Integrating and Comparing Optimization Methods for Cost-Efficient Reverse Supply Chain Design

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Abstract: The reverse supply chain (RSC) plays a critical role in industrial waste management by ensuring efficient waste collection, transportation, and recycling. A key

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aspect of RSC optimization involves determining the most cost-effective location for waste collection centers. This study evaluates three facility location optimization methods – Center of Gravity (CoG), Voronoi Diagrams, and k-Means Clustering – to determine the most effective approach for waste collection center placement in Kazakhstan's mining industry. Using geospatial data from 10 tailings dams, each method was applied to minimize transportation costs and optimize collection center locations. The findings reveal that the CoG method is the least effective, as it assumes a centralized facility, leading to longer travel distances and higher costs. The Voronoi method significantly reduces transportation distances by assigning mining sites to the nearest collection center but does not account for variations in waste generation. k-Means Clustering outperforms both methods, achieving the lowest transportation distance while maintaining a balanced distribution of waste across multiple facilities. The study highlights the advantages of multi-facility clustering techniques in RSC design, demonstrating that dynamic optimization approaches can significantly enhance cost efficiency and logistical feasibility in industrial waste management. These findings provide a basis for further research and practical implementation of data-driven facility location strategies in the mining sector.

Keywords: Reverse Supply Chain; Facility Location Optimization; k-Means Clustering; Voronoi Diagrams; Center of Gravity; Mining Waste Management

1 Introduction

Sustainability and the open issues of sustainability affect all sectors of the economy. Sustainable operation and management raises a number of issues and demands from individuals and companies alike [1], affecting almost the entire spectrum of decisions to be taken. The mining industry is one of the most resource-intensive sectors, generating vast amounts of waste that pose serious environmental and economic challenges [2]. As global sustainability efforts intensify, the need for efficient reverse supply chains (RSCs) in mining has become more evident [3].

Reverse supply chains, which focus on the collection, recycling, and reuse of waste materials, play a crucial role in reducing environmental impact and improving resource efficiency [4], [5]. In the automotive sector [6], [7], they facilitate the remanufacturing of used components, reducing production costs and minimizing waste. The electronics industry [8], [9] benefits from improved e-waste management, enabling the recovery of valuable materials such as rare earth metals. Additionally, in the pharmaceutical sector [10] reverse supply chains ensure proper disposal and recycling of expired or unused medications, preventing environmental contamination and promoting regulatory compliance. Despite their proven benefits in various industries, RSCs remain largely unexplored in the context of Kazakhstan's mining sector, where waste management practices are underdeveloped, and recycling efforts are minimal [11].

Previous studies demonstrate that reverse supply chain optimisation in mining generates substantial environmental benefits. At the operational level, proactive waste reprocessing reduces acid mine drainage risks and prevents uncontrolled metal leaching [12], [13]. Recycling and remanufacturing approaches within reverse logistics contribute to lowering carbon emissions, reducing landfill volumes, and conserving non-renewable resources [14]. Integrative strategies also highlight broader ecological advantages, including enhanced land rehabilitation and the creation of secondary raw materials for other industries [15]. Together, these findings reinforce the view that reverse supply chain models extend beyond cost-efficiency to deliver measurable pollution prevention and resource recovery outcomes, thereby aligning the mining sector with global sustainability goals.

Reverse supply chain optimisation in mining extends beyond operational efficiency to governance and policy-making. Studies highlight that incorporating recycling and secondary sourcing into mineral supply requires new forms of institutional coordination and international governance mechanisms to ensure responsible resource use [16]. At the national level, integrative mineral policy strategies demonstrate that reverse logistics can foster horizontal policy integration, participatory decision-making, and adaptive learning, thereby increasing both legitimacy and effectiveness of mineral governance [17]. Consequently, RSC optimisation serves not only as a technical solution but also as a governance instrument that supports sustainable regional development and regulatory innovation.

