Performance Analysis of Weighted Least Squares State Estimation in a Medium Voltage Distribution Network

Vit Krcal[1], Jan Koudelka[1,2] and David Topolanek[1]

[1]Brno University of Technology, Department of Electrical Power Engineering, Faculty of Electrical Engineering and Communication, Technicka 3082/12, 616 00 Brno, Czech Republic, vit.krcal@vut.cz, xkoude20@vut.cz, topolanek@vut.cz

[2]Jan Evangelista Purkyne University in Usti nad Labem, Faculty of Mechanical Engineering, Pasteurova 3544/1, 400 96 Usti nad Labem, Czech Republic, jan.koudelka@ujep.cz

Abstract: This paper presents the research of a weighted least squares (WLS) state estimation (SE) analysis, for a real medium voltage (MV) distribution network, equipped with several Remote Terminal Units (RTUs). The study examines the impact of various factors on SE performance, including the handling of virtual measurements, load estimation errors, the number and placement of real meters, and a comparison between radial and ring topology operation. For each analyzed factor, a simulation scenario is described and conducted using a numerical model of the network. A performance evaluation is carried out by comparing the estimation results, with load flow references. The results are presented using the root mean square error (RMSE) metric.

Keywords: state estimation; distribution network; wls algorithm; distributed measurement, medium voltage

1 Introduction

Distribution system operators (DSOs) need to have the credible representation of the system in real time. It is important not only regarding the operation and possible flexibility provision, but also for fault detection and system protection setting, as well as for the analysis of network operation. However, the measurement devices, providing relevant data, can be installed in the limited amount and usually only at specific network elements. Such restricted measurement options together with increased complexity and extent of the network lead to depreciated observability of such system. This particularly concerns distribution networks of medium voltage (MV) and low voltage (LV) levels.

Key characteristics of distribution networks are that they are typically large, operated as radial feeders and line parameters are very heterogeneous. Many operation problems are related to unbalanced loads, which are common especially in LV networks. Let us not forget also the possible changes of network topology as distribution networks, usually, contain large number of switches, whose status may on top of that be unknown or incorrectly interpreted in the DSO's systems.

The well-known method for obtaining the image of the power system is the state estimation (SE). State estimation is a process or technique by which the state of the entire system is obtained (estimated), based on the measurement data. It is a quite common tool in transmission system operation, where the measured data are collected from Phasor Measurement Units (PMUs) and Intelligent Electronic Devices (IEDs). However, regarding the aforementioned characteristics of distribution system, SE in these networks has certain limitations. The main problem is the availability of measurements, as there are not installed enough PMUs, IEDs or other telemetered measurements. Regarding specifically MV networks, relevant measurement data can be obtained from remote terminal units (RTU), usually installed in selected distribution transformer stations (DTS), reclosers or intelligent sectionalizers. The network topology and configuration pose problems for numerical stability of the SE algorithms, as network matrices may become sparse or ill-conditioned. The estimation accuracy may also be affected by emerging of bad data, caused by inaccurate metering instruments or telecommunication problems. Despite all the constraints, distribution system state estimation (DSSE) is an examined and living problem, and many DSOs as well as researchers are trying to find new or improved solutions to be able to use this method in practice.

A comprehensive review of existing DSSE can be found in the literature [1-4]. The traditional method used for DSSE is the Weighted Least Squares (WLS) method. Research in the field is mainly trying to modify WLS method to obtain numerical stability and convergence. However, other techniques such as Kalman filter or neural networks are also researched, but these methods are still marginal compared to WLS. Except developing the main DSSE technique, research scope is focused on optimal meter placement, measurement accuracy and bad data identification. As a general conclusion, WLS method is identified as a fundamental method for DSSE, it is capable to operate under the distribution system specifics, but obviously tends to have more uncertainties. It has the greatest number of pilot use cases and innovation projects across Europe, summarized in [3].

WLS method was used in the research being further presented. The aim of this paper is to test the application of the WLS method for a part of the typical MV distribution system and assess different aspects affecting its performance. After a brief description of WLS method in section 2, section 3 describes the methodology of the evaluation and considered aspects affecting the SE process. Section 4 presents the case in which the SE performance in the real MV distribution system is discussed and results are evaluated. Section 5 summarizes and concludes the work.

2 Weighted Least Squares Method for Distribution System State Estimation

The objective of SE is to obtain a vector describing the network state, usually voltage phasors at all nodes, at specific steady state based on accurate numerical model and a set of synchronized measurements. The method used for studies presented further is aforementioned WLS, which is described in general in this section. It is important to explain first the different types of measurements.

2.1 Measurement Data

As mentioned in the introduction, the distribution systems usually contain small number of relevant measurement devices leading to disabled network observability. As all real-world measured data are subject to measurement errors, the objective of SE procedure is to employ large number of measurements to filter out the bad data influence on the results. To substitute the missing real measurements virtual and pseudo measurements are introduced [5] allowing to meet the observability criterium of minimal number of measurements. The types of individual measurements are differentiated in the algorithm using different weights for each type, emphasizing more accurate data sources over the less accurate estimates.

