

Thermodynamic Embedding of the Temperature Increment of a Collapsing Oscillator

Katalin Gambár^{1,2}, Ferenc Márkus³

¹Department of Natural Sciences, Institute of Electrophysics, Kálmán Kandó Faculty of Electrical Engineering, Óbuda University, Tavaszmező u. 17, H-1084 Budapest, Hungary

²Department of Natural Sciences, National University of Public Service, Ludovika tér 2, H-1083 Budapest, Hungary

³Department of Physics, Budapest University of Technology and Economics, Műegyetem rkp 3, H-1111 Budapest, Hungary

gambar.katalin@uni-obuda.hu, markus.ferenc@ttk.bme.hu

Abstract: This paper examines the thermal behavior of the spontaneous wave-function collapse model heating in quantum harmonic oscillators. A peculiar result of the theory is that the obtained temperature increase is proportional to the square of the oscillator frequency. In the present work, we compare this temperature increment – during the dissipation of an oscillator – with a package of thermal excitation in the quantized thermal propagation. It seems that a thermodynamically consistent physical embedding of the collapsing propagation can be recognized.

Keywords: spontaneous collapse model; temperature increment; dissipation; Lagrangian formalism of Fourier heat conduction; quantized description; thermal excitation

1 Introduction

The laws of thermodynamics involve such fundamental physical relationships by which we can understand the dissipation and the measure of irreversibility (entropy production) in the different processes [1]. In addition, these concepts are essential not only because of the formulation of important conclusions from a purely scientific point of view. Knowing the direction of processes is essential because of technical applications [2], environmental phenomena, and sustainable development. The macroscopic world is indisputably irreversible, and one of the intensive quantities, the thermodynamic temperature has a crucial role in the description. In the case of microscopic phenomena, two questions arise in this regard. How can we

formulate the relationship between dissipation and reversibility [3]? It is clear that microscopic processes are also dissipative. These represent losses, but at the same time, we want to be aware of the possibilities of recovery [4-6]. In the processes that control the operation of quantum computers, stability, and controllability are indisputable [7-9]. The other question concerns the existence of the temperature created on a microscopic scale. How can we interpret temperature in quantum phenomena, or what corresponds to temperature? Here, single-particle events can be particularly interesting, i.e., those cases where we cannot speak of ensembles. For this reason, the topic of this article is the investigation of a phenomenon in which a dissipative event occurs during the movement of a single particle. Several double-slit experiments have been designed to observe the interference of matter waves, like electrons, neutrons, and even fullerenes. During the examination of the massive objects are in spatial superposition [10]. The collapse of the coherence means a turn of the considered system into a dissipative spreading process. The collapse models predict a temperature increment when the incoherent state is achieved [11].

The structure of the paper is as follows. The main steps in the mechanical model of a collapsing quantum oscillator are summarized in Section 2. Here, we focus on the existence of temperature increment without the details of the calculations. The appeared temperature is a motivation of the present paper since we aim for the thermodynamic formulation of this effect. The quantized description of the thermal propagation is shortly presented in Section 3. The two quantum effects on this level are compared and connected in Section 4. At this stage, we can identify this temperature increase by a packet of thermal excitation. It sounds relevant that the loss of coherence pertains to an irreversible alteration. In this way, we can take a look at how to introduce the laws of thermodynamics in such theories, which are far from them in the axioms. The conclusions are collected in Section 5.

2 Temperature Increment of a Collapsing Oscillator

In a macroscopic system, there is no superposition of different position states. In another approach, we may say that macroscopic superposition decays at universal rates. These phenomena pertain to the spontaneous collapse models [12-17]. Due to the consequence of the wave function collapse, an experimentally proved significant temperature increase appears due to a mechanical oscillator. We emphasize that in this study the considered oscillator is macroscopic, and we assume of its existence of quantum wave function. The collapsing process suggests a decay of the initial state, which is dissipative and irreversible. The framework of thermodynamics is an effective tool for interpreting such phenomena. During the collapse an energy transformation occurs. It seems to identify it as a heating effect. Accepting these antecedents, we focus on the above-mentioned temperature

increment to find the thermodynamic relation to it. If we can achieve this, we can place this quantum process in the thermodynamic framework.

