Evaluation and Selection of Design Solutions for a Small Laboratory Tensile Testing Device under a Type 2 Fuzzy Environment

Sonja Kostić[1], Danijela Tadić[2*], Jasmina Miljojković[3], Vladimir Kočović[4], Nikola Komatina[5] and Saša Jovanović[6]

[1]Academy of Professional Studies Sumadija, Technical College of Applied Studies, Kosovska 8, 34000 Kragujevac, Republic of Serbia
e-mail: skostic@asss.edu.rs

[2,3,4,5,6]University of Kragujevac, Faculty of Engineering, Sestre Janjić 6, 34000 Kragujevac, Republic of Serbia
e-mail: galovic@kg.ac.rs, jasmina.miljojkovic@fink.rs, vladimir.kocovic@kg.ac.rs, nkomatina@kg.ac.rs, dviks@kg.ac.rs
* Corresponding author

Abstract: In this research, evaluating and selecting design solutions for a small laboratory tensile testing device is stated as a multi-criteria, decision analysis problem, incorporating both quantitative and qualitative criteria. Fuzzy ratings of criteria values are provided by managers, students, and potential customers. Their assessments are described using linguistic expressions modelled by type 2 triangular fuzzy numbers. The ranking of various design solutions for the small laboratory tensile testing devices is carried out by applying the proposed two-stage fuzzy model. In the first stage, the weight vector is calculated by using Criteria Importance Through Intercriteria Correlation, which is extended with type 2 triangular fuzzy numbers. The ranking of small laboratory tensile testing device design solutions is obtained by applying the proposed Technique for Order Preference by Similarity to Ideal Solution with type 2 triangular fuzzy numbers. The proposed two-stage fuzzy model is tested on real-life data, originating from an industrial company, operating in the Republic of Serbia. By applying the proposed methodology, the best design solutions for the small laboratory tensile testing devices are selected in an exact manner. This solution is less burdened by subjective decision-makers' opinions, making it more accurate. In this way, the risk of diversification is reduced, while simultaneously enhancing the competitiveness and sustainability of the business in the long run.

Keywords: Small laboratory tensile testing device; Evaluation; Selection; Multi-Criteria Decision Analysis; Type 2 fuzzy numbers

1 Introduction

Tensile testing is the most widespread and simplest method used for testing the mechanical properties of materials, as it best describes the behavior of materials under load. Conventional tensile testing devices, usually of large dimensions and mass, are intended for use in special laboratory conditions and the geometry and shape of the test samples are proscribed by standards. Currently, the optimization of the production process, procurement and placement of products on the market concur with a tendency towards digitization, automation and implementation of artificial intelligence in industry. Manufacturers of testing devices are faced with an increasing demand for compact, reliable and easy-use products designed for applying low and medium forces when testing different materials and not requiring special conditions for installation in the production plant.

In addition, the development of science and technology has shown a clear trend of applying new, expensive materials in machine systems and reducing their dimensions due to both costs and the minimization of industrial components. That produced the need to produce small, non-conventional tensile testing devices, which enable working with samples of smaller cross-sections, non-standard shapes and dimensions, but still provide reliable and accurate results. Furthermore, small laboratory tensile testing devices can be used for educational purposes, considering the economic aspect and their technological and ergonomic advantages compared to conventional devices.

Today, special attention is paid to the development and production of low-cost, unconventional tensile testing devices with good technical-technological and ergonomic properties, which are, above all, reliable and multifunctional and can be used both in the industry for obtaining the essential mechanical characteristics of materials and in laboratories, for educational and research purposes. The design of a modern tensile testing device should focus on ease of use and light construction, whose strength and rigidness are high enough to ensure resistance to deformations when applying loads during the test, as this directly affects the accuracy of the elongation measurement. Grips for clamping the sample should be simple, easy to handle and prevent the ends of the sample from slipping during the test. The load transferred to the specimen must be applied steadily, without impacts, and if necessary, be kept constant for a long time, while the reading of force value with satisfactory accuracy should be enabled at any moment during the test.

The first unconventional tensile testing devices were created in the middle and end of the last century [1]. Afterwards, their development was directed towards the improvement of the measurement results accuracy and the seeking optimal solutions, both for the devices themselves and for the geometry of the test samples [2-7]. Lim and Kim [8] presented a device for tensile testing, the primary purpose of which was the education of students at technical faculties.

During product development, designers aim for the product to satisfy certain requirements regarding its functionality and design, manufacturing simplicity with minimal costs, safety, ease of use, etc. When designing a product, it is necessary to consider various limitations related to both the production system and the environment. This paper presents four different design solutions for a small laboratory tensile testing device (SLTTD), i.e., a device that can be used for educational and research purposes but also be applied in industry for tensile testing of various materials.

The changes occurring in the business environment make describing some criteria values by precise numbers challenging. In this case, uncertain criteria values can be adequately described by pre-defined linguistic expressions modelled by applying the type 2 fuzzy sets (IT2FSs). The concept of IT2FSs was introduced by Zadeh [9]. The use of IT2FSs has benefits such as a higher degree of freedom and flexibility, but at the same time, computational operations are significantly more complex. Therefore, the modelling of uncertain criteria values is based on the interval type 2 fuzzy numbers (IT2FNs) as a special case of IT2FSs. Many authors use type 2 fuzzy triangular fuzzy numbers (IT2TFNs) for handling uncertainties [10-13] because they have been successfully applied in perceptual computing.

In this research, the ranking of considered design solutions for SLTTD is achieved by integrating two Multi-Attributive Decision Making (MADM) techniques enhanced with Interval Type-2 Fuzzy Numbers (IT2TFNs). Initially, the weights vector is computed using the CRiteria Importance Through Intercriteria Correlation (CRITIC) with IT2TFNs (ITFCRITIC). As indicated in the paper Aleksić and Tadić [14] most authors recommend employing the Technique for Order of Preference by Similarity to Ideal Solution with IT2FNs (IT2FTOPSIS) for ranking alternatives. Hence, in the subsequent stage, the ranking of design solutions for SLTTD is conducted utilizing the proposed IT2FTOPSIS.