Pseudo measurements are used in SE algorithms to improve the accuracy and observability and stand for estimates of power injections at the nodes with defined loads or generation. The pseudo measurement data are derived as an assumption based on historical data, load profiles, short term predictions or other available sources. Pseudo measurements are typically implemented in the algorithm as active and reactive power injections (P_{i}, Q_{i}) with lower defined weights compared to the real measurements.

The virtual measurements are virtually placed in the nodes with no power injections. This model is derived using fundamental electric laws, known system constraints or other available data. In the studied cases, virtual measurements are used in nodes without any connected generation or load, i.e., they represent zero power injections (P_{i}, Q_{i}) which could be emphasized by assigning very heigh weights.

2.2 Weighted Least Squares Method Fundamental Algorithm

The fundamental state estimation WLS algorithm aims to find the solution for the state variables (voltage phasors) by minimizing the objective function $\boldsymbol{J}(\boldsymbol{x})$ (1).

$$\boldsymbol{J}(\boldsymbol{x}) = [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})]^{\mathrm{T}} \boldsymbol{R}^{-1} [\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] \tag{1}$$

In (1), $\boldsymbol{z}$ is the measurement vector; $\boldsymbol{x}$ is the state vector (voltages); $\boldsymbol{h(x)}$ is a vector of nonlinear functions that relate the measurements to the state vector and $\boldsymbol{R}$ is the covariance matrix of measurements errors.

Two additional matrices need to be introduced for the process – Jacobian matrix of the measurement vector $\boldsymbol{H}(\boldsymbol{x})$ (2) and gain matrix $\boldsymbol{G}(\boldsymbol{x})$ (3).

$$\boldsymbol{H}(\boldsymbol{x}) = \frac{\partial h(x)}{\partial x} \tag{2}$$

$$\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H} \tag{3}$$

The weight matrix $\boldsymbol{R}^{-1}(\boldsymbol{x})=\boldsymbol{W}$ (4) is defined as a diagonal matrix of variances reflecting the accuracy of each measurement. The standard deviation σ directly reflects properties of the meter considering normal distribution, and therefore allows to force heigh weights to virtual measurements and low weight to pseudo measurements, which is the key for correct SE algorithm operation.

$$\boldsymbol{W} = \boldsymbol{R}^{-1} = diag(\sigma^2) \tag{4}$$

Minimization of $\boldsymbol{J}(\boldsymbol{x})$ (1) can be done iteratively, and the state vector update increment in each iteration (from step k to $k+1$) can be obtained by solving equation (5), called the normal equation (NE).

$$\Delta\boldsymbol{x} = \boldsymbol{G}_k^{-1}\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{R}^{-1}[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}_k)] \tag{5}$$

Further description of the algorithm is available in the literature, e.g., in [5].

2.3 Weighted Least Squares Algorithm Application for Distribution System

Regarding the fundamental description of the WLS algorithm, especially the equation (5), one can identify the computational problems which may arise. Finding the matrix inverse of $\boldsymbol{G}$ and $\boldsymbol{R}$ is not only computationally expensive but also requires well-conditioned inputs. The computational instability may be caused mainly due to the characteristics of the distribution system:

- Large number of virtual measurements, small number of real measurements
- Large discrepancy between weights (variances) of real, virtual and pseudo measurements
- Radial topology leading to sparse matrices

Due to the limitations of NE formulation of WLS method, alternative formulations and methods are studied. Some approaches are aiming to mitigate the numerical instabilities, such as hybrid method or Peters and Wilkinson method [5]. Another way to improve performance is to avoid using high weights for virtual measurements by modelling them as explicit constraints in the WLS estimation. This approach uses Lagrange method and consequential set of nonlinear equations is solved e.g., with Gauss-Newton method. Other applied techniques include augmented approach [6] or hybrid methods.

In addition to more common node voltage-based method for SE (estimating the nodal voltages), branch current methods have been also researched, e.g., in [7] [8]. According to findings comparisons presented in [9] [10], both node voltage based and branch current based methods for distribution system SE perform with similar accuracy.

To sum up, the general approach of WLS method is applicable for SE in the distribution system. However, the computational performance can be affected by the characteristics of the distribution networks. Multiple techniques and modifications are therefore proposed in the literature.

3 Methodology

The main objective of the presented research was to test the WLS algorithm performance on a specific radial MV distribution network. This section presents the methodology used. It brings the description of the MV network model used, considered assumptions, and evaluated parameters of the SE performance with the defined metrics.

