The Application of Benford's Law on the Hourly Differences of Electricity Prices

František Kurimský, Marek Pavlík, Jaroslav Petráš and Ardian Hyseni

Department of Electric Power Engineering, Faculty of Electrical Engineering and Informatics, Technical University of Košice, Mäsiarska 74, 042 00 Košice, Slovakia; marek.pavlik@tuke.sk, frantisek.kurimsky@tuke.sk, jaroslav.petras@tuke.sk, ardian.hyseni@tuke.sk

Abstract: This study explores the applicability of Benford's Law, to the hourly differences of electricity prices in six European countries: Germany, France, Poland, Hungary, the Czech Republic and Slovakia. We analyze hourly price data from the day-ahead market from 2015 to 2024. By analyzing data from the day-ahead market over the period 2015 to 2024, we assess the degree of correspondence of the distribution of the first number, the second number and the first two numbers of absolute hourly price differences with predictions Benford's law. The research results show a high degree of agreement across all countries, as confirmed by chi-square tests. This can provide useful information for electricity price formation and contributes to a deeper understanding of electricity price dynamics. Our findings demonstrate the potential of Benford's law as a robust tool for anomaly detection, market manipulation analysis and improving the accuracy of electricity price prediction models. This approach allows regulators and market participants to better understand and monitor electricity market dynamics.

Keywords: Benford's law; electricity price; hourly price variation; day-head market; electricity market

1 Introduction

In recent years, electricity prices have become increasingly volatile and thus, many companies try to predict how market would react to certain events. As mentioned in [1] [2], various methods have been tried for electricity price forecasting, with goal for optimalization and financial gain. Optimalization of demand and supply is more challenging. Since renewables sources of electricity became popular, we can observe more common negative prices on the market, which were not very present in the past. The number of occurrences of negative prices in 2024 reached record levels in Europe. In the year 2022, negative prices appeared 261 in 2023 those prices appeared 3027 times, and in the year 2024 taking into account months January till August, negative prices appeared 7841 times [3].

Geopolitical changes influence energy markets, because of dependence on resources such as gas and others. After war on Ukraine, energy markets were more volatile, and prices had sharply risen [4]. Also, at the end of 2021 and early 2022, energy prices spiked, straining consumer purchasing power. Initially driven by post-pandemic demand recovery, the rise was even worse by supply issues and the Russian invasion of Ukraine, leading to sharp increases in gas and electricity prices across Europe [5].

Electricity prices in Europe are determined using the Merit Order Model, which ranks supply bids from the lowest to the highest marginal production cost to meet market demand. This system operates in the Day-Ahead Market (hereafter DAM), where suppliers submit bids reflecting their variable costs, and buyers may submit demand bids or rely on forecasted demand. Hourly prices for the following day are calculated based on these bids. The final clearing price is determined using the Euphemia algorithm, which optimizes social welfare by minimizing total system costs while accounting for transmission constraints across the coupled European electricity market. This article focuses on the application of Benford's law on hourly differences in electricity prices, with aim to determine, if war in Ukraine and Covid pandemics have impact on hourly differences for electricity prices. We observed results of Benford's law tests for first digit, second digit and first two digits on hourly difference values from DAMs in Germany, France, Poland, Hungary, Czech Republic and Slovakia.

2 Benford's Law

Benford's law, also known as the First-digit Law, is widely used across many fields such as fraud detection in accounting data, detecting anomalies or computer disk space allocation. Theodore P. Hill in [6] wrote, that the first-digit law is the empirical observation of numerical data, that the leading digits are not uniformly distributed, but they follow logarithmic distribution. Many researchers have confirmed the law's broad applicability but have also observed, that the commonly accepted explanations remain somewhat unconvincing [7]. As we go through most cited articles, application of Benford's is seen in many fields. Since 1998, China has faced falsification on gross domestic product (GDP). Conclusions in [8] stated, that application of law on GDP in China shows only few statistical anomalies, thus falsification was not that significant, or party responsible for falsification have knowledge about statistical regularities. Benford's law can be also used for detection of fabricated and falsified scientific data. Findings in [9] shows, that in order to detect fabricated data, first digit follows distribution of

Benford's law, but second, third and fourth digit were distributed less in accordance with Benford's law. In [10] authors applied Benford's law on daily closing values and log-return values of S&P500. Closing values showed significant deviations from Benford's law, but log-return values were more fit. In [11], Benford's law is used to determine whether the numbers reported by individual countries seem valid or potentially manipulated.

Phenomenon, that the first significant digit is more often 1 than others digits was firstly described in [12] by Newcomb. Later, Benford published more detailed article unaware of Newcomb's research. He tested his observations on collection of 20.229 numbers gathered from various diverse fields, such as lengths of rivers, addresses, human populations and others [13]. The probability of observing the first digit d_1 in a set of numbers conforming to Benford's Law (BL1 hereafter) is defined below [14].

$$P_1(d_1) = \log_{10}(\frac{d_1+1}{d_1}), \qquad d_1 = 1,2,3,4,5,6,7,8,9 \tag{1}$$

According to this equation, we can see that probability of observing the first digit 1 is 30.1% and probability of observing the first digit 9 is 4.58%. Furthermore, the probability of observing the second digit of d_2 is given by Benford's law for the second digit (BL2 hereafter):

$$P_2(d_2) = \sum_{k=1}^{9} \log_{10}(1+\frac{1}{10k+d_2}), \quad d_2 = 0,1,2,3,4,5,6,7,8,9 \tag{2}$$

and by extending BL1 we can get Benford's law for the first two digits (BL12 hereafter):

$$P_{12}(d_1d_2) = \log_{10}(1+\frac{1}{d_1d_2}), \qquad d_1d_2 = 10,11,...,99 \tag{3}$$

Calculated probabilities for BL1 and BL2 are in Table 1, and for BL12 in Table 2.