In the analyzed papers [15-19] decision matrix values are described as uncertain numbers. In this research, the elements of a fuzzy decision matrix can be either precise numbers or uncertain data, which can be identified as one of the differences and simultaneously an advantage of this research compared to the analyzed papers. In this research, uncertain criteria values are described using a seven-point scale as in [14, 15, 18, 20] and a five-point scale as in [16, 17, 19]. The authors (from TOPSIS) suggest that the normalization of the decision matrix should be carried out, using the proposed linear normalization procedure. However, in this research, the vector normalization procedure combined with type 2 fuzzy algebra rules is employed, representing one of the fundamental differences between the analyzed research papers and our manuscript.

The solutions obtained using CRITIC are more accurate than those based on evaluations made by decision-makers (DMs). Conversely, CRITIC is less complex than pairwise comparison MADM with fuzzy sets. In the literature, one can find papers where the CRITIC method has been expanded with type 1 fuzzy set numbers

[15] [16] or a decision matrix with higher-order fuzzy numbers [21] [22]. However, there are no papers in the literature where CRITIC is extended with IT2FNs, representing the main difference and advantage of our work compared to others where the CRITIC method for weight determination is presented. The calculation of correlation coefficients for each pair of criteria is based on the procedure proposed in the conventional CRITIC combined with distances between two fuzzy numbers [22], as in this research.

In all analyzed papers, the weighted normalized fuzzy decision matrix is constructed respecting the principle of added value [17-19] combined with type 2 fuzzy algebra rules, as in this research. The transformation of the weighted normalized fuzzy decision matrix into the weighted normalized decision matrix is performed in [19, 23], significantly simplifying the process of ranking, which can impact the accuracy of the solution. FPIS and FNIS are defined according to the procedure proposed by Kuo et al. [24] in [17-19],so that the calculated distances from FPIS and FNIS, as well as the closeness coefficient, are described using precise numbers. Determining FPIS and FNIS is based on the veto concept in [20] [25], so the distance from FPIS and FNIS, as well as the closeness coefficient, are calculated using type 2 fuzzy algebra rules [26]. The rank of considered issues is determined based on scalar values of the closeness coefficient obtained using the moment method in [20] and the defuzzification procedure proposed by Kahraman et al. [27], as in this research.

The motivation for this research comes from the fact that there are no papers dealing with the problem of evaluation and ranking of design solutions by employing IT2FMADM.

The broader objective of this research may be interpreted as the integration of embracing methods: (i) modelling of criteria values by IT2TFNs, (ii) determination of some uncertain criteria values stated as fuzzy group decision making, (iii) determination of criteria weights based on the proposed IT2FCRITIC, (iv) determination of the SLTTD design solutions rank by using the proposed IT2FTOPSIS. The paper is organized as follows. The proposed methodology for the problem of ranking the SLTTD design solutions is given in Section 2. Section 3 discusses the proposed two-stage fuzzy model illustrated by real-life data. The conclusion and discussion are presented in Section 4.

2 Methodology

This section shows the modelling procedure of uncertain data using type-2 fuzzy sets theory and the procedures for determining the weights vector and rank of design solutions of SLTTD by applying the proposed IT2FCRITIC and IT2FTOPSIS, respectively. The proposed model is depicted in Figure 1.

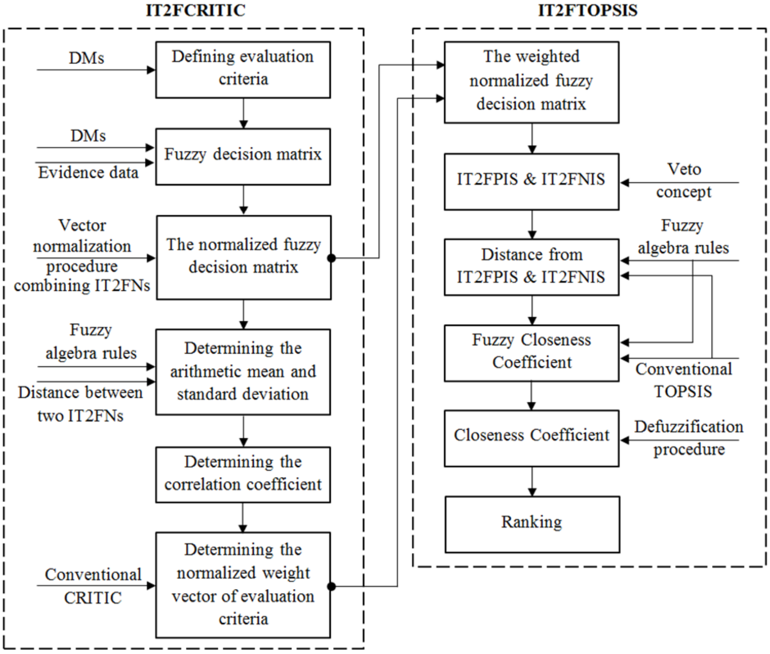


Figure 1

The proposed model

The proposed model is based on two stages. The first stage involves the application of the IT2FCRITIC method, while the second stage focuses on the application of the IT2FTOPSIS method. The objective of the first stage is to determine the weights of the evaluation criteria. These criteria are predefined by the decision-makers.

The result of the first stage, i.e., the criteria weights, serves as input data for the initiation of the second stage. The fuzzy decision matrix values are weighted according to the determined criteria weights. By applying the TOPSIS method, along with the use of fuzzy algebra rules, the second stage involves ranking the alternatives. In this case, these alternatives are SLTTD. A detailed step-by-step explanation of the proposed model follows in the continuation of this chapter.

2.1 Definition and Description of the Evaluation Criteria

The criteria for evaluating tensile testing devices are numerous and can be classified into different groups. One class of criteria is labelled as hard requirements. These criteria must be met; otherwise, the design is entirely unacceptable, i.e., the device is unsuitable for use [28]. These criteria were not considered when the SLTTD design solutions were evaluated, as all proposed solutions fully satisfy these requirements. The second class of criteria is labelled as minimum requirements. Another class of requirements consists of criteria that must be fulfilled to the greatest extent, considering the existing conditions and all the accompanying restrictions. Therefore, the evaluation and selection of design solutions for any device should be based on second-class criteria.