3.1 Medium Voltage Network Model

For the purposes of this study, a model of a typical MV (22 kV) distribution feeder was selected. This feeder is relatively extensive, with a total line length of 82 km, and 59 MV/LV DTS. Total number of 232 line sections and 95 switches complete the topological model. The feeder had previously been the subject of another study of fault localization based on distributed measurement units (RTUs), presented in [11]. A graphical representation of the feeder is provided in Figure 1. The network is normally operated in radial configuration, but there is a possibility of ring topology operation by closing the coupling switches. The coupling switch is highlighted in the network diagram (for simplicity, remaining switches are not plotted in the diagram). Presented network model offers possibility to perform various scenarios and simulation of miscellaneous scenarios. Although the distributed energy resources (DERs) were not considered explicitly, in further research, a large penetration of DERs, e.g., photovoltaics, can be considered as loads with reversed power flow.

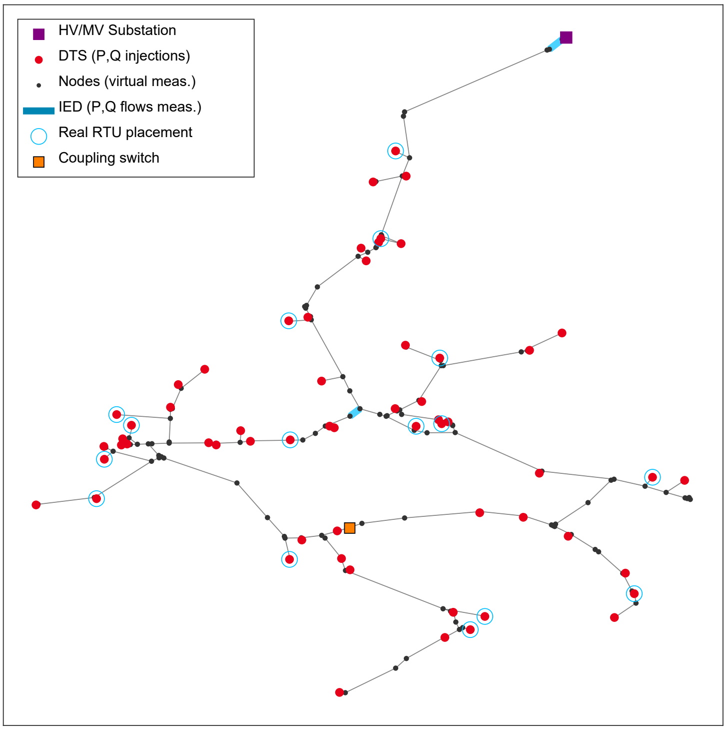

Figure 1
Single line diagram of model MV feeder

3.2 Modelling Assumptions

Certain assumptions regarding network topology, measurement placement and loads modelling in the considered MV network were carried out, and their description follows.

Topology

The MV feeder maintains a fixed configuration, with all switches retaining their initial statuses across all simulation scenarios. The one exception is the analysis of ring topology operation (Section 4.4), where a selected switch, highlighted in Figure 1 was closed. In its default radial configuration, the network consists of 233 electrical nodes. Many of these nodes are only line interconnections and are not assigned with any power injections (in DSSE, they are treated as virtual measurements). Another option is to eliminate the nodes without any injection, and thus create a simplified 124-node equivalent, which can enhance the computational efficiency and stability. The reduced 124-node equivalent was used for all presented simulations, except for virtual measurements analysis.

Real measurements

The primary real measurements in this study are provided by RTUs installed at DTS nodes. These units are designed to monitor voltage magnitudes (V_i) and power injections (P_i, Q_i). Another source of real measurement data are intelligent electronic devices (IEDs), which, in this context, are located in the substation and at the recloser in the network (see Figure 1). This setup enables the measurement of power flows in lines (P_{ij}, Q_{ij} – from i-th to j-th node) as well as bus voltages (V_i). Modern measurement devices being considered achieve uncertainty levels well within 1% [12], and accordingly, the standard deviations for all real measurements are also set to 1%.

Loads

A specific problem of SE in distribution system is the load estimation in unmeasured nodes, because the actual load may quite vary due to various customer composition and stochastic behavior. There do exist a vast number of methods dealing with this issue [13], starting form customers' billing data and load diagrams, going through machine learning [14], advanced metering infrastructure [15], or, last but not least using the correlation between real and pseudo measurements [12].

For simulations in this paper, the loads are derived from MV/LV transformers rated powers and biased with noise to emulate load estimation errors. The load values are set as random proportions from range 20-50% of rated power of each DTS, and the power factors are chosen random from range 0.97-1 inductive. A series of generated load values is uniformly distributed to capture the stochastic nature of the load. The range of transformers loading used is based on operation experience of subjected network. Loads are placed in 59 DTSs indicated in Figure 1, defining the steady state under test.

Load flow and state estimation algorithm set up

Load flow of the network needed to be computed as a reference for SE performance evaluation. A standard, simple Newton-Raphson load flow (NRLF) was used, considering the impedance network model and loads to be balanced. The supply station was selected as a slack bus, other nodes are of type PQ load (or zero load). No node was considered as a generator bus, with predominating current injection.

