Optimization of Mamdani-type Fuzzy Risk Assessment Models

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Abstract: Nowadays, risk assessment has become the focus of many research efforts, which is the result of environmental, health and/or war related situations. One of the most important trends for the risk assessment models is the use of different patient surveillance systems, which can be used for various purposes, from performance monitoring of athletes to remote monitoring of the elderly. If the aim of the monitoring is to determine the current risk levels, based on the measured values, special care is required. Two fundamental requirements for these kinds of patient monitoring systems are personalization and timely availability of results. However, the proper personalization and the appropriate accuracy, but quickly available result are still difficult issues for researchers. In this article, the authors provide an overview of the methods they propose to address the above questions. In terms of customization, a personal profile-based evaluation is proposed that works with the patient's health characteristics. The applicability of the statistics, generated from previous measurements, in the evaluation is also presented, as well as a method for handling interactions between the input factors. In order to improve the reaction time, the authors propose some methods modifying the traditional Mamdani evaluation, and applying the Higher Order Singular Values Decomposition (HOSVD) method.

Keywords: risk assessment; fuzzy inference; reduction; personalization; optimization

1 Introduction

Risk management has been of particular importance in the last decade. Due to environmental and climatic changes, the pandemic and the war situation, it is gaining more and more attention in everyday life and in research. One of the main directions of risk management is related to surveillance systems. Due to today's technological development and the appearance of smart devices, the possibilities for using monitoring systems for different purposes are expanding [1]. At the same time, in order to ensure reliability and customization of these systems, the demand for the use of computational intelligence methods has increased. The complexity of the systems has increased to such an extent that traditional methods are no longer sufficient in all cases. This is the reason for the increasing popularity of computational intelligence methods, as they enable the implementation of much more flexible and adaptable systems that intelligently adapt to the circumstances and manage the lack of information, the inaccuracy, uncertainty, and subjectivity that appear in the data and in the evaluation process [2].

Monitoring during physical activity is of great importance to avoid unexpected emergency situations. In medical applications, the treatment of blurred boundaries is particularly important, since in the case of physiological characteristics, a sharp boundary cannot be defined to describe the normal and abnormal ranges. As a result, the need for a fuzzy approach is obvious. Another advantage of the method is that not only the shape of the functions and the operators used during the evaluation can be chosen flexibly, but even in the case of the complete system, the function parameters, or even the number/type of the inputs can also be changed flexibly, according to the patient's capabilities [3].

In this type of system, a result determined on the basis of personalized value limits and available on time is essential [4]. In the literature there are several solutions proposed to solve these issues. M. Moazeni, L. Numan et al. proposed a personalized remote patient monitoring algorithm, in which the longitudinal measurements of stable HF patients were compared to the measured values of a patient and based on this comparison a personal threshold was defined [5]. In the study of Z. Jia, Y. Shi and J. Hu a metalearning-based algorithm is presented, which is able to generate a customized neuram network for each patient separately [6]. K. Wu et al. introduced an adaptive action-aware model to personalize the generally defined thresholds, taking into account the patients' reactions and the motion form as well [7]. T. M. Tuan, L. T. H. et al proposes a rule-base reduction technique to reduce the computational complexity of the Mamdani inference using similarity measures in granular computing [8]. In the paper of N. Rathnayake, T. L. Dang, and Y. Hoshino a cascaded ANFIS model is introduced, which consists of two parts, the first is used to select the best match for the inputs and the training part generates the output [9].

In this paper, a model is studied that determines the current risk level, based on measured physiological parameters focusing on personalization and complexity reduction. For this purpose, the authors propose a risk assessment framework that, with the help of a personal profile, enables flexible determination of both the choice of the factors to be measured and their limits. In order to further improve reliability, a method is also presented that refines the personalized value limits by taking into account statistics created from previous measurement data. The management of interactions between input factors is also an important issue, for this reason a possible way of handling this issue is also presented by the authors. In the aspect of improving the response time, the authors deal with methods related to reducing the computational complexity. In order to handle this issue a modified structure of the traditional Mamdani inference and an HOSVD-based reduction method are proposed.