The classical physical situation can be formulated via the quantum master equation [18, 19], by which we arrive at the Liouville equation that describes mechanical oscillator diffusion [11]

$$\frac{d\rho}{dt} = \{H, \rho\} = D_{sp} \frac{\partial^2 \rho}{\partial p^2}, \quad (1)$$

where $\rho(p, x)$ is the phase density, $H(p, x)$ is the Hamilton function of the oscillator, $\{H, \rho\}$ is the Poisson bracket, and D_{sp} is the diffusion parameter to the diffusion of ρ . It is relevant that the collapsing oscillator raises the environmental temperature by spontaneous heating [11, 12]. In the detailed analysis, the mechanical oscillator is placed in a thermal environment, which will modify the Liouville equation. Moreover, the classical and quantum noise also contributes to the process. For this reason, the above equation becomes somewhat more complex. Assuming a stationary state the phase density $\rho(p, x)$ approximates

$$\rho_{st}(p, x) \sim \exp\left(-\frac{H(p, x)}{k_B T}\right) \quad (2)$$

by the Gibbs canonical distribution, where k_B is the Boltzmann constant. After a detailed discussion, the related temperature increment for higher temperatures can be expressed as

$$\Delta T_{sp} = \frac{D_{sp}}{m k_B} \tau, \quad (3)$$

where τ is the relaxation (collapse) time of the oscillator, and m is the mass. The diffusion parameter for a massive oscillator (originally a gravity-related is considered) in a spontaneous collapse model restricted to a bulk matter is

$$D_{sp} = \frac{1}{2} \hbar m \omega^2, \quad (4)$$

by which the temperature increment can be obtained as

$$\Delta T_{sp} = \frac{\hbar \omega^2}{2 k_B} \tau. \quad (5)$$

We note that the formula has a general meaning, there is no restriction to the vibrating entity, i.e., it is independent of the actual physical nature of the oscillator. The temperature increase of ΔT_{sp} may depend on the mass of m and the spring constant of k_s of the oscillator via the angular frequency of ω . We see that the temperature appears explicitly here, so an obvious wish may be to find a thermodynamic connection point to this result. The frequency square in the expression means a peculiar effect of the collapsing-decaying process. The traveling oscillator energy is proportional to the frequency, but in the dissipation, the $k_B \Delta T_{sp}$ is proportional to the square of ω . We expect that the quantized thermalizing should give back this relation. It is encouraging that the entropy production calculated on the quantum scale of the frequency energy packet is also related to the square of the

frequency [20]. It seems that there is a connecting point among the processes in the deep. We draw attention, to that today there are several clarifying thoughts about the macroscopic thermodynamics and quantum transports [21], and the entropy limits for macroscopic and single particle systems [22].

3 The Thermal Excitation within the Thermodynamic Frame

To join with the thermodynamic aspects of the above dissipative process we need to introduce the quantum excitation of thermal energy [23]. The Lagrangian-Hamiltonian-based canonical formulation of thermal propagation enables us to introduce and elaborate the quantum description of temperature and thermal energy [24, 25]. We shortly summarize the model of the thermal field to see later the physical relation with the collapse process. The initial step is the formulation of the Lagrange density function

$$L = \frac{1}{2} \left(\frac{\partial \varphi}{\partial t} \right)^2 - \frac{1}{2} D^2 \left(\frac{\partial \varphi}{\partial x} \right)^2, \quad (6)$$

that can describe the thermal propagation. The introduced potential function $\varphi(x, t)$ generates the measurable thermal field – or proportional to the usual temperature of T – in the sense of the definition

$$T = -\frac{\partial \varphi}{\partial t} + D \frac{\partial^2 \varphi}{\partial x^2}. \quad (7)$$

In Eqs. (6) and (7), the definition of the diffusivity parameter of D is given by the parameters of Fourier heat conduction $D = \lambda / (\rho c_v)$; λ is the heat conductivity, ρ is the mass density and c_v is the specific heat. Applying the least action principle (Hamilton's principle) ($\delta S = \delta \int L dV dt = 0$) and the calculus of variations, the Euler-Lagrange equation can be obtained for the potential function

$$-\frac{\partial^2 \varphi}{\partial t^2} - D^2 \frac{\partial^4 \varphi}{\partial x^4} = 0. \quad (8)$$

We take into account the definition in Eq. (6), by which we obtain a differential equation for the temperature field of T

$$\frac{\partial T}{\partial t} - D \frac{\partial^2 T}{\partial x^2} = 0. \quad (9)$$