Obviously, the WLS algorithm used the same impedance network model as the load flow. The iterative NE formulation process according to equation (5) capped at 10 iterations and a termination error threshold of 10^{-6} pu. Such a threshold, applied to 22 kV level, represents the voltage change of 22 mV, which is way less than realistic measurement precision. The NE iteration process usually meets the threshold in less than 5 iteration steps, therefore, capping the iterations serves as an alarm of bad inputs conditioning. The measurement vector was obtained from the NRLF reference solution by adding noise, which was generated for each type of measurement within the bounds of standard deviations reflecting the measurement uncertainty. Modelling and calculation were carried out in the MATLAB environment.

3.3 Assessing the WLS Performance

The performance of WLS algorithm was evaluated by comparing the load flow results as a reference and SE results. For comparison of voltage and power deviations, the root mean square error (RMSE) was used, according to formula (6), in which N is the total number of nodes and subscripts refer to results origin.

$$\mathrm{RMSE}_{Vi} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left|\bar{V}_{i,SE} - \bar{V}_{i,NRLF}\right|^2} \tag{6}$$

The computational stability was evaluated using the condition of the gain matrix $\boldsymbol{G}$ – enumerating the condition number κ according to equation (7), in which ||·|| stands for the matrix norm. Condition number as a metric is proposed in [16]. The higher the value of κ, the more ill-conditioned the matrix is.

$$\kappa(\boldsymbol{G}) = \|\boldsymbol{G}\| \cdot \|\boldsymbol{G}^{-1}\| \tag{7}$$

Computational stability is among other things affected by virtual measurements as high weights of such measurement models devaluate conditions of matrices used in NE formulation.

3.4 Analyzed Aspects and Conducted Simulations

The conducted study examined several aspects influencing the SE performance. The focus is primarily on addressing the limitations of the WLS method in handling the unique characteristics of distribution systems, as outlined earlier in the paper. Additionally, the analyzed factors consider practical challenges arising from the specific features of MV distribution feeders. Each factor is analyzed separately to evaluate its individual impact and determine the necessary measures. The analysis explores the following effects:

- Virtual measurements effect on SE process – the comparison was done for a full model with virtual measurements (network of 233 nodes) and reduced model excluding most of virtual measurements (network model of 124 nodes)
- Load estimates errors effect on SE performance – simulations were done for varying load error, considering also different number of real measurements (RTUs)
- Real measurements penetration effect on SE performance – simulations were done for increasing penetration of real measurements up to 59 (all loads being measured)
- Real measurement configuration effect on SE performance – simulations were carried out for real RTUs placement in the network, considering significant measurement errors
- Switching radial topology to ring effect on SE performance – simulations with real RTUs placement in the network were carried out also for network being operated in ring topology

Various scenarios have been prepared and simulated for each of the aforementioned factor. In addition, some scenarios were tested also in specific cases to include further aspects in the evaluation. The summary of simulation cases and scenarios is given in Table 1. Results and their discussion are the content of the following section.

Table 1
Simulated scenarios and cases for SE performance analysis

Evaluated effect	Evaluation metric	Cases	Scenarios
Virtual measurements	Condition number (7)	Original and reduced model (233 vs 124 nodes); 59 and 16 RTUs	Varying virtual measurement weight
Pseudo measurements	RMSE (6)	5, 16 and 30 RTUs	Varying load error
Real measurement penetration	RMSE (6)	Varying number of RTUs (1 – 59)	Varying placement of RTUs
Real measurement placement	Difference NRLF - SE	Only fundamental, real RTUs placement	
Radial vs ring topology	Difference NRLF - SE RMSE (6) Condition number (7)	Only fundamental, real RTUs placement	

4 Results and Discussion

The analysis addresses the discussed challenges and examines the previously mentioned aspects individually. The results are organized based on the specific aspect under investigation, accompanied by a description of the simulation setup and the corresponding results.

4.1 Virtual Measurements

The condition of the gain matrix was assessed for the model feeder as a function of the standard deviation σ of virtual measurements. Simulations were conducted for both model variants (full 233-node, reduced 124-node equivalent), considering two variants of RTU penetration (real-placed 16 units, maximal 59 units). Deviations of real and pseudo measurements were set constant to 0.01 and 0.4 pu, respectively. Figure 2 shows the simulations results. The left-side limit of the curves represents the point at which the WLS algorithm failed to converge within 10 iterations.

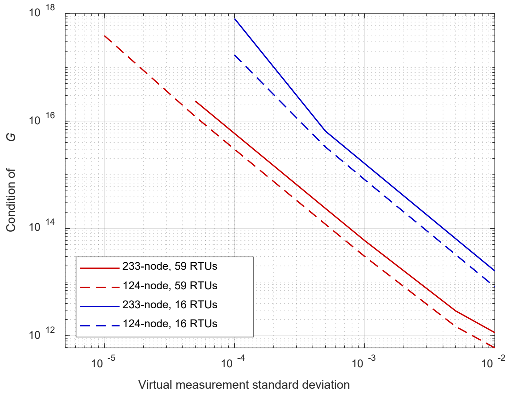

Figure 2
Virtual measurements effect on gain matrix condition

As shown by Figure 2 the reduced 124-node model yields lower condition numbers and accommodates smaller deviations in virtual measurements. The graph also demonstrates that a higher number of real measurements results in reduced condition numbers, enhancing calculation stability. The results show that only reduction of virtual measurements does not lead to significant stability improvement. Those would have to be completely eliminated to get a more solvable problem (which could be done by other WLS technique described in section 2.3 – by treating them as constraints). The results also suggest that the case study network requires values of σ higher than 10^{-4} pu to get stable calculations using NE formulation.