Table 1
Calculated probabilities for BL1 and BL2

Digit	0	1	2	3	4
BL1	-	30.10%	17.61%	12.49%	9.69%
BL2	11.97%	11.39%	10.88%	10.43%	10.03%
Digit	5	6	7	8	9
BL1	7.92%	6.69%	5.80%	5.12%	4.58%
BL2	9.67%	9.34%	9.04%	8.76%	8.50%

Table 2
Calculated probabilities for BL12

Digits	BL12	Digits	BL12	Digits	BL12	Digits	BL12
10	4.14%	33	1.30%	56	0.77%	78	0.55%
11	3.78%	34	1.26%	57	0.76%	79	0.55%
12	3.48%	35	1.22%	58	0.74%	80	0.54%
13	3.22%	36	1.19%	59	0.73%	81	0.53%
14	3.00%	37	1.16%	60	0.72%	82	0.53%
15	2.80%	38	1.13%	61	0.71%	83	0.52%
16	2.63%	39	1.10%	62	0.69%	84	0.51%
17	2.48%	40	1.07%	63	0.68%	85	0.51%
18	2.35%	41	1.05%	64	0.67%	86	0.50%
19	2.23%	42	1.02%	65	0.66%	87	0.50%
20	2.12%	43	1.00%	66	0.65%	88	0.49%
21	2.02%	44	0.98%	67	0.64%	89	0.49%
22	1.93%	45	0.95%	68	0.63%	90	0.48%
23	1.85%	46	0.93%	69	0.62%	91	0.47%
24	1.77%	47	0.91%	70	0.62%	92	0.47%
25	1.70%	48	0.90%	71	0.61%	93	0.46%
26	1.64%	49	0.88%	72	0.60%	94	0.46%
27	1.58%	50	0.86%	73	0.59%	95	0.45%
28	1.52%	51	0.84%	74	0.58%	96	0.45%
29	1.47%	52	0.83%	75	0.58%	97	0.45%
30	1.42%	53	0.81%	76	0.57%	98	0.44%
31	1.38%	54	0.80%	77	0.56%	99	0.44%
32	1.34%	55	0.78%				

2 Related Work

In order to visualize the research landscape surrounding Benford's law, we extracted relevant publications and using the topics "benford's law" or "Benford law". Based on Web of Science database search results since 1978, there have been 762 total publication related to Benford' law. As shown in Figure 1, Benford's law has become more popular over the years. Most publications were written in the year 2022, with a count of 69, which represents 9.1% from 762 publications. Search was performed in October 2024. Linear trendline in Figure 1 outlines increase in popularity over the years.

In order to uncover the most significant topics related to Benford's law, we performed a keyword co-occurrence analysis on extracted records from WOS search mentioned above, for conducting a bibliometric analysis. The VOSviewer tool from Clarivate was used [15]. Occurrence threshold for selecting keyword to be present in generated graph was set to 10.

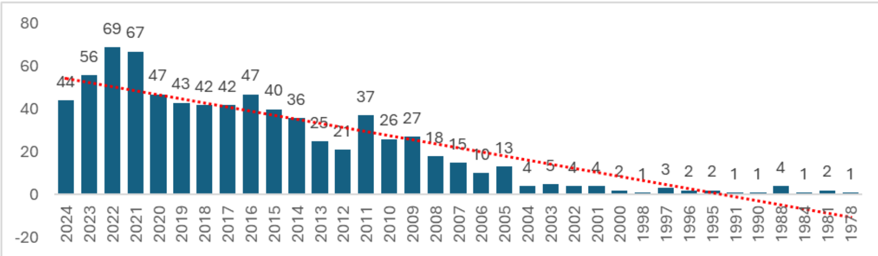

Figure 1
Publications count over years for the topics "benford"

From generated keyword co-occurrence analysis graph, see Figure 2, one can see, that the strongest connections to the most occurred keyword "benford's law", which appeared 375 times over extracted publications, has 'fraud' keyword with occurrence in 59 publications, numbers in 42 publications, 'fraud detection' with occurrence in 41 publications, '1st digit' in 30 publications and 'earnings management' with occurrence in 29 publications.

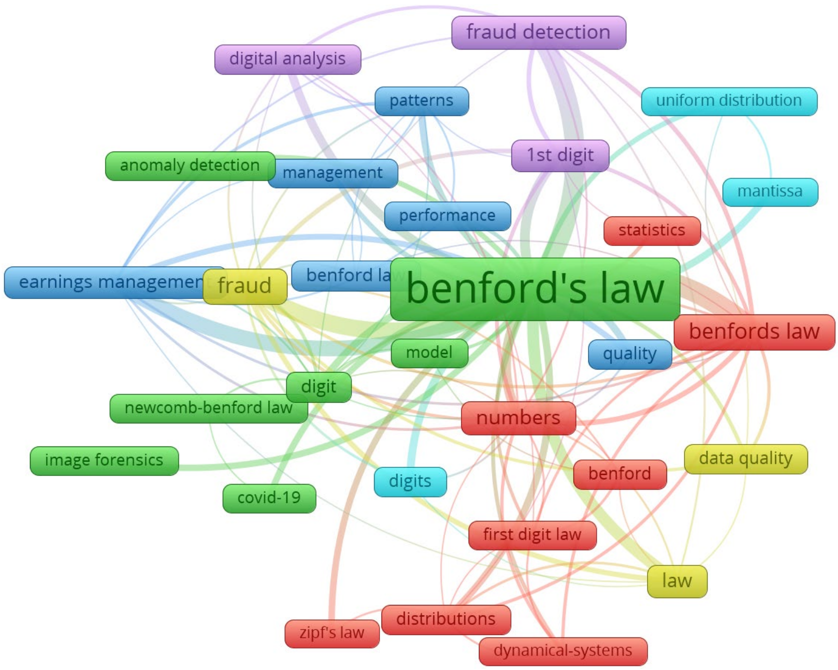

Figure 2
Keywords related to Benford's law

Most interesting keywords were 'covid-19', which occurred in 14 publications, and anomaly detection present in 14 publications. In conclusion, application of Benford's law seems to be useful in fraud detection and detection of questionable manipulation of report data as it is presented in more specifically in section 2.1. Keyword "earnings management" is most fit for topics with market data and finances.