The remainder of this paper is organized as follows: In Section 2 the applied basic definitions and the basic monitoring system model is presented. In Section 3 methods related to model personalization are proposed in three subsections: Section 3.1 illustrates the importance and use of the personal profile-base risk assessment framework, Section 3.2 presents the application of the previous measurement statistics in the personalization, while Section 3.3 describes the issue related to the interaction handling between the input factors. Section 4 is devoted to the complexity reduction methods. In Section 4.1 a modification of the traditional Mamdani inference structure is proposed, while in Section 4.2 a HOSVD-based method is introduced. Section 5 concludes and summarizes the proposed methods.

2 Preliminaries

2.1 Related Definitions

Fuzzy membership functions:

The fuzzy membership function, which is the basis of fuzzy logic, can be described by a mapping $\mu_A(x): R \rightarrow [0,1]$ that specifies the extent to which a given element belongs to a set [10]. One of its most frequently used forms is the trapezoidal function, which can be defined as follows:

$$\mu_A(x) = \begin{cases} 0 & \text{if} \quad x \le a \\ \dfrac{x-a}{b-a} & \text{if} \quad a \le x \le b \\ 1 & \text{if} \quad b \le x \le c \\ \dfrac{d-x}{d-c} & \text{if} \quad c \le x \le d \\ 0 & \text{if} \quad d \le x \end{cases} \tag{1}$$

where *a,b,c,d* are the membership function parameters $a \ne b$ and $c \ne d$.

Ruspini-partition:

$\{A_1,...,A_n\}$ set family belonging to an input variable is considered as a fuzzy partition of *X* if these sets cover the base set together, i.e. have positive membership information for all possible input values, $\forall x \in X$, $\exists i \in [1,n] : A_i(x) \ge \varepsilon$, $\varepsilon > 0$. In the case of the Ruspini-partition there are two additional conditions as follows [11]:

Sum normalization: $\sum_{i=1}^{n} \mu_i(x) = 1$

Non-negativity: $\mu_i(x) \ge 0, i = 1,2, \dots, n, \forall x \in X$

Aggregation operators: The function $h : [0,1]^n \to [0,1]$ can be considered as an aggregation operator on *n* fuzzy sets (*n≥2*) in the case when the arguments of the function are the fuzzy sets $A_1(x),...,A_n(x)$ on the base set *X*, and this function (*h*) generates a fuzy set for every x∈X using the membership values of the arguments, i.e., $A(x) = h(A_1(x),...,A_n(x))$. A well-defined aggregation operation must also satisfy the following axiomatic conditions [12]:

H1: $h(0,...,0) = 0$ and $h(1,...,1) = 1$

H2: h is monotonically incresing in its each argument, i.e., for two arbitrary n-tuple $\langle a_1,...,a_n \rangle$ and $\langle b_1,...,b_n \rangle$, $a_i, b_j \in [0,1]$ and $a_i < b_j$, $\forall i \in [1,n]$ then $h(a_1,...,a_n) \le h(b_1,...,b_n)$.

H3: h is a continuous function

In addition to the above conditions, further restrictions can be made:

H4: h is a symmetric function in its each argument, i.e., $h(a_1,...,a_n) = h(a_{p_1,...,p_n})$, where *p* is an arbitrary permutation of the numbers 1,…,n

H5: *h* is idempotnent, i.e., $h(a,...,a) = a$, $\forall a \in [0,1]$

Ordered Weighted Average (OWA) operator: OWA operator is a specific aggregation operator.

Definition: Let the weight vector be $\underline{w}=\langle w_1,\ldots,w_n\rangle$, for every $w_i\in[0,1]$, $i\in[1,n]$ it is fulfilled that $\sum_{i=1}^{n} w_i = 1$. The OWA operator associated with this weight vector is $h_{\underline{w}}: R^n \rightarrow R$ a $h_{\underline{w}}(a_1,\ldots,a_n)= w_1b_1,\ldots, w_nb_n$, where b_i is the i-th largest element of $(a_1,\ldots,a_n)$, which means that vector $\underline{b}=\langle b_1,\ldots,b_n\rangle$ is a permutation of $\underline{a}=\langle a_1,\ldots,a_n\rangle$ in descending order i.e., $b_i\geq b_j$ if $i<j$, $i,j\in[1,n]$ [12].

