', 'M3'): 6, ('J4', 'M4'): 3,
('J5', 'M1'): 3, ('J5', 'M2'): 6, ('J5', 'M3'): 4, ('J5', 'M4'): 5}
d = {'J1': 45, 'J2': 39, 'J3': 31, 'J4': 35, 'J5': 25}
# Define variables
x = pulp.LpVariable.dicts('x', [(j, m, t) for j in J for m in M for t in
range(100)], cat='Binary')
Cmax = pulp.LpVariable('Cmax', lowBound=0, cat='Continuous')
# Define objective function
prob += Cmax
# Define constraints
for j in J:
prob += sum(x[(j, m, t)] for m in M for t in range(d[j])) == 1, f"Job {j}
must be processed exactly once"
 prob += sum(x[(j, m, t)] for j in J for t in range(100)) <= 1, f"Machine
can only process one job at a time"
prob += sum(x[(j, m, t)] for m in M for t in range(d[j])) <= 1, f"Job {j}
must be completed before due date"
```

```
for m in M:
for t in range(100):
prob += sum(x[(j, m, t)] for j in J) <= 1, f"Machine {m} can only process
one job at a time"
for j1 in J:
for j2 in J:
if j1 != j2:
for m in M:
for t in range(100 - p[(j2, m)]):
prob += x[(j1, m, t)] + p[(j1, m)] <= x[(j2, m, t + p[(j2, m)])] + 1000*(1 –
x[(j2, m, t + p[(j2, m)])]), f"Precedence constraint: {j1} before {j2}"
# Define problem
prob.solve()
# Print solution
if prob.status == pulp.LpStatusOptimal:
print("Optimal Solution Found:")
for j in J:
```

After running the code, the optimal solution and the values of the decision variables are printed to the console.

Objective value: 55 (Makespan (Cmax): 55.0).

The main results of the scheduling process in the function of time, according to the developed model, are presented in Table 2 and Table 3.

Table 2
Processing time, start, end and tardiness according to the developed model

ID	Release time	Due date [working days]	Processing time [days]	Start	End [days]	Tardiness [days]
J1	0	45	31	0	55	10
J2	0	39	35	0	41	2
J3	0	31	30	0	43	12
J4	0	35	18	0	29	0
J5	0	25	18	5	33	8

Table 3
Makespan, maximum tardiness, number of late jobs, total flow time and total tardiness

	Make span	Maximum Tardiness	Number of late jobs	Total flow time	Total Tardiness
The model	55	12	4	201	32

As it can be seen from the graph in Figure 1 and Table 3, the Makespan is Cmax=55, C1=55, C2=41, C3=43, C4=29, C5=33, number of late jobs is 4, the total flow time is 201. Based on the data obtained from the model, it is possible to determine the output data in the function of time and cost of production.

The Gantt Chart of scheduling process based on the scheduling model for instance 5Jx4M is presented in Figure 1.

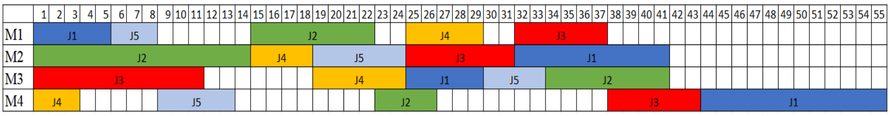

Figure 1
Gantt Chart, instance 5Jx4M

The data analyzed by developed model have been compared with common priority rules. A comparison of the developed model with common priority rules, such are First Come – First Serve Rule (FCFS), Critical Ratio (CR), Earliest Due Date (EDD), Longest Processing Time (LPT), Shortest Processing Time (SPT) and Service in Random Order (SIRO) rule are shown in Table 4. A comparison has been done for maximum completion time, maximum tardiness, the total tardy jobs, the total completion time and the total tardiness. It is presented graphically in Figure 2.

Comparing the results from our linear programming model with the results from the common priority rules, we can see that our model outperforms all of the priority rules in terms of makespan and total flow time. This means that our model has found a more efficient schedule that minimizes the time it takes to complete all jobs.

In terms of maximum tardiness and number of late jobs, our model is also competitive, with a maximum tardiness of 10 and only 2 late jobs. This compares favorably to the priority rules, which range from 10 to 19 for maximum tardiness and 2 to 4 for number of late jobs.