Formally, these criteria are presented by set $\{1, \ldots, k, \ldots, K\}$, where K denotes the total number of criteria and k is the index of criterion, $k = 1, \ldots, K$. The selection of criteria for evaluating the design solutions can be considered a separate problem. In this research, the goal was to select a design solution for SLTTD that can be produced in series. According to the defined goal, the general manager, supply manager, designer and sales manager, adopted a set of criteria that will be further described.

Measurement accuracy $(k = 1)$ is defined as the uncertainty in elastic modulus measurement within the interval 1.2%-5% (at the confidence level of 95%) [29].

Ease-of-use $(k = 2)$ can be defined as the convenience and ease of operation. For users of the considered device, it is very important to assess the stability of the structure, which, among other things, depend on the proportions of the device's supporting frame, and to evaluate the ease of handling the device during material testing.

One of the primary market requirements for any product, including the SLTTD, is its competitiveness. The unit price has the greatest influence on the competitiveness of the SLTTD. Therefore, decision-makers believe the third criterion should be the Unit price $(k = 3)$.

The design of SLTTD $(k = 4)$ is defined depending on the intended purpose [30]. The first considered design solution can be used to educate students of technical universities at all levels of studies. Other SLTTD design solutions can be used in industry and research. When designing new solutions, it was considered that adding new elements, such as a stepping motor and electronics, would not only automate the testing process but also improve the SLTTD design.

2.2 Defining the Set of Design Solutions for SLTTD

A set of design solutions can be represented by a set of indexes $\{1, \ldots, i, \ldots, I\}$. The total number of considered intervals is denoted as *I*. The index of alternative is $i,\ i = 1, \ldots, I$.

In this paper, four design solutions of SLTTD are considered. These SLTTDs were designed by [31], and their characteristics are described in the Case study.

2.3 Modelling of the Criteria Values

Uncertain and imprecise criteria values are assessed based on the subjective judgment of DMs. They can better express their opinions using natural language words than measurement scales. In this research, linguistic expressions are modelled by IT2TFNs, which do not require complex mathematical calculations and simultaneously capture natural language uncertainties adequately. There are no recommendations or rules on how to determine granularity. The number of linguistic expressions sufficient to describe existing uncertainties is typically determined relative to the size of the problem. The domain of the IT2TFNs is defined on the real line. In this research, it is assumed that the domains of the used IT2TFNs are within the interval [1-9], which is analogous to Saaty's measurement scale. The values 1 and 9 represent the smallest and the largest value of the considered criteria, respectively.

In order to describe the uncertainties that exist in the considered problem IT2TFNs were used. The measurement accuracy ($k = 1$) is determined based on experimental research. In practice, it is common to repeat the experiment several times. It is almost impossible to achieve identical measurement accuracy. In this paper, the measurement accuracy was determined by monitoring the uncertainty value of the elastic modulus [32]. The values of this criterion can be described using five different linguistic expressions modelled by IT2TFNs, as shown in Table 1, along with the description of each linguistic expression. The limit values of the uncertainty in the elastic modulus measurement are determined based on literature sources [29, 33-35]. Table 1 and Table 2 provide the linguistic expressions modelled by using IT2TFNs.

Table 1

The linguistic expressions describing the criteria values for SLTTD

Linguistic expressions	**Corresponding IT2TFNs**	**Description of the measurement accuracy**
very high (L1)	$((1, 2, 3; 1), (1.5, 2, 2.5; 0.8))$	Uncertainty in the determination of the elastic modulus is less than 1.6% of the reference value.
high (L2)	$((2.5, 4, 5.5; 1), (3, 4, 5; 0.8))$	Uncertainty in the determination of the elastic modulus is less than 2% of the reference value.
medium (L3)	$((3.5, 5, 6.5; 1), (4, 5, 6; 0.8))$	Uncertainty in the determination of the elastic modulus is less than 3% of the reference value.
low (L4)	$((4, 5.5, 7; 1), (4.5, 5.5, 6.5; 0.8))$	Uncertainty in the determination of the elastic modulus is less than 5% of the reference value.
very low (L5)	$((7, 8, 9; 1), (7.5, 8, 8.5; 0.8))$	Uncertainty in the determination of the elastic modulus is less than 10% of the reference value.

Table 2 provides a description of the linguistic expressions that can be used to evaluate the criteria values: ease-of-use ($k = 2$) and design ($k = 4$). Decision makers consider these two criteria can be described sufficiently well using seven different linguistic expressions.

Table 2
The linguistic expressions describing the ease-of-use and design of SLTTD

Linguistic expressions	**Corresponding IT2TFNs**	**Description of the ease-of-use**	**Descriptions of design**
very low (M1)	$((1, 1, 2.5; 1), (1, 1, 2; 0.8))$	very complicated handling-manual control of the testing process	barely acceptable design
low (M2)	$((1, 2, 3; 1), (1.5, 2, 2.5; 0.8))$	very complicated handling-automated control of the testing process	acceptable design
medium-low (M3)	$((2.5, 4, 5.5; 1), (3, 4, 5; 0.8))$	medium-complicated handling	good enough design
medium (M4)	$((4, 5.5, 7; 1), (4.5, 5.5, 6.5; 0.8))$	complicated handling	good design
medium-high (M5)	$((5.5, 7, 8.5; 1), (6, 7, 8; 0.8))$	medium-easy handling	very good design
high (M6)	$((7, 8, 9; 1), (7.5, 8, 8.5; 0.8))$	easy handling	excellent design
very high (M7)	$((7.5, 9, 9; 1), (8, 9, 9; 0.8))$	very easy handling	exceptional design

Generally, the values of lower, upper, and modal values of both membership functions of the used IT2TFNs can be determined exactly [36]. In the majority of papers analyzed in [14], these values are determined based on subjective assessment, as in this research.