Singular Value Decomposition (SVD): The method is based on the decomposition of a real-valued matrix as follows:

$$\underline{\underline{C}}_{(n_1\times n_2)} = \underline{\underline{A}}_{1,(n_1\times n_1)} \underline{\underline{B}}_{(n_1\times n_2)} \underline{\underline{A}}^T_{2,(n_2\times n_2)} \tag{2}$$

where matrices $\underline{\underline{A}}_k$ (*k=1,2*) are orthogonal, i.e., $\underline{\underline{A}}_k \underline{\underline{A}}_k^T = \underline{\underline{E}}$, and matrix $\underline{\underline{B}}$ is diagonal containing the singular values (λ_i) of the matrix $\underline{\underline{C}}$ in descending order. Singular values are intended to indicate the importance of the column $\underline{\underline{A}}_k$ to which they belong. The maximum number of the relevant singular values depending on the size of the matrix is defined as $n_{SVD}=min(n_1,n_2)$. The matrices obtained after the decomposition can be divided into two submatrices during the reduction as follows [11]:

$$\underline{\underline{A}}_k = \begin{vmatrix} \underline{\underline{A}}^r_{k,(n_k\times n_r)} & \underline{\underline{A}}^o_{k,(n_k\times(n_k-n_r))} \end{vmatrix} \tag{3}$$

$$\underline{\underline{B}} = \begin{vmatrix} \underline{\underline{B}}^r_{(n_r\times n_r)} & 0 \\ 0 & \underline{\underline{B}}^o_{((n_1-n_r)\times(n_2-n_r))} \end{vmatrix} \tag{4}$$

where *r* denotes the parts to be kept, while *o* is used to denote the parts that can be omitted. It is an important requirement that condition $n_r \leq n_{SVD}$ must always be met.

2.2 Risk Assessment Model

In this study a risk assessment model is examined that estimates the risk level based on the patient-related factors, including the real-time measured physiological parameters. For easy handling, the basic model has a hierarchical structure as illustrated in Fig. 1. This kind of structure ensures easy expansion and high adaptivity. In the system basic parameters of the patient are analyzed in the first subsystem, and in the second subsystem the characteristics of the activity are evaluated. These subsystems perform the evaluation offline, while the third subsystem is responsible for the real-time evaluation. Here inputs are the measured physiological parameters. The main goal of the system is to recognize a possible risky situation, i.e., if the measured values differ from the permissible ones. During the evaluation Mamdani-type inference was applied.

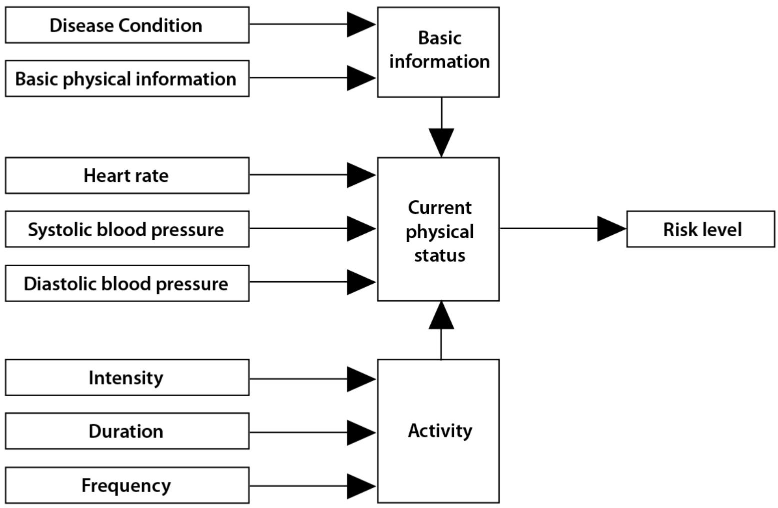

Figure 1
Basic model structure

3 Personalized Risk Assessment

In the case of patient monitoring systems, personalized evaluation is particularly important. The range of physiological values considered normal for a specific patient depends on several factors. In addition to the patient's gender and age, the patient's basic health condition and activity level should obviously be taken into account, among other things. In this section, the options for personalization are reviewed.