4.2 Load Estimates

The impact of load estimation errors on SE performance was assessed by introducing significant noise to the reference power injection values derived from NRLF. The analysis considered three levels of RTU penetration, with 5, 16 and 30 units, also determining the number of pseudo measurements assigned to the remaining DTS nodes. The pseudo measurement noise was generated as uniformly distributed random ratios relative to the reference load values at each DTS. The random load errors were categorized into five bins, each spanning ten percent, with the pseudo-measurement standard deviation set as the upper limit for each bin. The SE simulation was then carried out 1000 times for each RTU penetration case and each load error bin, varying pseudo measurements. This way the simulation was repeated 15000 times, each run incorporating randomly generated noise ratios. Real measurements were assumed to be stable and unbiased. The impact of load errors on the estimated parameters are shown in Figs. 3 and 4, where the columns are the mean RMSE values for all simulation runs and the error bars indicate variability in the results.

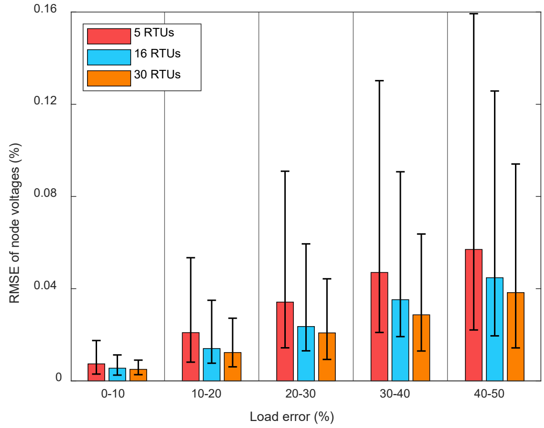

Figure 3

Load error impact on estimated voltages

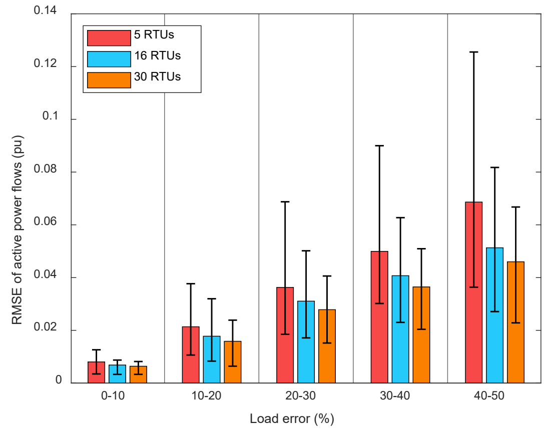

Figure 4

Load error impact on estimated power flows

From the results (Figs. 3 and 4), it is obvious that the dependence of voltage errors on load errors is approximately linear. The errors decrease with increasing number of real measurements, but the difference between 16 and 30 RTUs is not significant. This is due to RTU placement at the end of radial branches, where the effect on balancing the pseudo measurements errors, also placed at branch ends, is very little.

The voltage estimation errors seem quite small, but they cause significant errors in power flows. The RMSE of active power flows in all line sections is depicted in Figure 4. The base power was defined as 1 MVA, so the 0.01 pu is equivalent to 10 kW. The average active power flow value through all branches is 0.59 pu. The load errors affect mostly the estimated power injections and power flows near the load error location. The negative effect of load errors could be suppressed by placing more real power flows measurements in trunk power lines.

4.3 RTU Penetration

By varying the RTU penetration only the ratio of real and pseudo measurements is changed as both of those types are disjunctively placed in DTS nodes. For each case of RTUs amount (1-59 units), the simulation was repeated 100 times, with RTU placement scenario randomized for each run. The effect of number of real measurements on estimation error is depicted in Figure 5 and Figure 6. The line represents mean values of RMSE, and the filled area represents variance band obtained from repeated runs. In these simulations, the real measurements were considered noise-free and assigned with σ = 0.01; the pseudo measurements were biased with 30% random noise (load error) with σ = 0.3.

Figure 5 and Figure 6 show that estimation errors decrease approximately linearly with increasing number of RTUs. The variance band indicates the dependence on RTU placement which proves to be important in the presented case study. Optimal placement of a given number of RTUs can result in RMSE values that are significantly lower compared to non-optimal placement. This highlights the importance of addressing the meter placement issue, which was out of scope of the presented research.