2.4 The Proposed IT2FCRITIC

The CRITIC method [37] is one of the most commonly used objective MADM techniques. It falls under the category of correlation methods, utilizing standard deviations of normalized criterion values and correlation coefficients of all pairs of criteria. The steps of the proposed IT2FCRITIC are presented in the following.

Step 1. The fuzzy decision matrix is presented as

$$[\tilde{\tilde{x}}_{ik}]_{IxK} \tag{1}$$

Step 2. Transform the fuzzy decision matrix into the normalized fuzzy decision matrix $[\tilde{\tilde{r}}_{ik}]_{IxK}$ by using vector normalization procedure:

Step 3. Determine the standard deviation:

a) crisp value:

$$r_k = \frac{1}{I} \cdot \sum_{i=1,..,I} r_{ik} \tag{2}$$

$$\sigma_k = \sqrt{\frac{1}{I-1} \cdot \sum_{i=1,..,I} (r_{ik} - r_k)^2} \tag{3}$$

b) fuzzy value:

$$\tilde{\tilde{r}}_k = \frac{1}{I} \cdot \sum_{i=1,...,I} \tilde{\tilde{r}}_{ik} \tag{4}$$

$$\sigma_k = \sqrt{\frac{1}{I-1} \cdot \sum_{i=1,...,I} \left(d(\tilde{\tilde{r}}_{ik}, \tilde{\tilde{r}}_k)\right)^2} \tag{5}$$

where $d(\tilde{\tilde{r}}_{ik}, \tilde{\tilde{r}}_k)$ is the distance between the two IT2TFNs [38].

Step 4. Determine the correlation coefficients for each pair of criteria, $\rho_{kk'}$:

$$\rho_{kk'} = \frac{\sum_{i=1,..,I} d(\tilde{\tilde{r}}_{ik}, \tilde{\tilde{r}}_k) \cdot d(\tilde{\tilde{r}}_{ik'}, \tilde{\tilde{r}}_{k'})}{\sqrt{\left(d(\tilde{\tilde{r}}_{ik}, \tilde{\tilde{r}}_k)\right)^2 \cdot \left(d(\tilde{\tilde{r}}_{ik'}, \tilde{\tilde{r}}_{k'})\right)^2}} \tag{6}$$

Step 5. Determine the criteria weights:

$$W_k = \sigma_k \cdot \sum_{k'=1,..,K} (1 - \rho_{kk'}) \tag{7}$$

Step 6. The normalized weights vector is given by using linear normalization procedure:

$$[\omega_k]_{Kx1} \tag{8}$$

where:

$$\omega_k = \frac{W_k}{\sum_{k=1,..,K} W_k} \tag{9}$$

2.5 The Proposed IT2FTOPSIS

TOPSIS [39] is based on the concept that the best alternative should be closest to the positive ideal solution and farthest from the negative ideal solution. In the literature, numerous studies can be found where conventional TOPSIS is extended with IT2TFNs. IT2FTOPSIS can be successfully applied in solving various problems in a fuzzy environment. The six steps of the proposed IT2FTOPSIS are presented as follows.

Step 1. The weighted normalized fuzzy decision matrix is stated as:

$$[\tilde{\tilde{z}}_{ik}]_{IxK} \tag{10}$$

Where:

$$\tilde{\tilde{z}}_{ik} = \omega_k \cdot \tilde{\tilde{r}}_{ik} \tag{11}$$

Step 2. Fuzzy Positive Ideal Solution (FPIS), $\tilde{\tilde{z}}_k^+$ and Fuzzy Negative Ideal Solution (FNIS), $\tilde{\tilde{z}}_k^-$ is defined according to the veto concept so that:

$$FPIS = \big((1,1,1;1),(1,1,1;1)\big);\ FNIS = \big((0,0,0;1),(0,0,0;1)\big)$$

Step 3. Calculate the distances from FPIS (Eq. 12) and FNIS (Eq. 13) at the level of each alternative $i, i = 1,..,I$:

$$\tilde{\tilde{d}}_i^+ = \sum_{k=1,\dots,K} \frac{(\tilde{\tilde{z}}_k^+ - \tilde{\tilde{z}}_{ik})}{\tilde{\tilde{z}}_k^+ - \tilde{\tilde{z}}_k^-} \tag{12}$$

$$\tilde{\tilde{d}}_i^- = \sum_{k=1,\dots,K} \frac{(\tilde{\tilde{z}}_{ik} - \tilde{\tilde{z}}_k^-)}{\tilde{\tilde{z}}_k^+ - \tilde{\tilde{z}}_k^-} \tag{13}$$

Step 4. The relative closeness coefficient can be calculated by using the procedure defined in conventional TOPSIS combined with IT2FTFNs:

$$\tilde{\tilde{c}}_i = \frac{\tilde{\tilde{d}}_i^-}{\tilde{\tilde{d}}_i^- + \tilde{\tilde{d}}_i^+} \tag{14}$$

Step 5. The representative scalars of IT2FTFNs, c_i is given using the defuzzification procedure proposed by [27].

Step 6. The crisp values c_i are sorted in descending order. The SLTTD design solution with the highest value of the relative closeness coefficient is in the first place. This design solution should be adopted for series production.

3 Case Study

The proposed methodology was tested on data obtained from an industrial company operating in the Republic of Serbia. Among other products, the company manufactures measuring devices and educational equipment considered in this paper and places them in both domestic and foreign markets. Customers of the considered product type are manufacturing companies, material testing laboratories, and educational institutions. Therefore, company management must offer new products to the market to increase competitiveness and sustainability.

The company's design team has created four SLTTD design solutions that can be used for tensile testing of materials. These design solutions were evaluated according to the four previously defined criteria. For the values of the criteria to be obtained, the company's management decided that prototypes of each design solution must be created.