This is exactly the Fourier's heat equation. The quantization procedure of the thermodynamic theory starts with the basic modification of the least action principle. It states that the variation of the action is not zero:

$$\delta S = \delta \int L dV dt = \hbar. \quad (10)$$

The quantity \hbar shows the deviation from the classical paths. We emphasize that the least action principle, the variational calculus is not sensitive to the dissipative property of a process. We managed to apply the procedure for thermal conduction, despite of that the related quantum behavior differs from the classical one. The mathematical structure enables the use of the operator formalism [25]. Introducing the generalized momentum (conjugated to φ)

$$P = \frac{\partial L}{\partial \dot{\varphi}} (= \dot{\varphi}), \quad (11)$$

we can reformulate the variation from the classical path with the so-called commutation rule

$$\int [P, \varphi] dV = \hbar. \quad (12)$$

Mathematically this pertains to the total variation of action. We can express both the potential and the momentum in the Fourier series

$$\varphi = \sum_k \sqrt{\frac{2}{V}} (C_k \cos kx + S_k \sin kx), \quad (13)$$

$$P = \sum_k \sqrt{\frac{2}{V}} (\dot{C}_k \cos kx + \dot{S}_k \sin kx), \quad (14)$$

where the function coefficients C_k and S_k depend on time only, V is the volume, and k is the wave number. Substituting them in the Lagrangian in Eq. (6), and integrating the Lagrangian over the entire volume, we obtain

$$L = \frac{1}{2} \sum_k \left[(\dot{C}_k^2 + \dot{S}_k^2) + D^2 k^4 (C_k^2 + S_k^2) \right]. \quad (15)$$

The relevant momenta are

$$P_k^{(C)} = \dot{C}_k, \quad (16)$$

$$P_k^{(S)} = \dot{S}_k. \quad (17)$$

Now, we can write the Hamiltonian

$$\begin{aligned} H &= P_k^{(C)} \dot{C}_k + \dot{S}_k P_k^{(S)} - L \\ &= \sum_k \left[\left(\frac{1}{2} P_k^{(C)2} - \frac{1}{2} D^2 k^4 C_k^2 \right) + \left(\frac{1}{2} P_k^{(S)2} - \frac{1}{2} D^2 k^4 S_k^2 \right) \right]. \end{aligned} \quad (18)$$

At this point, we can introduce the operator formalism. The momenta ($P_k^{(C)}$, $P_k^{(S)}$), and the coefficients (C_k , S_k) are operators allowing the commutation rules

$$[P_k^{(C)}, C_l] = \hbar \delta_{kl}, \quad (19)$$

$$[P_k^{(S)}, S_l] = \hbar \delta_{kl}, \quad (20)$$

by which we can introduce creation and annihilation operators

$$C_k^\mp = \frac{P_k^{(C)}}{\sqrt{2}} \mp \frac{D k^2}{\sqrt{2}} C_k, \quad (21)$$

$$S_k^\mp = \frac{P_k^{(S)}}{\sqrt{2}} \mp \frac{D k^2}{\sqrt{2}} S_k. \quad (22)$$

These obey the following commutation rules

$$C_k^- C_l^+ - C_l^+ C_k^- = D k^2 \hbar \delta_{kl}, \quad (23)$$

$$S_k^- S_l^+ - S_l^+ S_k^- = D k^2 \hbar \delta_{kl}. \quad (24)$$

It is easy to check that the unit of $D k^2 \hbar$ is J, i.e., the quantum of energy stands on the right-hand side of these relations. The Hamiltonian operator – the entire energy on the system in quantized form – is

$$E = H = \frac{1}{2} \sum_k (C_k^+ C_k^- + C_k^- C_k^+ + S_k^+ S_k^- + S_k^- S_k^+). \quad (25)$$

The dissipative behavior of the heat conduction appears as a repulsive interaction in Eq. (18). However, the description can handle the unbounded spectrum from below due to the inverted parabola potential. Despite these difficulties, the mathematical formulation serves a quantized temperature increase $\Delta T = \Delta E / C_v$ for an elementary excitation of ΔE of one degree of freedom [26, 27]:

$$\Delta T(k) = \frac{2\hbar}{k_B} D k^2, \quad (26)$$

It depends on the wave number of k of the propagation mode. Here, we take into account the relation

$$C_v = \frac{1}{2} k_B. \quad (27)$$

The calculations reflect that the thermal energy propagation proceeds as if a repulsive potential would act. This means that the spectrum is continuous. The situation is very similar to the case of the study of harmonic oscillators with an inverted parabolic potential [28]. The interaction is repulsive, the wave number is arbitrarily continuous.