After updating search in Web of Science to "benford's law" (Topic) or "Benford law" (Topic) and "earnings management" (Author Keywords) or "earnings management" (Keyword Plus), we found 35 articles. Span of publication years for these articles are from 2005 to 2024. As we can see in Figure 3, some years do not have any publications. The year 2023 holds the most publications. Linear trendline present as red dotted line in Figure 3 suggest, that the topic is more popular over the years, but it is not as strong as in the Figure 1.

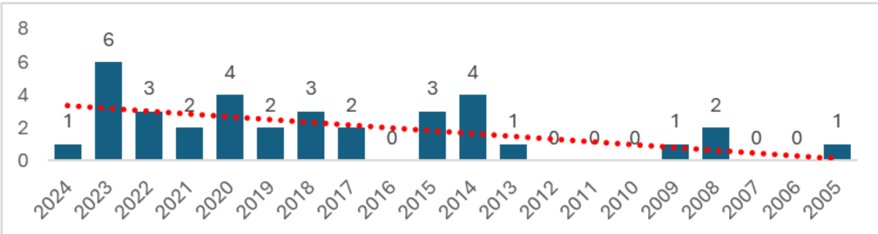

Figure 3
Publications count by year for topic "benford's law" and "benford law" and keywords "earnings management"

2.1 Related Work in Earning Management

In the Table 3 we provided top cited publications, which are related to benford's law application with earning management keyword. These studies utilized Benford's Law for anomaly detection, fraud detection, and identifying data manipulation. This approach has proven to be an effective tool for uncovering irregularities in financial data. Similarly, our analysis explores the application of Benford's Law in the electricity market. We analyzed articles and below we overviewed their approach, conclusions and contribution. Six articles were written by NATIONAL TAIPEI UNIVERSITY OF TECHNOLOGY, and the second biggest contributor is TEXAS A&M UNIVERSITY SYSTEM with 3 publications. The biggest contributor in terms of authors is Lin FY, with six publications under his name. Emerald Group Publishing is the biggest publisher in this area with 9 publications, and the second biggest publisher is Elsevier with 6 articles.

In [16], authors introduced the Financial Statement Divergence Score (FSD Score), a firm-year measure for identifying financial reporting errors based on deviations from Benford’s Law. Benford’s Law predicts the distribution of leading digits in datasets, where smaller digits occur more frequently. The FSD Score highlights errors such as revenue overstatements, expense understatements, and strategic earnings manipulation. When evaluating the conformity of individual firm-years shows 86% of firm-years conform to Benford’s Law. Firms that are smaller, younger, more volatile and growing tend to diverge from Benford’s law. When financial statements are restated, they align more with Benford’s Law than the original misstated numbers. The more a firm's financial statements deviate from the law, the lower their earnings persistence. The FSD Score helps detect earnings management and serves as a useful tool for identifying potential misstatements. It offers a practical method for assessing errors in financial reports, especially with electronic disclosures.

In [17], the authors used Benford's Law to study the connection between accounting conservatism, investor ownership, and earnings manipulation. They found that firms with conservative accounting practices were less likely to manipulate earnings, as their financial data followed Benford's Law closely. In contrast, less conservative firms with low institutional ownership were more likely to manipulate earnings, shown by deviations in digit patterns. The study emphasized the importance of institutional investors in monitoring practices and demonstrated how Benford's Law can help identify financial reporting issues and assess the reliability of disclosures.

The article [18] assesses the incidence of earnings management during the Enron-Andersen episode, particularly focusing on the years 2001 and 2002. It analyzes earnings reports using Benford's Law to detect manipulations in revenue and earnings per share (EPS) numbers. The findings indicate a noticeable increase in earnings management in 2002, characterized by upward rounding of revenue figures and a marked discontinuity in EPS numbers around zero, suggesting that companies were managing earnings to avoid losses and meet psychological thresholds. The study concludes that despite heightened scrutiny following the Enron scandal, earnings management practices persisted and even intensified.

Authors in [19] examined earning of publicly listed companies in Taiwan and United States from 1990 to 2011. Investigating whether managers adopted accounting adjustments to engaged in cosmetic earnings management, was done using Benford’s law. Results show that despite developed and emerging markets, the phenomenon of cosmetic earnings management still prevails. An important finding was that earning management was significantly lower after implementation of corporate governance regulations. The results were statistically analyzed using Chi-square test, Z statistics and Cramer’s V. They indicate that managers in the U.S. have strong incentives to manipulate earnings by exaggerating the earnings numbers.

Publication [20] applied Benford’s Law to detect accounting data manipulation in the banking industry, particularly before and during the global financial crisis in

2008. Quarterly data used in this study were divided into 2 parts, first representing before crisis state spanning from first quarter of 2003 to third quarter of 2007 and second representing crisis quarters spanning from fourth quarter of 2007 to fourth quarter of 2012. In this second period, multiple misstatements and frauds occurred in the banking industry. Study focused on the U.S. commercial and savings banking institutions that file a Report on Condition and Income. The authors used Benford's Law to examine the manipulation of a set of fundamental balance sheet and income statement data, both before and after the financial crisis. In conclusion, banks manipulated financial data, particularly during crises, to enhance reported earnings, where distressed institutions shown the most manipulation. Tools like Benford's Law can help detect and reduce such manipulation.

Table 3
Publications researching Benford's law with keyword earnings management

Authors Publication Year, Ref.	**Article Title**	**Conference/Journal/ Book**	**Times Cited, All Databases**
Amiram, D; Bozanic, Z; Rouen, E 2015, [16]	Financial statement errors: evidence from the distributional properties of financial statement numbers	REVIEW OF ACCOUNTING STUDIES	83
Lin, FY; Wu, CM; Fang, TY; Wun, JC 2014, [17]	The relations among accounting conservatism, institutional investors and earnings manipulation	ECONOMIC MODELLING	43
Nigrini, MJ 2005, [18]	An Assessment of the Change in the Incidence of Earnings Management Around the Enron-Andersen Episode	REVIEW OF ACCOUNTING AND FINANCE	43
Lin, FY; Wu, SF 2014, [19]	Comparison of cosmetic earnings management for the developed markets and emerging markets: Some empirical evidence from the United States and Taiwan	ECONOMIC MODELLING	21
Grammatikos, T; Papanikolaou, NI 2020, [20]	Applying Benford's Law to Detect Accounting Data Manipulation in the Banking Industry	JOURNAL OF FINANCIAL SERVICES RESEARCH	19

2.2 Related Work in Electricity Topic

When researching the application of Benford's law applied in electricity topic, we found only small number of published articles. In web of science search engine, we found 7 publications, but one of them was no related to this topic. Only one article was published in each of the years 2017, 2020, 2022, and 2023. In 2024, two publications were presented. This suggests that the application of Benford's Law in the electricity market is yet to be explored. Notably, three of these publications were written by the Technical University of Košice. In Table 4 we present all the publications and below we provide an overview of their methods, findings, and significance.