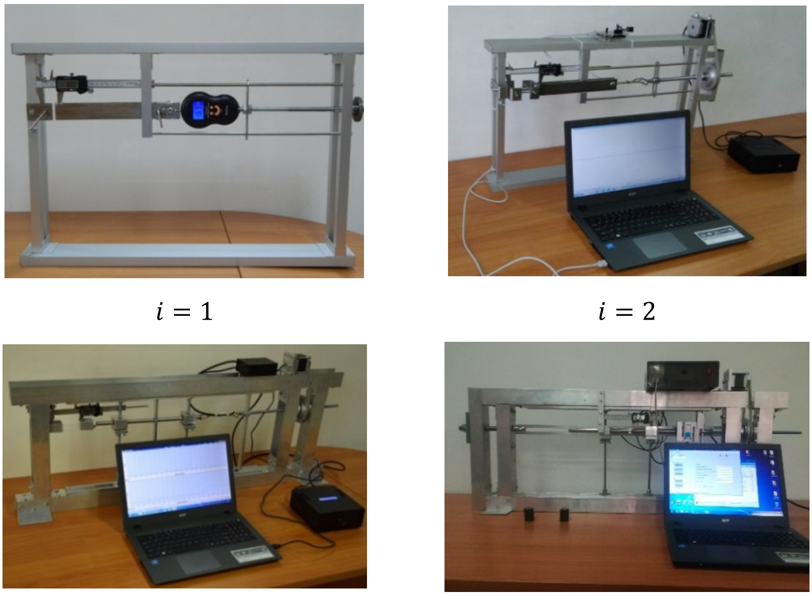
The criteria values for each SLTTD design solution were obtained in different ways. For example, the unit price ($k = 3$) is determined by the general manager based on information from the supply and sales managers. The accuracy of measurement ($k = 1$) at the level of each SLTTD design solution is described by the designer using one of five pre-defined linguistic expressions. The designer bases the assessments on the results obtained during experimental research.

The value of the second criterion, marked as ease-of-use ($k = 2$) at the level of each design solution, was obtained based on the interview method. One hundred students from the Republic of Serbia's higher education institutions participated in the interview. They performed tensile material testing on each prototype of SLTTD during one semester. After that, the students rated the ease of handling each prototype using one of seven pre-defined linguistic expressions. For each prototype of SLTTD, the value of the fourth criterion, denoted as Design ($k = 4$), was also calculated based on the interview data. Prototypes of SLTTD were exhibited at the Technology Fair in Belgrade, which was held in May 2023. Thirty potential customers were asked to rate the design of each prototype using one of seven linguistic expressions. The results of the survey are provided in Table 3. Figure 2 shows the appearance of the considered SLTTDs.

Table 3

Results of the ease-of-use assessment ($k = 2$) and results of the design assessment ($k = 4$)

	$\boldsymbol{k = 2}$	$\boldsymbol{k = 4}$
$i = 1$	$80\% \cdot L1 + 20\% \cdot L2$	$40\% \cdot M1 + 60\% \cdot M2$
$i = 2$	$60\% \cdot L3 + 35\% \cdot L4 + 5\% \cdot L2$	$45\% \cdot M3 + 35\% \cdot M2 + 20\% \cdot M1$
$i = 3$	$45\% \cdot L6 + 35\% \cdot L5 + 20\% \cdot L4$	$40\% \cdot M5 + 25\% \cdot M4 + 35\% \cdot M3$
$i = 4$	$35\% \cdot L7 + 30\% \cdot L6 + 35\% \cdot L5$	$20\% \cdot M7 + 60\% \cdot M6 + 20\% \cdot M5$



$i = 1$ $i = 2$

$i = 3$ $i = 4$

Figure 2

The considered SLTTD solutions

SLTTD solution ($i = 1$) is characterized by a measurement uncertainty of ≤ 10%, manual handling, no data acquisition system, and is used for testing the tensile strength of strings.

SLTTD solution ($i = 2$) is characterized by a measurement uncertainty of ≤ 5%, automated operation, and features a microcontroller with computer connection for data acquisition. It has a noise issue during operation and allows the choice between different strain rates. The design ensures equal ranges for both load application speed and release speed. Additionally, it enables spring stiffness testing.

SLTTD solution ($i = 3$) is characterized by a measurement uncertainty of ≤ 2%, automated operation, and features a microcontroller with computer connection for data acquisition. It operates noiselessly, offers improved structural stability, and enables reading of the load application speed.

SLTTD solution ($i = 4$) is characterized by a measurement uncertainty of ≤ 1.6%, automated operation, and improved structural stability. The shape of the grips and the clamping system are optimized. The control system and data acquisition system are unified, and it features a dynamometer with a larger measurement range. Additionally, it incorporates position sensors, software for automatic data processing, and allows for testing of samples with larger cross-sections and lengths. The release speed is higher than the load application speed.

3.1 Application of the Proposed IT2FCRITIC

This section explicates the procedure for determining criteria weights by applying the proposed IT2FCRITIC.

In the first step of the proposed Algorithm, the fuzzy decision matrix is stated and presented in Table 4.

Table 4

The fuzzy decision matrix

	$\boldsymbol{k=1}$	$\boldsymbol{k=2}$
$i=1$	$\begin{pmatrix}(7,8,9;1),\\(7.5,8,8.5;0.8)\end{pmatrix}$	$\begin{pmatrix}(1,1.20,2.60;1),\\(1.10,1.20,2.10;0.8)\end{pmatrix}$
$i=2$	$\begin{pmatrix}(4,5.5,7;1),\\(4.5,5.5,6.5;0.8)\end{pmatrix}$	$\begin{pmatrix}(2.95,4.42,5.90;1),\\(3.45,4.42,5.40;0.8)\end{pmatrix}$
$i=3$	$\begin{pmatrix}(2.5,4,5.5;1),\\(3,4,5;0.8)\end{pmatrix}$	$\begin{pmatrix}(5.87,7.15,8.42;1),\\(6.37,7.15,7.92;0.8)\end{pmatrix}$
$i=4$	$\begin{pmatrix}(1,2,3;1),\\(1.5,2,2.5;0.8)\end{pmatrix}$	$\begin{pmatrix}(6.65,8,8.82;1),\\(7.15,8,8.5;0.8)\end{pmatrix}$
	$\boldsymbol{k=3}$	$\boldsymbol{k=4}$
$i=1$	500	$\begin{pmatrix}(1,1.60,2.80;1),\\(1.30,1.60,2.30;0.8)\end{pmatrix}$
$i=2$	600	$\begin{pmatrix}(1.67,2.70,4.02;1),\\(2.07,2.70,3.52;0.8)\end{pmatrix}$
$i=3$	1000	$\begin{pmatrix}(4.82,6.20,7.57;1),\\(5.32,6.20,7.07;0.8)\end{pmatrix}$
$i=4$	1200	$\begin{pmatrix}(6.80,8,8.90;1),\\(7.30,8,8.50;0.8)\end{pmatrix}$

The fuzzy decision matrix is transformed into the normalized fuzzy decision matrix, as presented in Table 5.