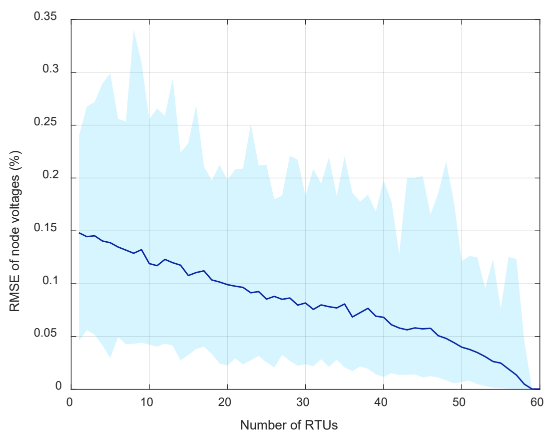

Figure 5
Impact of RTUs penetration on estimated voltages

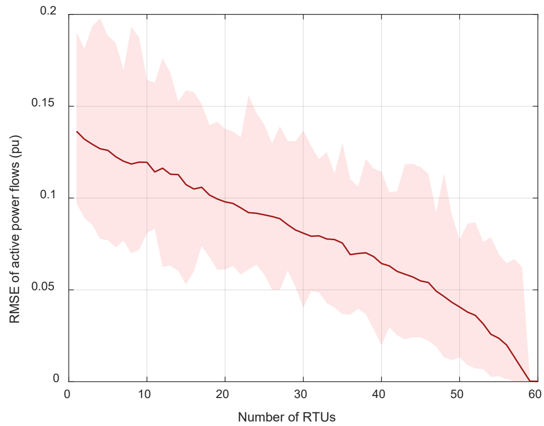

Figure 6
Impact of RTUs penetration on estimated power flows

4.4 Real RTU Placement

This studied case examines SE performance with real placement of the 16 RTUs (see Figure 1) and both pseudo and real measurement biased with noise representing measurement errors and load estimation errors, respectively. The real measurements errors were set to ±1% with respect to referential values of both voltage magnitudes and powers. The load estimation errors were generated as random ±30%. The standard deviations were set to 0.01, 0.3 and 10^{-4} for real, pseudo and virtual measurements. For this setup, the RMSE of voltages is 0.0011 pu (0.11%), RMSE of active power flows 0.06 pu and RMSE of active power injections 0.02 pu. Detailed results of SE performance are shown in Figure 7 and Figure 8, where comparisons between referential, measured and estimated values are illustrated. This analysis was inducted only for the reduced 124-node model.

Figure 7 shows node voltages sorted by magnitude for better readability. Despite relatively high weights of real measurements, the voltage measurement errors do not bias the estimation results much and estimation errors are much lower than voltage measurements errors. Figure 8 shows active power injections in nodes, which are grouped by measurement type and sorted by the referential value of active power, with positive values of power injections corresponding to loads. Therefore, the node order (x-axis) is different from Figure 7. The real and virtual measurement nodes experience negligible estimation errors with maximums of ca. 10^{-3} pu and 10^{-8} pu respectively. On the other hand, the pseudo measurement nodes dispose with significant estimation errors caused among others by simulated level of 30% load errors.

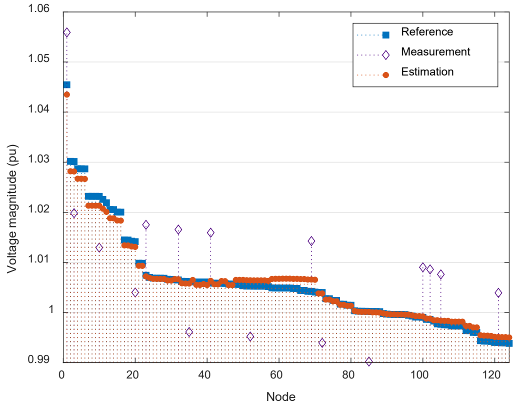

Figure 7
Comparison of referential and estimated voltage magnitudes (radial topology)

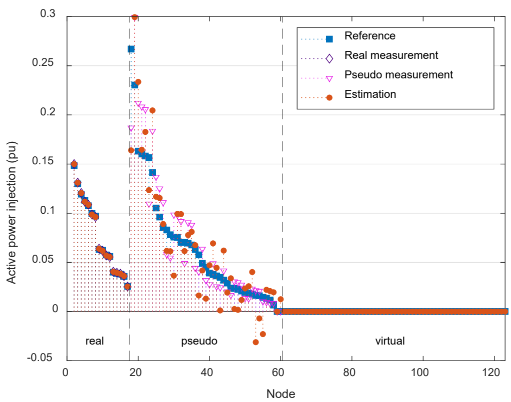

Figure 8
Comparison of referential and estimated node power injections (radial topology)

The estimation errors are in some nodes larger than the noise level and even reach values over 0.05 pu, which is equivalent to 50 kW. Also, the WLS results even indicate reverse directions of power injections in a few nodes misrepresenting the actual power flow.