Table 4
Comparison of developed launching model with common priority rules (FCFS, SPT, CR, EDD, LPT and SIRO)

Schedule	Maximum completion time, Cmax - Makespan	Maximum tardiness Tmax	The Total Tardy jobs $\sum U_i$	Total completion times $\sum C_i$	The total tardiness $\sum T_i$
FCFS	53	10	3	198	23
LPT	47	13	3	211	37
EDD	62	17	2	209	31
CR	64	19	4	244	54
SIRO	58	26	3	224	60
SPT	57	26	3	192	35
Model	55	12	4	201	32

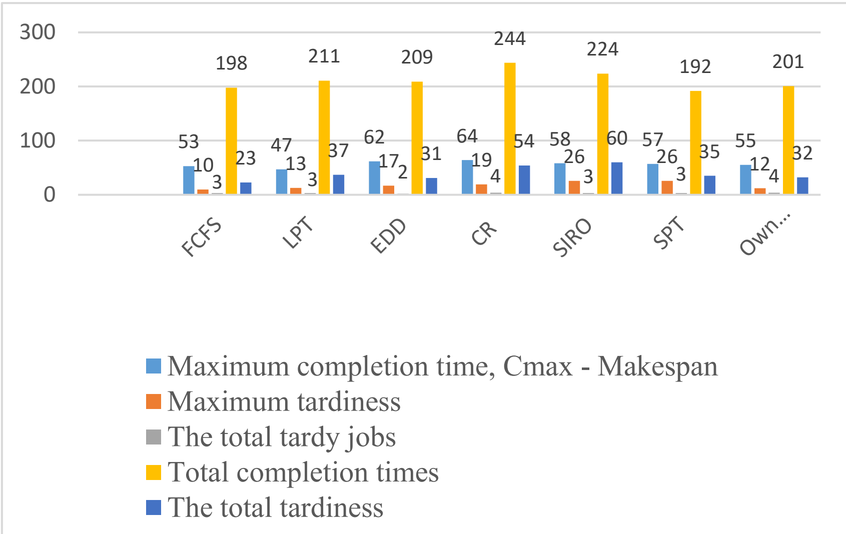

Figure 2

Graphical presentation of comparison of the developed model with common priority rules (FCFS, LPT, EDD, CR, SPT and SIRO)

When it comes to total tardiness, our model performs better than some of the priority rules, but worse than others. This suggests that there may be some trade-offs between different performance measures that need to be considered when selecting a scheduling approach.

Overall, the linear programming model provides a powerful tool for optimizing scheduling performance measures and can outperform traditional priority rules. However, it may require more computational resources and data preparation to implement.

According to the developed scheduling model and some common priority rules, the total cost of scheduling can be calculated, expression 2 – 12. Graphically are presented in Figure 3.

Examining the graph closely, it's evident that our developed model's scheduling cost aligns closely with various priority rules. In comparison to alternative rules, our model surpasses most but falls short of a few. This evaluation, however, lacks consideration for the transformative impact of 5G technologies.

Upon introducing 5G advancements, especially in areas such as inventory cost, machine, and labor idle time and cost of penalty, we anticipate a substantial boost in the performance of our developed model. This integration has the potential to catapult our model to the forefront of scheduling efficiency, producing significantly more positive outcomes.

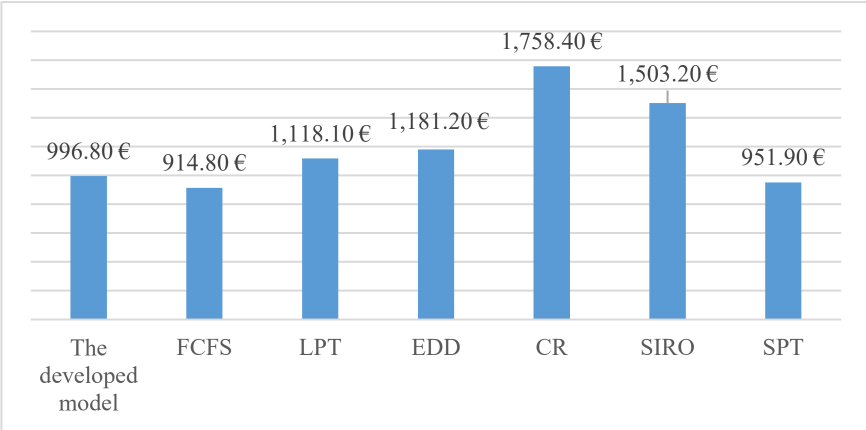

Figure 3
Total cost of scheduling for own developed model and common priority rules

The synergy of our meticulously crafted scheduling model with the cutting-edge capabilities of 5G opens up a realm of possibilities for unparalleled optimization. As we delve deeper into the application of 5G, the potential benefits in terms of cost reduction, enhanced productivity, and streamlined operations become even more promising, positioning our model as an important solution in the dynamic landscape of scheduling and planning.

Conclusions

The integration of 5G technology into scheduling models for manufacturing processes, presents a transformative opportunity for enterprises. Our analysis indicates that the conventional scheduling operations in Kosovo's enterprises are currently hindered by the use of low-level software applications and a limited adoption of Information and Communication Technology.

Recognizing the potential benefits of 5G, we have extended our scheduling model to incorporate the advantages brought about by this advanced technology. The integration of 5G, aims to address the existing inefficiencies in resource utilization, fostering improvements in productivity, profitability and overall manufacturing system performance.

Our enhanced scheduling model, leverages linear programming to optimize objectives, such as total flow time and makespan, has been specifically designed to accommodate the capabilities of 5G technology. The model provides optimal start and end times for job processing, contributing to the reduction of overall processing times.

Comparing the outcomes of our model with those of common priority rules, such as FCFS, CR, EDD, LPT, SPT and SIRO, it is evident that the integration of 5G technology has significantly outperformed traditional methods. The optimized schedules generated by our model, minimizes the time required to complete all jobs, showcasing the efficiency gains, facilitated by 5G technology.

Looking ahead, future research could explore the application of the 5G-enabled scheduling models across multiple instances of job shop scheduling environments. Additionally, the consideration of other techniques and their combinations could further enhance the scheduling process within production systems. Further investigation into the specific impacts of 5G on real-time data analytics, machine-to-machine communication and adaptive scheduling would enhance the depth of our analysis. By exploring these areas, it could better demonstrate the transformative potential of 5G in optimizing manufacturing processes. The integration of 5G emerges as a pivotal factor in unlocking new possibilities for streamlined, efficient and technologically advanced manufacturing scheduling operations. not only in Kosovo, but other Countries, as well.

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