Table 5

The normalized fuzzy decision matrix

	$\boldsymbol{k=1}$	$\boldsymbol{k=2}$
$i=1$	$\begin{pmatrix}(0.096,0.208,0.340;1),\\(0.149,0.208,0.273;0.8)\end{pmatrix}$	$\begin{pmatrix}(0.073,0.103,0.277;1),\\(0.084,0.103,0.205;0.8)\end{pmatrix}$
$i=2$	$\begin{pmatrix}(0.124,0.303,0.595;1),\\(0.195,0.303,0.456;0.8)\end{pmatrix}$	$\begin{pmatrix}(0.214,0.379,0.628;1),\\(0.266,0.379,0.527;0.8)\end{pmatrix}$
$i=3$	$\begin{pmatrix}(0.158,0.417,1.190;1),\\(0.253,0.417,0.684;0.8)\end{pmatrix}$	$\begin{pmatrix}(0.426,0.613,0.896;1),\\(0.491,0.613,0.774;0.8)\end{pmatrix}$
$i=4$	$\begin{pmatrix}(0.288,0.833,2.381;1),\\(0.507,0.833,1.369;0.8)\end{pmatrix}$	$\begin{pmatrix}(0.482,0.686,0.938;1),\\(0.551,0.686,0.830;0.8)\end{pmatrix}$
	$\boldsymbol{k=3}$	$\boldsymbol{k=4}$
$i=1$	0.687	$\begin{pmatrix}(0.079,0.151,0.327;1),\\(0.110,0.151,0.246;0.8)\end{pmatrix}$

$i = 2$	0.573	$\begin{pmatrix}(0.132, 0.255, 0.470; 1),\\(0.175, 0.255, 0.376; 0.8)\end{pmatrix}$
$i = 3$	0.344	$\begin{pmatrix}(0.380, 0.585, 0.884; 1),\\(0.450, 0.585, 0.756; 0.8)\end{pmatrix}$
$i = 4$	0.286	$\begin{pmatrix}(0.537, 0.755, 1.04; 1),\\(0.617, 0.755, 0.908; 0.8)\end{pmatrix}$

Arithmetic mean and standard deviation are calculated at the level of each criterion by applying the proposed Algorithm (Step 3 to Step 4), as presented in Table 6.

Table 6

The fuzzy mean and standard deviation at the level of each criterion

$\tilde{\tilde{r}}_1 = \begin{pmatrix}(0.166, 0.440, 1.126; 1),\\(0.276, 0.440, 0.809; 0.8)\end{pmatrix}$ $\sigma_1 = 0.419$	$\tilde{\tilde{r}}_2 = \begin{pmatrix}(0.299, 0.455, 0.685; 1),\\(0.348, 0.455, 0.584; 0.8)\end{pmatrix}$ $\sigma_2 = 0.258$
$\bar{r}_3 = 0.472$ $\sigma_3 = 0.189$	$\tilde{\tilde{r}}_4 = \begin{pmatrix}(0.282, 0.436, 0.680; 1),\\(0.338, 0.436, 0.571; 0.8)\end{pmatrix}$ $\sigma_4 = 0.299$

Correlation coefficients, calculated based on the proposed Algorithm (Step 5), are presented in Table 7.

Table 7

The correlation coefficients

	$\boldsymbol{k = 1}$	$\boldsymbol{k = 2}$	$\boldsymbol{k = 3}$	$\boldsymbol{k = 4}$
$k = 1$	1	0.875	-0.917	0.936
$k = 2$	0.875	1	-0.982	0.884
$k = 3$	-0.917	-0.982	1	-0.955
$k = 4$	0.936	0.883	-0.955	1

The applying procedure (Step 6 to Step 7 of the proposed Algorithm), the normalized weights vector of criteria (Step 7 of the proposed Algorithm) is $[0.28, 0.18, 0.35, 0.2]$.

3.2 Application of the Proposed IT2FTOPSIS

The proposed IT2FTOPSIS is explicated in this section.

The proposed Algorithm (Step 1 to Step 2) was applied, and obtained results are presented in Table 8.

Table 8

The weighted normalized fuzzy decision matrix, FPIS and FNIS

	$\boldsymbol{k = 1}$	$\boldsymbol{k = 2}$
$i = 1$	$\begin{pmatrix}(0.027, 0.058, 0.095; 1),\\(0.042, 0.058, 0.076; 0.8)\end{pmatrix}$	$\begin{pmatrix}(0.013, 0.019, 0.050; 1),\\(0.015, 0.019, 0.037; 0.8)\end{pmatrix}$

$i = 2$	$\begin{pmatrix}(0.035, 0.085, 0.476; 1),\\(0.055, 0.085, 0.128; 0.8)\end{pmatrix}$	$\begin{pmatrix}(0.039, 0.068, 0.133; 1),\\(0.048, 0.068, 0.095; 0.8)\end{pmatrix}$
$i = 3$	$\begin{pmatrix}(0.044, 0.117, 0.333; 1),\\(0.071, 0.117, 0.192; 0.8)\end{pmatrix}$	$\begin{pmatrix}(0.077, 0.110, 0.161; 1),\\(0.088, 0.110, 0.139; 0.8)\end{pmatrix}$
$i = 4$	$\begin{pmatrix}(0.081, 0.233, 0.667; 1),\\(0.142, 0.233, 0.383; 0.8)\end{pmatrix}$	$\begin{pmatrix}(0.087, 0.123, 0.169; 1),\\(0.099, 0.123, 0.149; 0.8)\end{pmatrix}$
FPIS	$\big((1, 1, 1; 1), (1, 1, 1; 0.8)\big)$	$\big((1, 1, 1; 1), (1, 1, 1; 0.8)\big)$
FNIS	$\big((0, 0, 0; 1), (0, 0, 0; 0.8)\big)$	$\big((0, 0, 0; 1), (0, 0, 0; 0.8)\big)$