4 Comparison of the Temperature Increment and the Thermal Excitation

The spontaneous collapse model suggests the idea that the macroscopic oscillation mode turns into a quantum thermal effect, i.e., comparing the results in Eqs. (5) and (26), we should find the same measure of temperature increment. To achieve this aim, we need to create a relevant, comparable physical model. The oscillator in Eq. (4) is originally a gravitational oscillator. (These studies were started to verify the assumption that the gravitational field destroys the coherent quantum states.)

A small particle with a mass of m is moving radially inside the Earth as Fig. 1 shows. The effective force – at the instantaneous position, the radius (r) of the mass-point, where the gravitational interaction originates from the internal sphere with a radius of r –, $F(r)$ which is acting on the particle

$$F(r) = -\gamma \frac{mM \frac{r^3}{R^3}}{r^2} = -\gamma \frac{mM}{R^3} r, \quad (28)$$

where M is the mass of the Earth, R is the radius, and γ is the gravitational constant. The equation of motion of the particle is

$$m\ddot{r} = -\gamma \frac{mM}{R^3} r, \quad (29)$$

which pertains to a harmonic oscillator. The angular frequency ω is

$$\omega = \sqrt{\frac{\gamma M}{R^3}}. \quad (30)$$

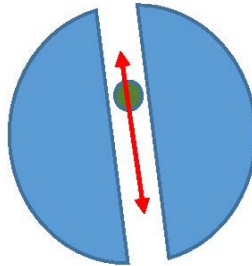


Figure 1

Gravitational oscillator: a tiny particle of mass of m oscillates inside an Earth-like mass of M . The acting gravitational force on the particle is proportional to the displacement of r .

Due to the big mass of the Earth, we may replace this model with a fixed at-one-end suspended oscillator: A mass of m oscillates at the end of a spring; k_s denotes the stiffness constant. This elastic coefficient can be identified as

$$k_s = \gamma \frac{mM}{R^3}. \quad (31)$$

It ensures the required angular frequency, but this physical model is somehow far from the picture that the oscillator can move freely. The fixed end is a rough mechanical connection to the environment. However, from the viewpoint of the thermal quantum, it seems better to imagine the vibration system as a coupled oscillator, a pair of two $m' = m/2$ masses bounded with the same spring. Naturally, the spring constant, k_s is the same.



Figure 2

Free-ended oscillators with the total mass of m . The stiffness constant of the spring is k_s . The angular frequency of the vibration is $\omega' = 2\sqrt{k_s/m}$.

In this modified situation the vibration is going around the center of mass, thus the effective spring constant becomes, $k'_s = 2k_s$. Thus, the frequency formula of the suspended oscillator $\omega = \sqrt{k_s/m}$ turns into

$$\omega' = \sqrt{\frac{k'_s}{m'}} = \sqrt{\frac{2k_s}{m/2}} = 2\omega, \quad (32)$$

due to the vibration around the equilibrium point. This means that the relation in Eq. (4) should be rewritten in a modified form as

$$D_{mod-sp} = 2\hbar m\omega^2, \quad (33)$$

fitted to the present model. We need to mention that the calculation of the effective mass is not so trivial in general. Sometimes the approximation requires careful consideration [29].

Then the related temperature increment is

$$\Delta T_{mod-sp} = \frac{2\hbar\omega^2}{k_B} \tau. \quad (34)$$

At this point, the temperature increments in Eqs. (5) and (26) can be compared. To this, we consider – on the one hand – the phase speed c of the propagation mode k is simply

$$c = \frac{\omega}{k}. \quad (35)$$

On the other hand, according to the hypothesis of “*Extended Irreversible Thermodynamics*” the finite speed of thermal signals (pulses) is ensured by the telegrapher equation [1, 30]

$$\tau' \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} - D \frac{\partial^2 T}{\partial x^2} = 0. \quad (36)$$

The first term of the equation comes from the inertia of the thermal effect suggested by Maxwell first, then much later by Cattaneo, and independently Vernotte, too. The parameter τ' expresses the relaxation time. Now, we focus on the dispersion relation of the above differential equation

$$-\tau'\omega'^2 + i\omega' + Dk'^2 = 0, \quad (37)$$

where ω' is the (real) frequency and k' is the (complex) wave number. The calculated phase velocity is

$$c = \frac{\omega'}{\operatorname{Re} k'} = \frac{\sqrt{2D\omega'}}{\sqrt{\tau\omega' + \sqrt{1 + \tau'^2 \omega'^2}}}. \quad (38)$$