4.5 Ring Topology Effect

To assess the effect of ring topology, all parameters were kept consistent with the previous case of real RTU placement. By closing the coupling switch (see Fig. 1), the two radial branches of the network become connected, resulting in slightly altered power flows distribution. The coupling also reduces the number of nodes, enhancing observability due to an improved ratio of real measurements to number of nodes. However, the impact of this adjustment is limited, as a single switch has little influence on a network of this scale with this configuration. The simulation results are presented in Figure 9 and Figure 10, following the same format as Figure 7 and Figure 8. Observing these figures, the switching to ring topology may have improved the SE performance in several pseudo measurements nodes but the overall impact is not obvious.

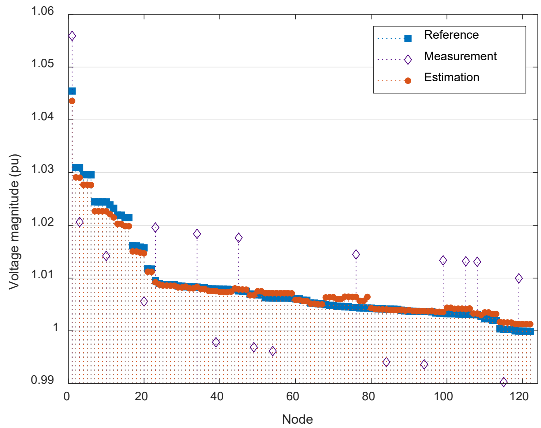

Figure 9
Comparison of referential and estimated voltage magnitudes (ring topology)

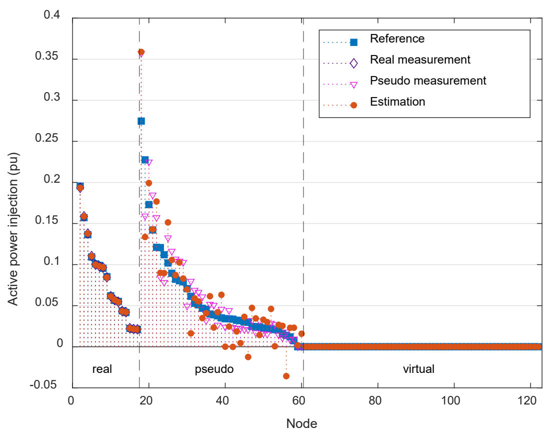

Figure 10
Comparison of referential and estimated node power injections (ring topology)

A numerical comparison between radial and ring topologies is provided in Table 2, detailing metrics RMSE for selected parameters and the condition number of gain matrix. The results indicate that SE performs slightly better with the ring topology, as evidenced by overall lower RMSE values. Additionally, the interconnected system demonstrates improved observability and stability, supported by a lower condition number of the gain matrix. While more simulations across varying scenarios would be required for a comprehensive conclusion, the interconnected system appears to perform slightly better and does not cause additional errors.

Table 2
Selected SE results for radial and ring network topology

Topology	RMSE of V_i	RMSE of P_i	RMSE of Q_i	RMSE of P_{ij}	RMSE of Q_{ij}	Condition of G
Radial	0.00111	0.01869	0.01280	0.06466	0.07359	$5.6784 \cdot 10^{16}$
Ring	0.00103	0.01781	0.01282	0.06002	0.07255	$3.5288 \cdot 10^{16}$

Conclusions

This study applied the node voltage based WLS method, for state estimation to a radial MV feeder, in an existing RTU installation, examining various factors affecting SE performance. Multiple simulations were carried out to assess the stability and accuracy of the SE. For the studied feeder, the NE formulation of the WLS method requires setting the standard deviations of virtual measurements no lower than 10^{-4} pu to prevent numerical instabilities. Furthermore, while reducing the number of virtual measurements through network topology adjustments proved beneficial, a more effective solution would involve incorporating virtual measurements as explicit constraints.

Simulating erroneously estimated load values revealed that SE performance is highly sensitive to pseudo measurement errors. With minimal load errors, the SE performed well even with a limited number of real measurements. However, as load errors increased, the RMSE of voltages and powers rose approximately linearly, leading to considerable deviations in true injected powers at some nodes. The results also underscored the influence of RTU penetration and placement. The RMSE is influenced linearly by the number of RTUs, compounded by the effects of pseudo-measurement errors; however, their placement has a substantial impact on overall performance. This highlights the need for future research into optimal placement strategies.

The simulation integrating measurement errors, load errors, and actual RTU placement revealed significant errors in estimated power injections in pseudo measurement nodes, with several nodes even indicating power flow in the opposite direction. However, nodes with real or virtual measurements exhibited only negligible deviations from the reference values. Evaluating deviations in power injections and power flows is crucial, as even small voltage deviations can lead to substantial power errors. In comparison, errors from real measurements were significantly smaller and had a much lesser impact overall. Switching the radial topology to ring proved to have only little effect, which is given by extent of the network. However, a slightly better SE performance was reached with improved RMSE metrics and lower condition number of the gain matrix.