	$k = 3$	$k = 4$
$i = 1$	0.240	$\begin{pmatrix}(0.016, 0.030, 0.065; 1),\\(0.022, 0.030, 0.049; 0.8)\end{pmatrix}$
$i = 2$	0.200	$\begin{pmatrix}(0.026, 0.051, 0.094; 1),\\(0.035, 0.051, 0.075; 0.8)\end{pmatrix}$
$i = 3$	0.120	$\begin{pmatrix}(0.076, 0.117, 0.177; 1),\\(0.090, 0.177, 0.151; 0.8)\end{pmatrix}$
$i = 4$	0.100	$\begin{pmatrix}(0.107, 0.151, 0.208; 1),\\(0.123, 0.151, 0.182; 0.8)\end{pmatrix}$
FPIS	1	$\big((1, 1, 1; 1), (1, 1, 1; 0.8)\big)$
FNIS	0	$\big((0, 0, 0; 1), (0, 0, 0; 0.8)\big)$

The proposed Algorithm (Step 3 to Step 5) was applied, and the obtained results are presented in Table 9.

Table 9

The fuzzy closeness coefficient values and representative scalars

	$\tilde{\tilde{c}}_i$	c_i	**Rank**
$i = 1$	$\big((0.071, 0.087, 0.117; 1), (0.078, 0.087, 0.103; 0.8)\big)$	0.082	4
$i = 2$	$\big((0.065, 0.101, 0.258; 1), (0.081, 0.101, 0.130; 0.8)\big)$	0.112	3
$i = 3$	$\big((0.071, 0.116, 0.224; 1), (0.087, 0.116, 0.159; 0.8)\big)$	0.117	2
$i = 4$	$\big((0.079, 0.152, 0.321; 1), (0.107, 0.152, 0.223; 0.8)\big)$	0.209	1

By taking into account all criteria and their weights and using the two-stage fuzzy model, the fourth design solution ($i = 4$) was singled out as the best one for SLTTD. The obtained result should help the general manager decide on the production program's diversification. Values of the closeness coefficients of SLTTD design solutions ($i = 2$) and ($i = 3$) are almost equal. Serial production for these two design solutions could be organised if there were no available resources (capacity, investments, workforce training) to produce the design solution ($i = 4$). The SLTTD design solution ($i = 1$) was ranked last, so it should not be taken into consideration by the general manager when deciding on the diversification of the company's production program.

Conclusions

This research proposes a fuzzy, two-stage model, whose application should lead to a ranking of technical solutions for SLTTD under type 2 uncertainties. The management should base a strategic decision regarding the diversification of the company's production program on the obtained results.

The DMs should assess the criterion values based on their knowledge, experience, and current information about competing companies producing similar products.

The main contributions of the presented research are:

- Modelling of criteria values by using IT2TFNs as well as precise numbers
- The weight vector of the criteria is determined by the proposed IT2FCRITIC, which offers certain advantages in direct application, and by using the fuzzy pairwise comparison MADM
- The design solutions for SLTTD are ranked by using the proposed IT2FTOPSIS

The practical implications of this research are primarily oriented towards the general manager, who decides on the production program's diversification to achieve competitive advantage and sustainability of the company.

The main advantage of the proposed fuzzy two-stage model over existing models lies in the combination of CRITIC and TOPSIS, extended with type 2 fuzzy set theory, to accurately obtain the best design solution for SLTTD. In this way, the risk of making a decision on diversification is significantly lower. The proposed model is sufficiently flexible in terms of changing the number and type of criteria and design solutions. Therefore, the proposed model can be extended to the analysis of different design solutions in different industries.

The main limitation of the fuzzy two-stage model is that the presence of subjectivity in criteria values evaluation cannot be entirely avoided.

Future research should include a sensitivity analysis of the obtained solution and the development of a software tool, that would enable Business Analysts and General Managers to make easier and better decisions, without being influenced by their possible subjective views.

References

[1] LaVan DA. Microtensile properties of weld metal. *Experimental Techniques* 1999; 23: 31-34

[2] Ma Z, Zhao H, Huang H, et al. A Novel Tensile Device for In Situ Scanning Electron Microscope Mechanical Testing. *Experimental Techniques* 2015; 39: 3-11

[3] Bai QS, Cheng K, He B, et al. Design of a novel tensile testing device and its application in tensile testing experiments on copper micro wires.

Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 2012; 226: 1594-1600

[4] Baharuddin B, Yani Y. Implementation of Design and Development Tensile Testing Machine for Application in Soft Material. *Journal of Multidisciplinary Engineering Science and Technology* 2019; 5: 9263-9267

[5] Ezzouine Z, Nakheli A. Development of a Measuring Sensory System Based on LabVIEW for Determining Elastic Proprieties of Solid Materials. *IJECE* 2016; 6: 2096

[6] Zhou J, Tagarielli VL, Heisserer U, et al. An Apparatus for Tensile Testing of Engineering Materials. *Exp Mech* 2018; 58: 941-950

[7] Bergonzi L, Vettori M, Pirondi A, et al. Numerical and experimental validation of a non-standard specimen for uniaxial tensile test. *Procedia Structural Integrity* 2018; 12: 392-403

[8] Lim W, Kim HK. Design and development of a miniaturised tensile testing machine. *Global journal of engineering education* 2013; 15: 48-53

[9] Zadeh LA. The concept of a linguistic variable and its application to approximate reasoning—I. *Information Sciences* 1975; 8: 199-249

[10] Ayyildiz E, Taskin Gumus A, Erkan M. Individual credit ranking by an integrated interval type-2 trapezoidal fuzzy Electre methodology. *Soft Comput* 2020; 24: 16149-16163