Authors in [21] proposed and implemented Distributed Intelligent Framework for Electricity Theft Detection (DIFETD). They equipped this framework with Benford's analysis and Stackelberg game-theoretic model, which is used to analyze strategic interactions between utility and electricity thieves. Benford's analysis is used to detect electricity theft as initial testing. Authors used hourly real-world data from 103 advanced metering infrastructure (AMI) [22] smart meters from an electric utility in Florida for may in 2013. Results demonstrated high detection rates with minimal false alarms, confirming the framework's effectiveness in identifying electricity theft.

Paper [23] constructed data quality inspection model based on Benford's law and the technique for order of preferences by similarity to ideal solution (TOPSIS). Model was applied to test reliability of data in coal, electricity and other sectors. Electricity alongside coal and steel performed poorly and showed significant quality issues. Authors stated that this occurred because of the government interference, market inefficiencies, and data manipulation. Sectors like non-ferrous metals and building materials performed much better in data quality.

In [24], authors used Benford's law to evaluate data quality of Chinese Industrial Census (CIC). Chinese economic data has been debated for a long time on their accuracy. Data were collected from this source and completed from the annual Chinese City Statistical Yearbook (CCSY). Processed data covered 195 cities in 30 provinces spanning from 1999 to 2011. To evaluate empirical distribution, authors used Pearson's Chi-square test. To further analyze magnitude and direction of data manipulation, authors utilized electricity and employment data to estimate city-level industrial output and calculate the residuals of industrial output at the city level. Abnormal residuals are those not explained by employment and electricity. Analysis shows no significant correlation between Benford's indices and residuals for state-owned enterprises (SOEs), but private firms with higher Benford's indices report lower residuals, indicating downward manipulation to reduce tax liabilities.

Authors in [25][26] applied Benford's Law's first-digit probability distribution test to electricity metering datasets obtained from smart electricity meters, specifically analyzing natural electricity consumption data recorded over a defined time interval. In conclusion, this paper highlights the effectiveness of Benford's law application for analyzing electricity consumption data from smart meters to find anomalies caused by artificial manipulation or electricity theft. Datasets with no manipulation closely followed Benford's law. Violation was simulated by adding +1 to dataset values, dividing values by 2, multiplying values by 2 and supplementary violation operations were performed, where 75%, 50%, 25% and 10% were replaced by pseudo-random numbers. Research showed diversion of manipulated data from Benford's law and pointed out the effectiveness of usage of Benford's law in this topic.

Research [27] pointed the fact that the year 2022 saw a dramatic rise in electricity prices in Germany, reaching extreme levels due to a combination of geopolitical and climatic factors. Analysis of price trends showed value of price is most frequent in interval between 40-50 over the years from 2015 to 2024, where year 2022 showed significant rice in histogram, where many values are between 180 – 240 EUR/MWh, year 2023 was slightly lower and year 2024 is slowly returning to previously known prices 60-80 EUR/MWh. The research aimed to investigate whether Benford's Law is suitable for analyzing electricity prices. The researchers found that while Benford's Law did not fully predict the distribution of electricity prices, the frequency of individual digits provided valuable data for predictive modeling. They also suggested that this approach could be applied to other commodities, such as gas, though further research is needed to assess its effectiveness in these markets.

Table 4

Publications researching Benford's law with the topic of electricity

Authors Publication Year, Ref.	**Article Title**	**Conference/Journal/ Book**	**Times Cited, All Databases**
Wei, LF; Sundararajan, A; Sarwat, AI; Biswas, S; Ibrahim, E 2017 , [21]	A Distributed Intelligent Framework for Electricity Theft Detection Using Benford's Law and Stackelberg Gam	2017 RESILIENCE WEEK (RWS) Page 5-11	36
Wang, DL; Chen, F; Mao, JQ; Liu, NN; Rong, FY 2022, [23]	Are the official national data credible? Empirical evidence from statistics quality evaluation of China's coal and its downstream industries	ENERGY ECONOMICS	4
Huang, YS; Niu, ZY; Yang, C 2020, [24]	Testing firm-level data quality in China against Benford's Law	ECONOMICS LETTERS	4
Petras, J; Pavlik, M; Zbojovsky, J; Hyseni, A; Dudiak, J 2023, [28]	Benford's Law in Electric Distribution Network	MATHEMATICS	2

Pavlík, M; Beres, M; Hyseni, A; Petrás, J 2024, [27]	Statistical Analysis of Electricity Prices in Germany Using Benford's Law	ENERGIES	0
Hyseni, A; Medved, D; Petras, J 2024, [26]	Benford's Law in Electric Power Engineering	24TH INTERNATIONAL SCIENTIFIC CONFERENCE ON ELECTRIC POWER ENGINEERING, EPE 2024	0

3 Data and Processing

In this article we work with hourly day-ahead market electricity price data from Energy Charts [29] for Germany, France, Poland, Hungary, Czech Republic and Slovakia. Extracted data spans from 2015 to 2024. In our research, we are interested in hourly price differences. Values are calculated as the absolute value between two consecutive hourly price values. This approach allows us to capture the market's volatility. Missing values are sometimes in data. To ensure, that only complete pairs of consecutive values are used, we excluded differences, where one of values at hour h_1 or h_2 is missing. Slovakia has 120 missing values (0.14%), in Hungary and Czech Republic there are 0 missing values, In France and Germany 95 values are missing (0.11%) and in Poland 5703 values are missing (6.5%). In case of Germany, market changed over the years. Bidding zone for Germany changed in September 2018 from DE-AT-LU, where Germany, Austria and Luxembourg participated, to DE-LU, when Austria detached. In the data obtained from Energy-charts, we observed a notable change on 2018-09-30 at 21:00, where the DE-AT-LU price is recorded. By 22:00 on the same date, the price is instead defined for the DE-LU market. We concatenated data defined in DE-AT-LU bidding zone and in DE-LU zone into one vector for our study.