Kazakhstan is one of the world's leading producers of raw minerals, yet its mining industry lacks an established framework for waste recovery and reutilization [18]. Most mining waste is either stockpiled or discarded, contributing to environmental degradation and inefficiencies in resource use [19]. Unlike manufacturing or consumer goods industries, where reverse logistics is well studied, the mining sector faces unique challenges such as hazardous materials handling, regulatory constraints, and high transportation costs [20]. However, no existing research systematically investigates the potential for reverse supply chain optimization in Kazakhstan’s mining sector. This study aims to fill this gap by exploring how data-driven optimization methods can enhance waste collection logistics, making RSCs both feasible and cost-effective [11].

One of the key challenges in RSC implementation is the strategic placement of waste collection centers to minimize transportation costs while ensuring accessibility to mining sites. Optimizing collection center locations is essential to improve logistical efficiency and reduce the financial burden of waste recovery operations [21], [22]. This study explores three well-established optimization techniques:

- Center of Gravity (CoG) Method – A mathematical approach that determines the optimal facility location based on demand and transportation costs [22];

- Voronoi Diagrams – A spatial partitioning technique used to allocate regions to the nearest collection center efficiently [23];
- K-Means Clustering Algorithm – A machine learning method that groups mining sites based on their proximity, optimizing collection center placement [24].

By applying these methodologies to Kazakhstan's mining industry, this study seeks to demonstrate how advanced optimization techniques can make reverse supply chains viable. The research also aims to identify best practices from successful implementations in other industries and adapt them to the unique constraints of the mining sector. Given the lack of prior studies in this area, the findings could serve as a foundation for future policy recommendations and practical implementations of RSCs in Kazakhstan.

Efficient supply chain management is critical for reducing costs, improving sustainability, and optimizing logistics networks. One key aspect of supply chain optimization is facility location selection, which significantly impacts transportation costs, resource utilization, and environmental impact [10]. Several models have been proposed to determine the number and placement of facilities to minimize operational costs [25]. These models consider both forward and reverse supply chain dynamics, incorporating multiple objectives such as cost efficiency, environmental impact, and service quality. Ene & Öztürk (2014) [26] developed a multi-stage, multi-period optimization model for an open-loop RSC, determining the optimal location of collection, recycling, and disposal centers. Their findings emphasize the importance of strategic facility placement in minimizing operational costs and improving efficiency. Facility Location Problems (FLPs) focus on determining the most strategic positions for distribution centers, warehouses, or collection hubs to enhance efficiency and minimize transportation costs [21].

In reverse supply chains, particularly in industrial waste management, selecting the optimal waste collection center location is crucial for ensuring cost-effective logistics. Traditional methods for facility placement include mathematical programming, heuristic models, and clustering techniques. Among these, the Center of Gravity (CoG) method, Voronoi diagrams, and k-Means clustering have gained popularity due to their efficiency in handling large datasets and real-world logistics constraints [27], [28].

This section reviews relevant studies that have applied these three optimization techniques in supply chain logistics, highlighting their strengths and limitations.

2 Methodological Background

2.1 The used Methodology from the Literature

2.1.1 Center of Gravity (CoG) Method in Supply Chain Optimization

The Center of Gravity (CoG) method is widely used for facility location problems, as it identifies the optimal location of logistics hubs by minimizing total transportation costs. This method has been applied in various industries, including reverse logistics and waste management [21]. Choudhary and co-authors [27] proposed a carbon-sensitive optimization model, integrating both forward and reverse logistics to reduce costs and emissions. Their study demonstrated how CoG-based facility placement can contribute to sustainability by optimizing not just financial efficiency but also carbon footprint reduction. Li [29] enhanced the CoG method by integrating clustering techniques, which improved cost efficiency by incorporating geographical pricing considerations. Similarly, Cai and co-authors [28] combined k-Means clustering with CoG, dividing demand points into clusters before applying CoG for better accuracy in facility placement.

$$X_c = \frac{\sum_{i=1}^{n} x_i * \omega_i}{\sum_{i=1}^{n} \omega_i}, \qquad Y_c = \frac{\sum_{i=1}^{n} y_i * \omega_i}{\sum_{i=1}^{n} \omega_i} \tag{1}$$

where:

x_i,y_i – Cartesian coordinates of mining site i;
n – total number of sites (tailing dams);
ω_i– waste volume at site i;