3.1 Risk Assessment Framework

A risk assessment framework greatly supports customization during patient monitoring. The basis of this framework is the personal profile, which contains all the important factors that should be considered related to the patient. This includes basic personal characteristics, such as the patient's age, gender, and other health-related characteristics, as well as the information necessary to identify the patient (e.g., Social Security Number). In the profile, the characteristic data of chronic diseases and the related medical recommendations regarding the normal range of physiological values are stored, as well as the factors recommended to be measured. In addition, the typically performed physical activities must also be stored, including the forms of movement, their typical duration, frequency, and intensity. The available measuring devices are also important factors, therefore their registration in the personal profile is also essential. The structure of the relational database representing the user profile is presented in detail in [13].

The applied model is an improvement of the basic model presented in Fig. 1 in Section 2.2, and it can be considered a framework in the sense that it can be flexibly customized based on the personal profile. On the one hand, the number and type of inputs can be changed according to needs, i.e., based on the medical recommendations (which factors should be measured), and/or based on the available devices. In order to determine the factors to be measured, the currently chosen form of movement must also be taken into account, since in the case of movements with different intensities, other factors contain relevant information. On the other hand, the value limits of the measured physiological characteristics must be set in accordance with the medical recommendation. It is important to keep in mind that even for patients of the same age and gender, the limits of the normal range can differ significantly, since an athlete and a patient with chronic diseases cannot be loaded to the same extent. Even in the case of the same person, it may be necessary to modify the limits of the normal range depending on the intensity of the chosen activity (e.g., walking, running, cycling, etc.). The most important relations of the database are summarized in Table 1. Of course, the personal profile must always be up-to-date so that the system can flexibly adapt to changes in the patient's condition.

Table 1
Relations in the database

Relation name	**Attributes**	**Explanation**
Users	TAJ, Name, Address, Birth, Gender, Height, Weight	basic data of the user
Sports	TAJ, sport	user’s typical activity forms
Monitored parameter	TAJ, Name, Address, Birth, Gender, Height, Weight	the parameters to be tested are user- and sport-specific
Antecedent_number	param, MFnumber	varies depending on the characteristics of the given parameter

Antecedent name	param, MFname	specifying the names of antecedent sets
Parameter limits	TAJ, MFname, limit, sport	user-specific limits of parameters according to medical recommendations
Rules	param1,…,paramn, output	related to each parameter combination
Interactions	TAJ, sport, index, value	the relative importance of the parameters

3.2 Statistics-based Evaluation

The flexible risk assessment framework described in the previous subsection provides a high degree of customization, but it can be further improved. The basic idea of the statistics-based approach is that previous measurement results, stored in the personal profile, can also be taken into account during the evaluation. These values represent the typical reactions of the patient under given conditions. It is important that measurements carried out under the same conditions (duration, intensity, sample frequency, resting heart rate) as the current ones can be taken into account. The essence of the method is to create statistics, more precisely a histogram, from previous measurements. These statistics can be used to further refine medical recommendations. If these values are typically lower than the highest value allowed in the medical recommendation, then the value limits must be modified accordingly in order to ensure a safer assessment. In the case that the measured value is still adequate compared to the original medical recommendation, but is higher than recorded in the statistics, it cannot be considered a normal value, as it differs from the patient's typical reaction.