Calculated differences between consecutive price values pointed us to construct a histogram, aiming to identify typical patterns. For histogram creation, we chose the maximal value 20 and number of bins or divisions also 20. We checked that histogram columns after value 20 were fading towards zero, thus we did not create a bigger histogram for sufficient clarity. Across all markets, the histogram in Figure 4 and Figure 5 shows a steep decline from the first bin, indicating that small price differences are the most frequent. The trend gradually decreases for higher difference values, showing a long tail. All markets exhibit a similar distribution, suggesting consistency in market behaviors.

Once the data were processed, we applied Benford's law on price differences. We performed separate analyses for the first digit BL1, second digit BL2 and first two digits BL12, to examine whether data are in accordance with the expected probabilities defined by Benford's law. Firstly, we applied Benford's law for each market (Germany, France, Poland, Hungary, Czech Republic and Slovakia) over the entire span between years 2015 – August 2024. This country-level analysis allowed us to examine specific patterns for each country in electricity price volatility. Our analysis also observed the application of Benford's law on yearly basis for each market, with the aim, whether different years exhibit varying levels of conformity to Benford's law, due to market changes, policy shifts or significant events (e.g., COVID-19 pandemic, war in Ukraine).

A potential area for research could involve applying Benford's Law to electricity prices in different currencies. Since the number of significant digits might vary depending on the currency, this could influence the application of Benford's Law and result in different patterns.

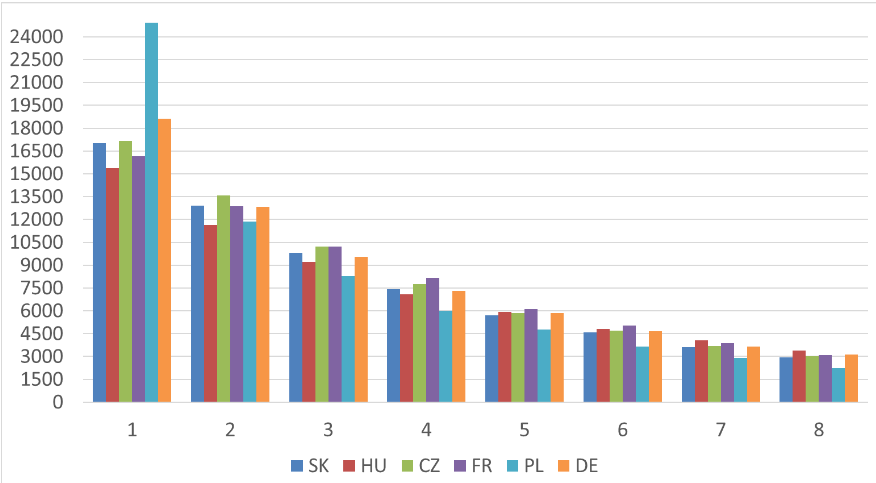

Figure 4

Histogram of absolute differences between two consecutive prices I

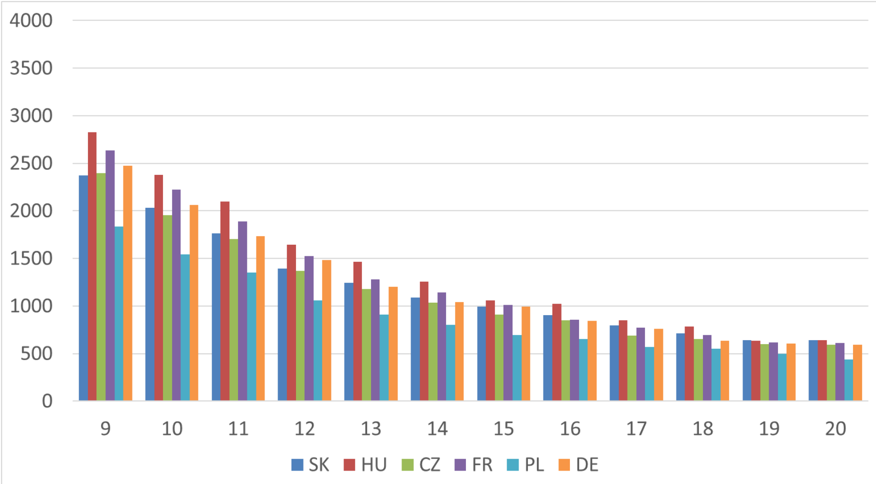

Figure 5

Histogram of absolute differences between two consecutive prices II

4 Results

We conducted results from application BL1, BL2 and BL12 law for each country over entire span. A chi-square test was performed to compare the observed percentages of digit occurrences with the expected values according to Benford's Law. The p-values from the chi-square test are summarized in the Table 5 below. These values indicate the probability that the observed distributions align with the theoretical expectations of Benford's Law. The p-values for BL1 and BL12 are consistently close to or equal to 1 across all countries, suggesting a strong conformity of the first digit and first two digits distributions to Benford's Law. For BL2, the p-values are slightly lower but still very high, indicating that the second digit distribution also aligns well with the expected values. Overall, these results suggest that the digit distributions of electricity hourly price differences closely follow the theoretical predictions of Benford's Law, reinforcing its applicability in analyzing price anomalies in the electricity market.