[11] Celik E, Gul M. Hazard identification, risk assessment and control for dam construction safety using an integrated BWM and MARCOS approach under interval type-2 fuzzy sets environment. *Automation in Construction* 2021; 127: 103699

[12] Komatina N, Tadić D, Aleksić A, et al. The assessment and selection of suppliers using AHP and MABAC with type-2 fuzzy numbers in automotive industry. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability* 2023; 237: 836-852

[13] Aleksić A, Milanović DD, Komatina N, et al. Evaluation and ranking of failures in manufacturing process by combining best-worst method and VIKOR under type-2 fuzzy environment. *Expert Systems* 2023; 40: e13148

[14] Aleksić A, Tadić D. Industrial and Management Applications of Type-2 Multi-Attribute Decision-Making Techniques Extended with Type-2 Fuzzy Sets from 2013 to 2022. *Mathematics* 2023; 11: 2249

[15] Haleem A, Khan S, Pundir H, et al. Investigating Barriers Toward the Implementation of Circular Economy: A Fuzzy CRITIC Approach. *J Ind Intg Mgmt* 2021; 06: 107-139

[16] Amin FU, Dong QL, Grzybowska K, et al. A Novel Fuzzy-Based VIKOR–CRITIC Soft Computing Method for Evaluation of Sustainable Supply Chain Risk Management. *Sustainability* 2022; 14: 2827

[17] Zhang H, Bai X, Hong X. Site selection of nursing homes based on interval type-2 fuzzy AHP, CRITIC and improved TOPSIS methods. *IFS* 2022; 42: 3789-3804

[18] Komatina N, Djapan M, Ristić I, et al. Fulfilling External Stakeholders' Demands—Enhancement Workplace Safety Using Fuzzy MCDM. *Sustainability* 2021; 13: 2892

[19] Zhang Z, Zhao X, Qin Y, et al. Interval type-2 fuzzy TOPSIS approach with utility theory for subway station operational risk evaluation. *J Ambient Intell Human Comput* 2022; 13: 4849-4863

[20] Sharaf IM. An interval type-2 fuzzy TOPSIS for MAGDM applied to solar power systems. *Pak.j.stat.oper.res* 2021; 559-575

[21] Kamali Saraji M, Streimikiene D, Kyriakopoulos GL. Fermatean Fuzzy CRITIC-COPRAS Method for Evaluating the Challenges to Industry 4.0 Adoption for a Sustainable Digital Transformation. *Sustainability* 2021; 13: 9577

[22] Kahraman C, Onar SC, Öztayşi B. A Novel spherical fuzzy CRITIC method and its application to prioritization of supplier selection criteria. *IFS* 2021; 42: 29-36

[23] Tabatabaee S, Ashour M, Sadeghi H, et al. Towards the adoption of most suitable green walls within sustainable buildings using interval type-2 fuzzy best-worst method and TOPSIS technique. *ECAM* 2024; 31: 1145-1164

[24] Kuo MS, Liang GS. A soft computing method of performance evaluation with MCDM based on interval-valued fuzzy numbers. *Applied Soft Computing* 2012; 12: 476-485

[25] Aleksic A, Runic Ristic M, Komatina N, et al. Advanced risk assessment in reverse supply chain processes: A case study in Republic of Serbia. *Adv produc engineer manag* 2019; 14: 421-434

[26] Mendel JM. *Uncertain Rule-Based Fuzzy Systems: Introduction and New Directions, 2nd Edition*. Cham: Springer International Publishing. Epub ahead of print 2017. DOI: 10.1007/978-3-319-51370-6

[27] Kahraman C, Öztayşi B, Uçal Sarı İ, et al. Fuzzy analytic hierarchy process with interval type-2 fuzzy sets. *Knowledge-Based Systems* 2014; 59: 48-57

[28] Jovičić N, Marjanović N. *Basics of Construction*. University of Kragujevac, Faculty of Engineering, 2021

[29] EN ISO 6892-1:2019. Metallic materials - Tensile testing - Part 1: Method of test at room temperature

[30] Practical Teaching Equipment. Official Site (online), available on <https://www.tecquipment.com>

[31] Kostić S. *Development and Optimization of a Small Laboratory Tensile Testing Device from the Aspect of Minimising Measurement Errors and Costing*. Doctoral dissertation, University of Kragujevac, Faculty of Engineering, 2021

[32] Kostic S, Miljojkovic J, Simunovic G, et al. Uncertainty in the determination of elastic modulus by tensile testing. *Engineering Science and Technology, an International Journal* 2022; 25: 100998

[33] Viala R, Placet V, Cogan S. Identification of the anisotropic elastic and damping properties of complex shape composite parts using an inverse method based on finite element model updating and 3D velocity fields measurements (FEMU-3DVF): Application to bio-based composite violin soundboards. *Composites Part A: Applied Science and Manufacturing* 2018; 106: 91-103

[34] Bucciarelli F, Malfense Fierro GP, Zarrelli M, et al. A Non-Destructive Method for Evaluation of the Out of Plane Elastic Modulus of Porous and Composite Materials. *Appl Compos Mater* 2019; 26: 871-896

[35] Lord JD, Morrell R. Measurement good practice guide. In: *No. 98: elastic modulus measurement*. National Physical Lab, 2006, pp. 41-65

[36] Zhang H. The multiattribute group decision making method based on aggregation operators with interval-valued 2-tuple linguistic information. *Mathematical and Computer Modelling* 2012; 56: 27-35

[37] Diakoulaki D, Mavrotas G, Papayannakis L. Determining objective weights in multiple criteria problems: The critic method. *Computers & Operations Research* 1995; 22: 763-770

[38] Sharaf IM. An interval type-2 fuzzy TOPSIS using the extended vertex method for MAGDM. *SN Appl Sci* 2020; 2: 87

[39] Hwang CL, Yoon K. Methods for multiple attribute decision making. Multiple attribute decision making: methods and applications a state-of-the-art survey. 1981, pp. 58-191