Previous statistics are taken into account by first fitting a piecewise-linear function to the histogram using (5) as illustrated in Fig. 2 [14].

$$\mu_H(x) = \begin{cases} 1 \quad \text{if} \quad H(x_i) = \sup\left(H((x))\right) \\ \mu(x_i) + \dfrac{\mu(x_i) - \mu(x_{i+1})}{x_i - x_{i+1}}\left(x - x_i\right) \quad \text{otherwise} \end{cases} \tag{5}$$

where $i=1,\dots,n$, n is the number of histogram values; x_i, $\mu(x_i)$ and x_{i+1}, $\mu(x_{i+1})$ are the coordinates of the adjacent breakpoints.

The function fitted to the histogram must be normalized in accordance with the fuzzy membership functions so that they fall into the range [0,1] using $\mu(x_i)=y_i/max(H(x))$, where y_i represents the histogram value of the interval i. Furthermore, the interpretation range is given as a percentage of the patient's maximum value.

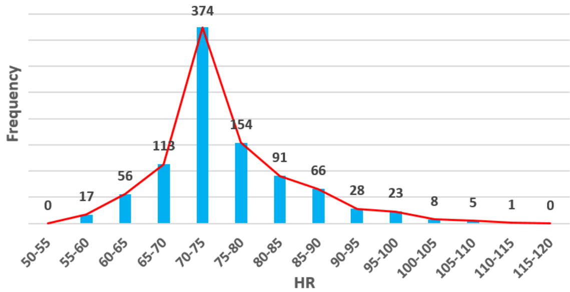

Figure 2
Histogram and the fitted function [15]

After the statistics-based membership function has been generated, it should be used to tune the membership function available based on medical recommendations using (6), (7), (8) [15].

$$\mu_{SA_i}(x) = \begin{cases} \mu\left(x_S + \left(x_M - x_S\right)/2\right) & \text{if} \quad c_1 \\ 1 & \text{if} \quad c_2 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where x_M, x_S, x, $x \epsilon X$ denote the points where the values of the medical recommendation-based, the statistics-based and the improved function are the same, the conditions c_1, c_2 can be defined as follows:

$$c_1 : \frac{a_{A_i} + x_{10}}{2} < x < \frac{b_{A_i} + x_{\max(h)_1}}{2} \quad \text{or} \quad \frac{c_{A_i} + x_{\max(h)_2}}{2} < x < \frac{d_{A_i} + x_{r0}}{2} \tag{7}$$

$$c_2 : \frac{b_{A_i} + x_{\max(h)_1}}{2} \le x \le \frac{c_{A_i} + x_{\max(h)_2}}{2} \tag{8}$$

where $a_{A_i}, b_{A_i}, c_{A_i}, d_{A_i}$ are the parameters of the membership function for the medical recommendation, $x_{10}, x_{\max(h)_0}, x_{\max(h)_1}, x_{r0}$ are the parameters of the statistics-based function.

The above method describes the modification of the membership function representing the normal range, but it is also necessary to modify the adjacent functions. Taking advantage of the fact that the original membership functions formed a Ruspini partition (see Section 2.1), this property should be kept even after the modification, so it is necessary to modify the parameters of the adjacent functions accordingly.

3.3 Interaction Handling

In the case of patient monitoring systems, not only the definition of input factors and their value limits can cause difficulties, but also the management of interactions between input factors. Regarding heart rate, which is one of the most important factors, about thirty other factors can be mentioned that influence its value, the most important of which are illustrated in Table 2.

Table 2
The effect of other parameters on the heart rate value

Parameter name	Parameter value	Effect on heart rate
Age	increasing	Decreasing
Gender	woman	Higher
Weight	overweight	Increasing
Part of the day	morning	lower (then gradually increasing)
Medicines	anti-inflammatories, stimulants, sedatives	depending on the drug effect
Smoking	yes	Increasing
Special conditions	pregnancy	Increasing
Activity	during/after	Increasing
Fitness	good	Decreasing
Environmental conditions	extreme air temperature, high humidity	Increasing

Taking into account the cumulative effect of various factors is a big challenge even for experts. A possible solution is to examine the factors in pairs, thereby simplifying the complex relationship system. during the process, the pairwise comparison is done using the well-known aggregation operators [16]. The evaluation structure is shown in Fig. 2. Traditional Mamdani inference (fuzzification, firing strength calculation, implication, aggregation, defuzzification) is supplemented with a preprocessing step, where the aggregation operator is applied to tune the input membership functions. The membership functions used in the system are trapezoidal and can be calculated using (1). The input membership functions of the system are the aggregated membership functions representing the interaction. Due to the different weight of the influence of the factors, the use of OWA operators (see Section 2.1) can be the most effective during aggregation in this type of system.