Table 5

P-values of Chi-square test

	SK	HU	CZ	FR	PL	DE
BL1	1.000000	1.000000	0.999999	1.000000	0.999999	1.000000
BL2	0.988724	0.997346	0.979442	0.999871	0.986619	0.997682
BL12	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Below we show results of the application of Benford's Law for the first-digit (BL1) distribution of electricity price differences across six countries: Slovakia, Hungary, the Czech Republic, France, Poland, and Germany. The Figure 5 displays the percentage distribution of the first digit for each country, while the Table 6 highlights the deviations between observed and expected values. Specifically, the values in Table 6 represent the absolute difference between the observed percentage of numbers starting with a given digit and the theoretical percentage defined by Benford's Law. For instance, for digit 1 in Germany, the deviation of 1.020829 indicates that the observed percentage of numbers starting with 1, at 29.08217% differs from the theoretical value of 30.103% by this amount. All countries are closely following the BL1 law distribution. Biggest difference with value 1.020829 is between percentage of numbers starting with digit 1 in Germany. In conclusion, the results confirm that the first-digit distribution of electricity price differences adheres closely to Benford's Law across all countries.

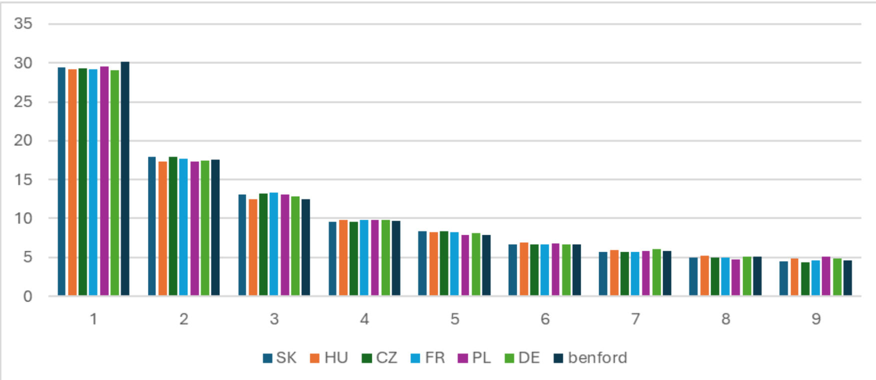

Figure 6
Application of BL1

Table 6
Absolute deviations between observed and expected values for BL1

	SK	HU	CZ	FR	PL	DE
1	0.717522	0.966039	0.853933	0.980725	0.614043	1.020829
2	0.331043	0.34372	0.343	0.052265	0.231393	0.185057
3	0.611619	0.035158	0.73559	0.774869	0.563773	0.392665
4	0.129317	0.161904	0.145121	0.096492	0.111598	0.149107
5	0.404789	0.348286	0.44998	0.349986	0.012854	0.251952
6	0.081731	0.24869	0.017533	0.053987	0.043906	0.040935
7	0.095064	0.134272	0.062882	0.141159	0.014525	0.285637
8	0.190273	0.139939	0.183388	0.160592	0.366903	0.068455
9	0.133543	0.241509	0.265713	0.062852	0.494733	0.235914

The results for BL2 are shown in Figure 6 and Table 7. Compared to the application of BL1, the deviations are noticeably larger. The most significant differences occur for numbers with the second digit 0, particularly in Slovakia and the Czech Republic. Across all countries, digit 0 shows the highest deviation. Also digit 9 shows also large difference with expected value of BL2 law in Poland. However, for other digits, the distribution remains consistent with BL2 expectations, as supported by the Chi-square test results. This suggests that while BL2 generally aligns with Benford's Law, certain digits exhibit more variability.

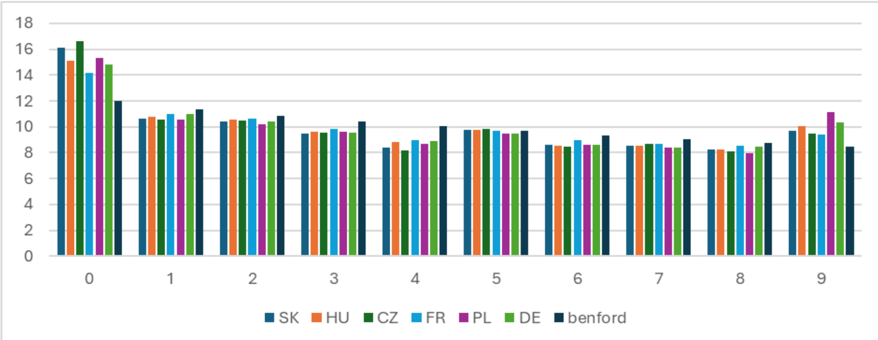

Figure 7
Application of BL2

Table 7
Absolute deviations between observed and expected values for BL2

	SK	HU	CZ	FR	PL	DE
0	4.163869	3.134392	4.630739	2.198424	3.365881	2.831807
1	0.722853	0.593221	0.836415	0.385551	0.82228	0.398462
2	0.471851	0.30706	0.37232	0.24417	0.647542	0.446749
3	0.939578	0.818082	0.903257	0.592595	0.845112	0.914008
4	1.631496	1.191284	1.860361	1.058185	1.335711	1.119782
5	0.111613	0.078876	0.190227	0.051961	0.172275	0.156804
6	0.741336	0.777567	0.882683	0.391273	0.734759	0.754477
7	0.468004	0.519201	0.339999	0.324606	0.640997	0.624252
8	0.525552	0.542592	0.628155	0.18318	0.769841	0.262327
9	1.225188	1.535738	1.002224	0.929174	2.602636	1.845053

In the case of applying BL12, we found interesting deviations for specific first two digits. These deviations may be attributed to the fact that electricity prices are typically rounded to two decimal places, resulting in relatively small differences between consecutive hourly prices (e.g., 0.05). In our dataset, such differences are scaled to 50, potentially influencing the observed distribution. Table 8 presents the results of the BL12 application, highlighting only the top deviations for clarity. The highlighted deviations were selected based on the highest average deviation values across the six countries. The average deviation was calculated for each two-digit combination across all datasets. Poland exhibits the highest deviation 0.92 followed by Czech Republic with 0.89 deviation for two-digit combination 10. Digit combination 10 exhibits the highest average deviation 0.69, indicating the largest departure from expected values.