In the high-frequency limit ($\omega' \rightarrow \infty$) the propagation speed can be obtained as

$$c = \sqrt{\frac{D}{\tau'}}. \quad (39)$$

At this point, we may identify this propagation time τ' with the relaxation time τ of the oscillator in Eq. (5), i.e., $\tau' = \tau$. Thus, the diffusivity parameter can be formulated as – taking into account the phase speed in Eq. (35) –

$$D = c^2 \tau = \frac{\omega^2}{k^2} \tau. \quad (40)$$

Here, this coefficient can be substituted into the quantized temperature increase given in Eq. (26). Now, the temperature increment during the dissipative process can be immediately obtained as

$$\Delta T = \frac{2\hbar\omega^2}{k_B} \tau, \quad (41)$$

which is exactly the expression of the spontaneously collapsed oscillator as is shown in Eq. (34). In this way, the thermodynamic background was successfully matched to the spontaneous decaying, collapsing, or absorbing process of the oscillators. At this point, we can reverse the thinking. If the spontaneous collapse of an oscillator generates a thermal process, reversely, the thermal process also relates to the collapses of wave functions of oscillators. In this way, we can understand why the Fourier heat conduction turns into a perfect irreversible process. The spontaneous processes cannot be reversed. As we know it from thermodynamic studies.

The spontaneous oscillator collapsing and the related irreversible effects may cause unpredictable changes in the signal propagation, e.g., in teleoperation [31], or nanoscale electronic devices [32]. It is just a suggestion that the discussed phenomenon can participate in various electronic devices in influencing its dispersion relations, thereby tuning the physical properties [33, 34].

Conclusions

We examined the relation between the spontaneously collapsing macroscopic quantum oscillators and the related thermal excitations from a thermodynamic point of view. It has been shown that the calculated temperature increment during the oscillator wave function collapse is proportional to the square of the oscillator frequency. In the comparison, we consider the expression of the deduced quantum thermal excitation, originating from the quantization procedure of thermal propagation, the Fourier's heat conduction. These excitations are also quadratic in the wave number. As an example, we apply the model for a mechanical oscillator, by which we can find a thermodynamic connection between the two kinds of approaches. The discussed description is not restricted to the peculiar type oscillator

cases, so the physical application is widely applicable. It means that any oscillator collapse involves the described thermodynamical process. The presented model shows a possible way how to apply the concept of dissipation (irreversible properties) in the field theories.

Acknowledgment

TKP2021-NVA-16 has been implemented with the support provided by the Ministry of Innovation and Technology of Hungary from the National Research, Development and Innovation Fund.

References

- [1] G. Lebon, D. Jou, J. Casas-Vázquez: *"Understanding non-equilibrium thermodynamics"* Springer, Berlin (2008)
- [2] N. Li, J. Ren, L. Wang, G. Zhang, P. Hänggi, B. Li: *Colloquium: Phononics: Manipulating heat flow with electronic analogs and beyond*. Rev. Mod. Phys. **84** (2012) 1045
- [3] B. G. Márkus, F. Márkus: Quantum Particle Motion in Absorbing Harmonic Trap. Indian J. Phys. **90** (2016) 441
- [4] G. Csósz, B. Dóra, F. Simon: Entropy in Spin Relaxation, Spintronics, and Magnetic Resonance. Phys. Status Solidi B **257** (2020) 2000301
- [5] G. Csósz, L. Szolnoki, A. Kiss, B. Dóra, F. Simon: Generic Phase Diagram of Spin Relaxation in Solids and the Loschmidt Echo. Phys. Rev. Res. **2** (2020) 033058
- [6] F. Márkus, K. Gambár: Quantized, Damped Transversal Single Particle Mechanical Waves. Quantum Rep. (2023)
- [7] G. M. Palma, K.-A. Suominen, A. K. Ekert: Quantum Computers and Dissipation. Proc. Roy. Soc. A **452** (1996) 567
- [8] B. G. Márkus, P. Szirmai, K. F. Edelthammer, P. Eckerlein, A. Hirsch, F. Hauke, N. M. Nemes, J. C. Chacón-Torres, B. Náfrádi, L. Forró, T. Pichler, F. Simon: Ultralong Spin Lifetime in Light Alkali Atom Doped Graphene. ACS Nano **14** (2020) 7492
- [9] B. G. Márkus, M. Gmitra, B. Dóra, G. Csósz, T. Fehér, P. Szirmai, B. Náfrádi, V. Zólyomi, L. Forró, J. Fabian, F. Simon: Ultralong 100 ns Spin Relaxation Time in Graphite at Room Temperature. Nat. Commun. **14** (2023) 2831
- [10] O. Romero-Isart: Quantum superposition of massive objects and collapse models. Phys. Rev. A **84** (2011) 052121
- [11] L. Diósi: Testing spontaneous wave-function collapse models on classical mechanical oscillators. Phys. Rev. Lett. **114** (2015) 050403