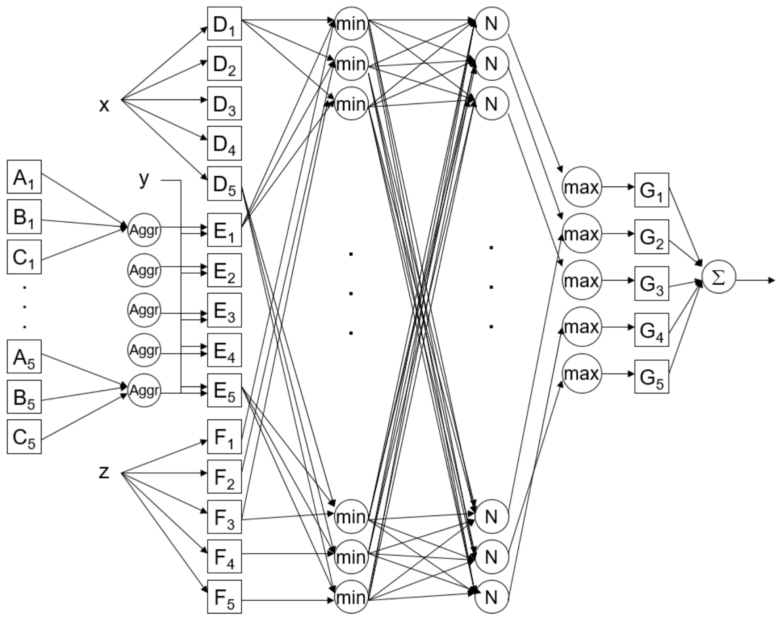

Figure 2
Evaluation structure

4 Reduction of Computational Requirements

In the case of real-time evaluation, in addition to customization, the availability time of the result is also of fundamental importance. Reducing this time is possible by reducing the computational demand of the mathematical model. In this section, the investigated ways to reduce the computational complexity are presented.

4.1 Modification of the Mamdani Inference Structure

The great advantage of the Mamdani-type inference system used in this study is that it is very close to the human way of thinking. However, the high computational demand of defuzzification can cause problems in the case of tasks requiring real-time evaluation. However, the proposed Mamdani-like structure presented in this section is able to combine the advantageous properties of the Mamdani and Sugeno models, while the obtained results are equivalent to the result of the original Mamdani structure. The essence of the discretization is that the defuzzification is done separately for each rule output, followed by the aggregation of the resulting crisp values. Consequently, compared to traditional Mamdani-type inference, the evaluation steps are reversed. It is important to note that in this case the defuzzification is reduced to a simple calculation, since the rule outputs are typically simple, piecewise linear functions. Another important aspect that the simplification can be performed equivalently only for certain operator combinations. The equivalence of the traditional Mamdani-type and the Mamdani-like structure with discretized output has been proven in the literature, if product implication, sum aggregation and Centre of Gravity (COG) defuzzification are used during the evaluation [17].

The authors proved (see details in [18]) that if the Zadeh norms are applied during implication and aggregation, and the rule-by-rule defuzzification is performed using the Mean of Maxima (MOM) method, then an equivalent evaluation is also obtained. The MOM values of the trapezoidal consequent sets can be calculated by (9), while for triangular sets it is simply equal to c_i.

$$MOM_i = \frac{b_i + c_i}{2} \tag{9}$$

where $i=1,\dots,n$, n is the number of the rules, b_i and c_i are the membership function parameters belonging to the highest function values of the consequent set belonging to rule i.