2.1.2 Voronoi Diagram-Based Approaches

Voronoi diagrams offer a spatial partitioning method for logistics optimization by dividing a region into service areas based on proximity. This method is useful in scenarios where demand points must be allocated to the nearest facility [23]. Huo and Zhang [30] improved k-Means clustering by incorporating Voronoi diagrams to optimize logistics network design. Their model resulted in more balanced cluster formations, reducing supply chain inefficiencies. Zheng and co-authors [31] applied Voronoi diagrams in metro-based freight logistics and successfully reducing transportation costs. Their findings show that Voronoi-based facility allocation significantly improves cost efficiency in urban logistics settings.

The Voronoi method divides the region into Voronoi polygons, where each mining site is assigned to the nearest facility. This ensures that each site transports waste to its closest collection center, reducing overall costs. Let S={s1,s2,...,sk} be the set of candidate facility locations. The Voronoi region Vk assigned to each facility Sk is defined as:

$$V_k = \{p \in R^2 | \, d(p, S_k) \leq d(p, S_j), \forall_j \neq k\} \tag{2}$$

where:

$d(p, S_k)$ is the Euclidean distance between any point and facility S_k;

Each site belongs to the nearest facility's Voronoi region.

2.1.3 K-Means Clustering for Supply Chain Optimization

K-Means clustering is a machine learning method widely used in logistics to group locations into clusters for improved facility placement [24]. It helps in demand point segmentation, which is essential for reverse logistics and waste collection planning. Yin and co-authors [32] proposed a k-Means clustering approach for supply chain risk management, embedding network connectivity constraints to identify at-risk clusters in large supply networks. Their work highlights how clustering techniques can enhance resilience in supply chains, particularly in high-risk industries like mining.

Unlike CoG and Voronoi method the k-clustering used for multi-facility placement scenarios. Given k clusters, the objective is to minimize the intra-cluster sum of squared distances:

$$min \sum_{j=1}^{k} \sum_{i \in C_j} d(x_i, \mu_j)^2 \tag{3}$$

where:

Cj = Cluster j containing sites assigned to collection center j.
μj = Centroid (optimal facility location) of cluster j.
d(xi,μj) = Distance between mining site i and cluster center j.

The k-Means method requires pre-defining the number of clusters (k). To determine the optimal number of collection centers, the Elbow Method [33] was applied, plotting the total transportation cost against different k-values. The optimal k was chosen at the point where adding more facilities did not significantly reduce cost. Additionally, the Silhouette Score [34], was calculated to measure clustering efficiency, ensuring that facilities were optimally placed with minimal intra-cluster transportation distances. The final selection of k=2 was based on the balance between cost reduction and infrastructure feasibility.

Beyond cost-efficiency, RSC optimisation carries broader implications for sustainable regional development. Effective facility location contributes not only to reducing operational expenditures but also to mitigating environmental risks by lowering carbon emissions and preventing uncontrolled waste accumulation.

In the context of emerging economies, such as Kazakhstan, optimised RSC networks may also serve as instruments of governance, enabling authorities to establish transparent monitoring systems and integrate waste recovery into national sustainability strategies. Previous studies have noted that infrastructural development, regulatory enforcement, and community engagement are critical elements linking reverse logistics practices with long-term socio-economic and environmental benefits.

2.2 Research Gap

Despite the extensive body of literature on supply chain optimization, several critical gaps remain in the context of reverse logistics for the mining industry, particularly in Kazakhstan. The following key distinctions and limitations provide the foundation for the present study:

2.2.1 Lack of Research on Reverse Logistics in Mining

Most existing studies primarily focus on traditional supply chains rather than reverse supply chains in mining operations [35]. The unique challenges of handling mine tailings, including material transport, environmental constraints, and long-distance logistics, are rarely addressed in optimization models.