- [12] A. Bassi, K. Lochan, S. Satin, T. P. Singh, H. Ulbricht: Models of wave-function collapse, underlying theories, and experimental tests. *Rev. Mod. Phys.* **85** (2013) 471
- [13] Diósi, L.: Gravity-related wave function collapse: mass density resolution. *J. Phys. Conf. Ser.* **442** (2013) 012001
- [14] Nimmrichter, K. Hornberger, K. Hammerer: Optomechanical sensing of spontaneous wave-function collapse. *Phys. Rev. Lett.* **113** (2014) 020405
- [15] L. Diósi: Gravity-related wave function collapse: Is superfluid He exceptional? *Found. Phys.* **44** (2014) 483
- [16] L. Diósi: Gravity-related spontaneous wave function collapse in bulk matter. *New J. Phys.* **16** (2014) 105006
- [17] L. Diósi: Schrödinger–Newton Equation with Spontaneous Wave Function Collapse. *Quantum Rep.* **4** (2022) 566
- [18] D. Manzano: A short introduction to the Lindblad master equation. *AIP Advances* **10** (2020) 025106
- [19] J. Keizer: On the solutions and the steady states of a master equation. *J. Stat. Phys.* **6** (1972) 67
- [20] F. Márkus, K. Gambár: Minimum entropy production effect on a quantum scale. *Entropy* **23** (2021) 1350
- [21] C. Strunk: Quantum Transport of Particles and Entropy. **23** (2021) 1573
- [22] U. Hohm, C. Schiller: Testing the Minimum System Entropy and the Quantum of Entropy. *Entropy* **25** (2023) 1511
- [23] K. Gambár, F. Márkus: Hamilton-Lagrange formalism of nonequilibrium thermodynamics. *Phys. Rev. E* **50** (1994) 1227
- [24] F. Márkus, K. Gambár: Temperature and entropy production operator in Fourier heat conduction. *Phys. Rev. E* **52** (1995) 623
- [25] K. Gambár, F. Márkus: On an Unusual Quantization Procedure of Heat Conduction. *Open. Sys. Inf. Dyn.* **8** (2001) 369
- [26] F. Márkus, K. Gambár: Derivation of the upper limit of temperature from the field theory of thermodynamics. *Phys. Rev. E* **70** (2004) 055102
- [27] F. Márkus, K. Gambár: Quasiparticles in a thermal process. *Phys. Rev. E* **71**(2005) 066117
- [28] H. Risken: *"The Fokker-Planck equation"* pp. 245-249, Springer, Berlin (1996)
- [29] L. Cveticanin, Gy. Mester: Theory of acoustic metamaterials and metamaterial beams: An overview. *Acta Polytechn. Hung.* **13** (2016) 43

- [30] D. Jou, J. Casas-Vázquez, G. Lebon: "*Extended Irreversible Thermodynamics*" (Second revised and enlarged edition) pp. 167-172, Springer, Berlin (1996)
- [31] R. Muradore, P. Fiorini: A review of bilateral teleoperation algorithms. *Acta Polytechn. Hung.* **13** (2016) 191
- [32] A. Ürmös, Z. Farkas, M. Farkas, T. Sándor, L. T. Kóczy, Á. Nemcsics: Application of self-organizing maps for technological support of droplet epitaxy. *Acta Polytechn. Hung.* **14** (2017) 207
- [33] M. Fried, A. van Silfhout: Optical dispersion relations in two types of amorphous silicon using Adachi's expression. *Phys. Rev. B* **49** (1994) 5699
- [34] T. Lohner, E. Szilágyi, Zs. Zolnai, A. Németh, Zs. Fogarassy, L. Illés, E. Kótai, P. Petrik, and M. Fried: Determination of the complex dielectric function of ion-implanted amorphous germanium by spectroscopic ellipsometry. *Coatings* **10** (2020) 480