The comparison of computational requirements is illustrated in Table 3. It is taken into account that in the case of the traditional evaluation structure, the aggregation must be applied to an equidistant division of a suitable fineness. Let Y be the input domain $[y_{min}, y_{max}]$ and the set containing its equidistant base points:

$$Y_i = \left[y_1; y_1 + \Delta; y_1 + 2\Delta; \dots; y_N \right] \tag{10}$$

where N is the number of equidistant base points and $\Delta=(y_{max}-y_{min})/(N-1)$ is the distance between the points. Aggregation must be performed in the points, obtained this way.

Table 3
Comparison of the computational requirements
(n – number of the rules, N – number of the discrete points)

Inference step	**Traditional Mamdani-type structure**	**Mamdani-like structure**
Aggregation	$N(n-1)$	$n-1$
Defuzzification (+,-)	$2(N+1)$	n
Defuzification (*,/)	5	n

It is clear from the comparison that the operation requirements of the aggregation can be significantly reduced by using the modified structure. In the case of defuzzification, the improvement is not so obvious, but taking into account the fact that it is necessary to use a large number of basis points for a sufficiently fine division, it can be stated that *N* is significantly larger than *n* (number of rules). Consequently, it can be concluded that the number of multiplicative operations is slightly greater in the case of the modified structure, but a significant improvement can be achieved in the case of additive operations.

4.2 HOSVD-based Reduction

The reduction option presented above serves to speed up the general operation of the system. However, there are situations where additional reduction is necessary to adapt to a special situation. In the case of a patient monitoring system, such an unexpected situation can occur when some deviation can be detected based on the risk level calculated from the measured characteristics. In such a case, a faster assessment of the situation is necessary to assess how dangerous the situation is and whether intervention is necessary. One possible method for this type of reduction is the Higher Order Singular Value Decomposition (HOSVD) method. By using this method, redundancies inherent in the evaluation and parts that play a less relevant role in determining the result can be filtered out, thereby reducing the complexity of the calculation and the time of the evaluation.

In Section 2.1 the SVD method was presented. However, in the case of fuzzy systems, usually matrices with a higher number of dimensions are applied, which require an extension of the SVD algorithm. This extended method, the so-called HOSVD is presented below. The precondition of the reduction method described below is that the antecedent sets are in a Ruspini partition (see Section 2.1). The t-norm used in the algorithm is the product operator, the t-conorm is the sum operator, and the defuzzification method is the COG method. The matrix $\underline{\underline{C}}$ which is the input of the algorithm contains the center of gravity and area of the consequent sets.

The reduction is performed step by step (number of steps is N), reducing one dimension of the matrix $\underline{\underline{C}}$ at each step according to the algorithm below. Input of the algorithm is $\underline{\underline{C}}_i$, the size of the matrix $\underline{\underline{C}}_i$ in step *i* is $n_1^r \times \ldots \times n_{i-1}^r \times n_i \times n_{i+1} \times \ldots \times n_n$, respectively.

Algorithm 1
The HOSVD algorithm

Loop from i=1 to N

$\underline{\underline{C}}_i$ *is transformed to a 2-dimensional matrix* $\underline{\underline{S}}_i : \left(s_{i_{j,k}} \right)_{n_i \times \left(n_1^r * \ldots * n_{i-1}^r * n_{i+1} * \ldots * n_n \right)}$

Reduction: $\underline{\underline{S}}_i \approx \underline{\underline{A}}_i \underline{\underline{B}}\underline{\underline{A}}'^T_i = \underline{\underline{A}}_i \underline{\underline{S}}^*_i$, $\underline{\underline{\mathrm{A}}}_{\mathrm{i}} : \left(\mathrm{a}_{\mathrm{i}_{\mathrm{j,k}}}\right)_{\mathrm{n_i} \times \mathrm{n_i^r}}$, $\underline{\underline{\mathrm{S}}}_{\mathrm{i}} : \left(\mathrm{s}_{\mathrm{i}_{\mathrm{j,k}}}\right)_{\mathrm{n_i^r} \times \left(\mathrm{n_1^r} * \ldots * \mathrm{n_{i-1}^r} * \mathrm{n_{i+1}} * \ldots * \mathrm{n_n}\right)}$