Table 8

Absolute deviations between observed and expected values for BL12

	SK	HU	CZ	FR	PL	DE	Average
10	0.81	0.64	0.89	0.43	0.92	0.45	**0.69**
50	0.81	0.53	0.88	0.46	0.41	0.52	**0.60**
30	0.67	0.44	0.70	0.37	0.56	0.46	**0.53**
60	0.48	0.42	0.52	0.28	0.50	0.36	**0.43**
20	0.52	0.29	0.68	0.25	0.28	0.32	**0.39**
39	0.34	0.31	0.30	0.32	0.64	0.43	**0.39**
99	0.23	0.27	0.19	0.18	0.87	0.32	**0.34**

The BL1, BL2, and BL12 tests were applied to annual datasets for each country, spanning the years 2015 to 2024. It is important to note that the volume of data available for individual years is considerably smaller compared to the entire dataset. Following the chi-square test, the resulting p-values were observed to be close to 1, indicating a strong alignment with the expected distributions. However, for Slovakia and the Czech Republic, the p-values for BL2 were notably lower, measuring 0.581 and 0.486, respectively. Despite the disruptions associated with the COVID-19 pandemic and the war in Ukraine, no significant irregularities were detected in the digit distributions during these years. Figure 7 presents the results of the BL1 test for each year from 2015 to 2024 for Germany.

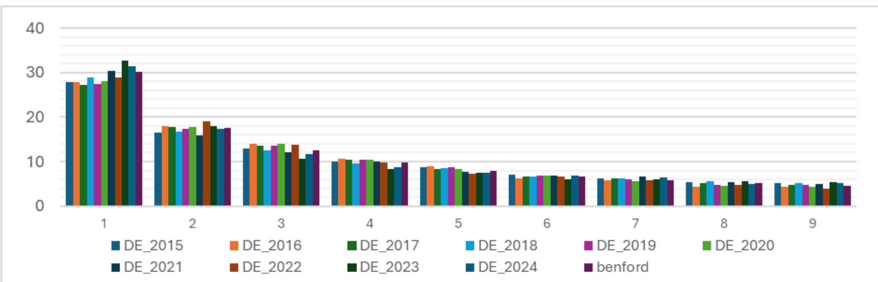

Figure 8

Application of BL1 for each ear spanning from 2015 to 2024 for Germany

After closer look to difference between expected percentage values defined by BL1 and observed percentage of numbers starting with a given digit, highest deviations were observed for Poland in year 2015 with value 4.247. Next biggest deviation was observed for France in year 2020 with value 4.232.

For BL2, the most significant deviations were observed in the Czech Republic and Slovakia in 2016, with values of 8.651 and 8.056, respectively. These findings indicate that the second-digit distribution for these countries in that year deviates more from theoretical expectations, possibly due specific factors affecting market behavior.

In contrast, for BL12, the deviations between the observed and expected percentage values remained below 2 across all countries and years analyzed. This indicates a consistently strong alignment of the combined first and second-digit distributions with the theoretical predictions of Benford's law.

The results of applying BL1, BL2, and BL12 highlight notable patterns in the digit distributions of electricity price differences across the analyzed countries. BL1 and BL12 consistently show strong conformity to Benford's Law, with minimal deviations and high p-values. BL2 reveals slightly larger deviations for specific digits. Overall, the results demonstrate a strong alignment of electricity price differences, with the expected distributions of Benford's Law across all tested scenarios.

Conclusions and Discussion

The research results show that the frequency of digits in hourly electricity price differences, contain useful information for predictive modelling. Benford's law can be integrated as part of sophisticated algorithms for price prediction or for detecting abnormalities in market data, which could lead to improved prediction accuracy and increased reliability of decision-making processes in energy markets. At the same time, using Benford's law, it is possible to detect abnormal behavior in the electricity market and help in fundamental market analysis. This research opens up opportunities for deeper analysis of electricity price dynamics, using advanced statistical methods and machine learning.

The application of Benford's law demonstrates a strong alignment between observed and expected digit distributions in electricity price differences for all analyzed countries. The chi-square test results confirm this conformity, with p-values consistently close to 1. External disruptions like the COVID-19 pandemic and geopolitical tensions did not result in significant irregularities.

The results also show that Benford's law can be applied to a variety of electricity markets. For our research we focused on the countries Germany, France, Poland, Slovakia, Czech Republic and Hungary. These are countries whose energy mix varies considerably. For example, the share of Renewable Energy Sources (RES) in Germany is considerably higher than the share of RES in, for example, Poland or Slovakia. France and Slovakia, on the other hand, have a significant share of electricity generation from nuclear power plants in their energy mix, which Germany, on the other hand, does not. Nevertheless, our research confirms that changing the energy mix, does not change the results of the research on the application of BL to the price of electricity. It can therefore be concluded, that BL can be applied in different markets under different changes.

The research results shown in Table 5 further confirmed to us that Benford's law, as a statistical tool, can be applied to electricity price and its possible prediction. The results show a very low value of the difference between the frequency of digits in our data sample and the frequency defined by Benford's law. This means that if the electricity price is, for example, 25.1 EUR/MWh, there is a 30.1% probability that in the next hour the price will start with digit 1. However, this could mean a price of, for example, 15.1 EUR/MWh but also 151.1 EUR/MWh, which is a significant difference. However, an analysis of historical price differences, in form of a histogram, suggests that price changes in the hundreds (e.g., from 15.1 EUR/MWh to 151.1 EUR/MWh) are far less likely. Instead, it is more probable that the price remains below 1 EUR/MWh, even though both scenarios begin with digit 1.

It is worth mentioning the planned transition to 15-minutes SPOT price resolution. In future analysis we could explore whether the shift to a finer time resolution introduces systematic changes in the distribution of first significant digits. Currently, we do not have the option to perform such study, due to the absence of such data.

Overall, this study demonstrates the robustness of Benford's Law as an effective tool for analyzing electricity price dynamics, offering valuable insights for predictive modeling and market anomaly detection.

Acknowledgement

This research was funded by Early Stage Grants (ESG) no. 56/2024/ESGTUKE:Prediction studies on evolving electricity market.