Based on concluded results, future research should focus mainly on strategies of precision load estimations and optimal meter placements, as well as other approaches and advanced methods that are specifically tailored for DSSE.

Acknowledgement

This research work has been carried out in the Centre for Research and Utilization of Renewable Energy (CVVOZE). Authors gratefully acknowledge financial support from the Technology Agency of the Czech Republic (project No. TN02000025).

References

[1] B. Hayes and M. Prodanovic, "State Estimation Techniques for Electric Power Distribution Systems," *2014 European Modelling Symposium*, Pisa, Italy, 2014, pp. 303-308, doi: 10.1109/EMS.2014.76

[2] J. Zhu, B. Ramachandran, "Review of Trends in State Estimation of Power Distribution Networks," *Journal of Power and Energy Engineering*, Vol. 8, No. 8, pp. 1-10, August 2020, doi: 10.4236/jpee.2020.88007

[3] M. Fotopoulou, S. Petridis, I. Karachalios, and D. Rakopoulos, "A Review on Distribution System State Estimation Algorithms," *Applied Sciences*, Vol. 12, No. 21, pp. 11073, 2022, doi: 10.3390/app122111073

[4] J. Vijaychandra, B. R. V. Prasad, V. K. Darapureddi, B. V. Rao, and Ł. Knypiński, "A Review of Distribution System State Estimation Methods and Their Applications in Power Systems," *Electronics*, Vol. 12, No. 603, 2023, doi: 10.3390/electronics12030603

[5] A. Abur and A. G. Expósito, "Power System State Estimation: Theory and Implementation," 1st ed., CRC Press, 2004, doi: 10.1201/9780203913673

[6] F. Therrien, I. Kocar and J. Jatskevich, "A Unified Distribution System State Estimator Using the Concept of Augmented Matrices," in *IEEE Transactions on Power Systems*, Vol. 28, No. 3, pp. 3390-3400, Aug. 2013, doi: 10.1109/TPWRS.2013.2248398

[7] Y. Deng, Y. He and B. Zhang, "A branch-estimation-based state estimation method for radial distribution systems," in *IEEE Transactions on Power Delivery*, Vol. 17, No. 4, pp. 1057-1062, Oct. 2002, doi: 10.1109/TPWRD.2002.803800

[8] H. Wang and N. N. Schulz, "A revised branch current-based distribution system state estimation algorithm and meter placement impact," in *IEEE*

Transactions on Power Systems, Vol. 19, No. 1, pp. 207-213, Feb. 2004, doi: 10.1109/TPWRS.2003.821426

[9] M. Pau, P. A. Pegoraro and S. Sulis, "Performance of three-phase WLS Distribution System State Estimation approaches," *2015 IEEE International Workshop on Applied Measurements for Power Systems (AMPS)*, Aachen, Germany, 2015, pp. 138-143, doi: 10.1109/AMPS.2015.7312752

[10] A. Primadianto and C. -N. Lu, "A Review on Distribution System State Estimation," in *IEEE Transactions on Power Systems*, Vol. 32, No. 5, pp. 3875-3883, Sept. 2017, doi: 10.1109/TPWRS.2016.2632156

[11] D. Topolanek, V. Vycital, V. Krcal, J. Grossmann and M. Jurik, "Pilot test of the method VDIP for an earth fault localization," *27th International Conference on Electricity Distribution (CIRED 2023)*, Rome, Italy, 2023, pp. 423-427, doi: 10.1049/icp.2023.0337

[12] C. Muscas, M. Pau, P. A. Pegoraro and S. Sulis, "Effects of Measurements and Pseudomeasurements Correlation in Distribution System State Estimation," in *IEEE Transactions on Instrumentation and Measurement*, Vol. 63, No. 12, pp. 2813-2823, Dec. 2014, doi: 10.1109/TIM.2014.2318391

[13] A. K. Ghosh, D. L. Lubkeman and R. H. Jones, "Load modeling for distribution circuit state estimation," in *IEEE Transactions on Power Delivery*, Vol. 12, No. 2, pp. 999-1005, April 1997, doi: 10.1109/61.584427

[14] J. Wu, Y. He and N. Jenkins, "A robust state estimator for medium voltage distribution networks," in *IEEE Transactions on Power Systems*, Vol. 28, No. 2, pp. 1008-1016, May 2013, doi: 10.1109/TPWRS.2012.2215927

[15] R. Arritt and R. Dugan, "Comparing load estimation methods for distribution system analysis," *22nd International Conference and Exhibition on Electricity Distribution (CIRED 2013)*, Stockholm, 2013, pp. 1-4, doi: 10.1049/cp.2013.0869

[16] R. Ebrahimian and R. Baldick, "State Estimator Condition Number Analysis," in *IEEE Power Engineering Review*, Vol. 21, No. 5, pp. 64-64, May 2001, doi: 10.1109/MPER.2001.4311389