Transformation on $\underline{\underline{\mathrm{A}}}_{\mathrm{i}}$ *to fulfill Sum normalization and Non-negativity conditions*

Generate $\underline{\underline{\mathrm{C}}}_{\mathrm{i+1}} : \left(\mathrm{c}_{\mathrm{i+1}_{\mathrm{j,k}}}\right)_{\mathrm{n_1^r} \times \ldots \times \mathrm{n_{i-1}^r} \times \mathrm{n_i^r} \times \mathrm{n_{i+1}} \times \ldots \times \mathrm{n_n}}$ *by transforming* $\underline{\underline{\mathrm{S}}}_{\mathrm{i}}$ *into n-dimensional*

Loop end

The output $\left(\underline{\underline{\mathrm{C}}}_{\mathrm{n}}\right)$ produced by the algorithm contains the consequence parts of the reduced rule base. As a result of the reduction, new membership functions must be defined, which have the following form:

$$\mu_{A'_{k,i}}\left(x_k\right) = \sum_{i=1}^{n_i^r} \sum_{j=1}^{n_i} \mu_{A_{k,j}}\left(x_k\right) a_{k,j,i} \tag{11}$$

where x_k is the input k, $\mu_{A_{k,j}}\left(x_k\right)$ is the membership function j belonging to input k, and $a_{k,j,i}$ is an element of the matrix $\underline{\underline{A}}_1$.

While the size of the reduced rule base is $n_1^r * \ldots * n_n^r$ instead of the original $n_1 * \ldots * n_n$.

The extent of the reduction depends on the boundary conditions, i.e., the accuracy requirement of the reduction and the acceptable error level. The reduction is exact if the $\underline{\underline{\mathrm{B}}}^{\mathrm{o}}$ given in (4) contains only zero singular values. In other cases, only an approximate result can be given within the defined error level, which can be calculated based on the abandoned singular values using (12).

$$\mathrm{E_{RSVD}} \leq \sum_{\mathrm{j=1}}^{\mathrm{o}} \lambda_{\mathrm{j}} \tag{12}$$

where *o* is the number of the omitted singular values.

The reduction can be performed offline, so it does not increase the computation time of the real-time evaluation. In the case of a potential emergency, simply the reduced evaluation is executed instead of the original one.

Conclusions

There is an increasing demand for patient monitoring systems, and as a result, related research, is increasing. The two main issues related to these kinds of systems are the reliability of the result and the availability, in time.

In this paper the authors proposed some solutions for the above-mentioned issues. In order for the evaluation to be sufficiently accurate, it is necessary to work with personalized parameters and value limits as much as possible. For this reason, as the authors highlighted, the use of a well-designed and up-to-date personal profile is essential. In this personal profile, in addition to the basic characteristics, it is possible to store medical recommendations, typical forms of activity, and even available devices. The greatest advantage of the evaluation framework created on the basis of this profile is that the number and type of factors to be measured, as well as the value range of the physiological parameters, can be flexibly changed according to the recommendation and the form of movement. These personalized thresholds can be further improved using the statistics created from the previously measured values. The authors also presented proposals related to the other basic requirement, the improvement of the system's reaction time. In the first solution, the traditional Mamdani inference process was modified, by swapping the order of the aggregation and defuzzification, i.e., the discretization of the rule consequents is used to reduce the computational requirements. Operational requirements of the traditional and the modified inference were also compared and the modified structure proved to be the better one. The second solution, is based on the HOSVD-based reduction, where the redundant or less relevant parts of the system are eliminated. It has been shown that in this way the number of evaluation rules can be significantly reduced. This method can be used as an alternative solution in the critical situation, when the quickly available result has of vital importance in avoiding a potential emergency. Consequently, the latter method is particularly advantageous for any time systems.

The proposed methods can be used separately, or for greater reliability, the personalized methods should be used together.

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