References

[1] R. Weron: Electricity price forecasting: A review of the state-of-the-art with a look into the future, Vol. 30, No. 4, pp. 1030-1081, Oct. 2014

[2] J. Lago, G. Marcjasz, B. De Schutter, and R. Weron: Forecasting day-ahead electricity prices: A review of state-of-the-art algorithms, best practices and an open-access benchmark, Vol. 293, p. 116983, Jul. 2021

[3] S. Tani and R. Millard: Negative European energy prices hit record level, *Financial Times*, Sep. 14, 2024, Accessed: Oct. 28, 2024 [Online] Available: https://www.ft.com/content/1f94d0b4-c839-40a2-9c8d-782c00384154

[4] J. F. Adolfsen, F. Kuik, T. Schuler, and E. Lis: The impact of the war in Ukraine on euro area energy markets, Jun. 2022 Accessed: Oct. 28, 2024 [Online] Available: https://www.ecb.europa.eu/press/economic-bulletin/focus/2022/html/ecb.ebbox202204_01~68ef3c3dc6.en.html

[5] F. Kuik, J. F. Adolfsen, A. Meyler, and E. Lis: Energy price developments in and out of the COVID-19 pandemic – from commodity prices to consumer prices, Jun. 2022 Accessed: Oct. 28, 2024 [Online] Available:

https://www.ecb.europa.eu/press/economic-bulletin/articles/2022/html/ecb.ebart202204_01~7b32d31b29.en.html

[6] T. P. Hill: A statistical derivation of the significant-digit law, Vol. 10, No. 4, pp. 354-363, Nov. 1995

[7] R. M. Fewster: A Simple Explanation of Benford's Law, Vol. 63, No. 1, pp. 26-32, Feb. 2009

[8] C. A. Holz: The quality of China's GDP statistics, Vol. 30, pp. 309-338, Sep. 2014

[9] A. Diekmann: Not the first digit! Using Benford's law to detect fraudulent scientific data, Vol. 34, No. 3, pp. 321-329, 2007

[10] M. Ausloos, V. Ficcadenti, G. Dhesi, and M. Shakeel: Benford's laws tests on S&P500 daily closing values and the corresponding daily log-returns both point to huge non-conformity, Vol. 574, p. 125969, Jul. 2021

[11] L. Butgereit: Covid-19 New Cases Measurements and Benford's Law with Specific Focus on South Africa, in *2021 International Conference on Artificial Intelligence, Big Data, Computing and Data Communication Systems (icABCD)*, Aug. 2021, pp. 1-5, Accessed: Sep. 25, 2024 [Online] Available: https://ieeexplore.ieee.org/abstract/document/9519321

[12] S. Newcomb: Note on the Frequency of Use of the Different Digits in Natural Numbers, Vol. 4, No. 1, pp. 39-40, 1881

[13] F. Benford: The Law of Anomalous Numbers, Vol. 78, No. 4, pp. 551-572, 1938

[14] S. J. Miller: *Benford's Law*. Princeton University Press, 2015

[15] VOSviewer - Visualizing scientific landscapes, VOSviewer. Accessed: Oct. 09, 2024 [Online] Available: https://www.vosviewer.com//

[16] D. Amiram, Z. Bozanic, and E. Rouen: Financial statement errors: evidence from the distributional properties of financial statement numbers, Vol. 20, No. 4, pp. 1540-1593, Dec. 2015

[17] F. Lin, C.-M. Wu, T.-Y. Fang, and J.-C. Wun: The relations among accounting conservatism, institutional investors and earnings manipulation, Vol. 37, pp. 164-174, Feb. 2014

[18] M. J. Nigrini: An Assessment of the Change in the Incidence of Earnings Management Around the Enron-Andersen Episode, Vol. 4, No. 1, pp. 92-110, 2005

[19] F. Lin and S.-F. Wu: Comparison of cosmetic earnings management for the developed markets and emerging markets: Some empirical evidence from the United States and Taiwan, Vol. 36, pp. 466-473, Jan. 2014

[20] T. Grammatikos and N. I. Papanikolaou: Applying Benford's Law to Detect Accounting Data Manipulation in the Banking Industry, Vol. 59, No. 1, pp. 115-142, Apr. 2021

[21] L. Wei, A. Sundararajan, A. I. Sarwat, S. Biswas, and E. Ibrahim: A distributed intelligent framework for electricity theft detection using benford's law and stackelberg game, in *2017 Resilience Week (RWS)*, Sep. 2017, pp. 5-11, Accessed: Dec. 17, 2024 [Online] Available: https://ieeexplore.ieee.org/abstract/document/8088640

[22] What Is Advanced Metering Infrastructure? | IBM. Accessed: Dec. 17, 2024 [Online] Available: https://www.ibm.com/topics/advanced-metering-infrastructure

[23] D. Wang, F. Chen, J. Mao, N. Liu, and F. Rong: Are the official national data credible? Empirical evidence from statistics quality evaluation of China's coal and its downstream industries, Vol. 114, p. 106310, Oct. 2022

[24] Y. Huang, Z. Niu, and C. Yang: Testing firm-level data quality in China against Benford's Law, Vol. 192, p. 109182, Jul. 2020

[25] J. Petráš, M. Pavlík, J. Zbojovský, A. Hyseni, and J. Dudiak: Benford's Law in Electric Distribution Network, Vol. 11, No. 18, Art. No. 18, Jan. 2023

[26] A. Hyseni, D. Medved, and J. Petras: Benford's Law in Electric Power Engineering, in *2024 24th International Scientific Conference on Electric Power Engineering (EPE)*, May 2024, pp. 1-6, Accessed: Dec. 17, 2024 [Online] Available: https://ieeexplore.ieee.org/document/10559527

[27] M. Pavlík, M. Bereš, A. Hyseni, and J. Petráš: Statistical Analysis of Electricity Prices in Germany Using Benford's Law, Vol. 17, No. 18, Art. No. 18, Jan. 2024

[28] Benford's Law in Electric Distribution Network. Accessed: Dec. 17, 2024 [Online] Available: https://www.mdpi.com/2227-7390/11/18/3863

[29] P. D. B. Burger: Energy-Charts. Accessed: Sep. 27, 2024 [Online] Available: https://www.energy-charts.info/index.html