2.2.2 Geographical and Logistical Complexity in Kazakhstan

Unlike previous studies that analyze supply chain networks in compact industrial areas, our research focuses on widely dispersed mining sites across Kazakhstan. The long distances between tailing dams and processing facilities introduce significant transportation costs and logistical challenges that are often overlooked in conventional models. Kazakhstan lacks an established practice for optimizing tailings transportation, making this study an essential contribution to developing a structured, cost-efficient reverse logistics framework.

2.2.3 Variation in Waste Generation Across Sites

Prior studies typically assume uniform waste generation rates, whereas in our case, each mining site produces a different volume of waste, ranging from small-scale to large-scale outputs. This variability affects the optimal location of collection centers, requiring a dynamic and adaptable model that can accommodate different production levels.

2.2.4 Integration of Multiple Optimization Methods

Few studies combine multiple facility location optimization techniques, such as the Center of Gravity (CoG) method, Voronoi diagrams, and k-Means clustering, to improve collection efficiency [28]. Existing research often applies these methods separately, while our study aims to evaluate their comparative performance and potential integration for enhanced accuracy and cost reduction.

2.3 Research Design

This study employs a quantitative optimization approach to determine the optimal locations for waste collection centers in Kazakhstan's mining sector. The research integrates three facility location methods – the Center of Gravity (CoG) method, Voronoi diagrams, and k-Means clustering – to minimize transportation costs and improve reverse supply chain efficiency. The study follows a comparative modeling framework, where each method is applied to the same dataset, and their performance is evaluated based on cost efficiency, geographical coverage, and clustering effectiveness.

2.4 Research Framework

The research is structured into the following stages:

- Data Collection – Gathering geographic and operational data from Kazakhstan's mining sector;
- Data Preprocessing – Preparing and normalizing location and waste volume data;
- Application of Optimization Methods – Implementing CoG, Voronoi diagrams, and k-Means clustering;
- Performance Evaluation – Comparing results based on cost minimization and logistical efficiency.

2.4.1 Data Collection and Processing

The study considers 10 tailings dams (mining sites) and a central processing facility in Kazakhstan's mining sector. Table 1 presents the distances from each tailing dam (mining site) to the central processing facility, along with the annual waste generation for each site. These parameters are key for assessing transportation costs and optimizing facility location decisions.

Table 1
Mining Site Distances, Waste Generation, and Processing Facility Data [36]

Tailing Dam(Mining site)	Mining site index, n	Distance to Processing Facility (km)	Waste generated (Mt/year)
Ridder	1	1205.5	7
Jairem	2	1149.7	5,8
Maleevka	3	1263.9	1,9
Kokshetau	4	1541.2	1,2
Ust'-Kamenogorsk	5	1081.7	1,9
Temirtau	6	1053.5	16,3
Balkhash	7	637.7	0,6
Aktogai	8	688.4	53,4
Bozshakol	9	1292.1	32,4
Bozymchak	10	1092.5	1,2

2.4.2 Data Preprocessing

The mining site locations were initially recorded using geographic coordinates (latitude and longitude). However, for accurate distance calculations in the optimization models, it is necessary to convert these coordinates into a Cartesian coordinate system.

While latitude and longitude are effective for mapping, they are based on a spherical representation of the Earth, making direct Euclidean distance calculations inaccurate due to the curvature of the Earth. The UTM system divides the globe into a series of zones, each using a locally optimized Mercator projection, ensuring that measurements within a zone are more precise. The conversion follows the standard formulas:

$$X=R\times\lambda\cos(\varphi\_0) \tag{4}$$

$$Y=R\times\ln\tan(\pi/4+\varphi/2) \tag{5}$$

where

R is the Earth's radius (≈ 6371 km),
λ is longitude,
φ is longitude,
φ_0 is the reference latitude.

This transformation ensures that distance calculations remain accurate when applying the CoG and clustering models. The conversion from latitude/longitude to UTM follows a standard mathematical transformation, which is implemented using geospatial libraries such as pyproj in Python. Each site’s location is projected into UTM Zone [XX], the appropriate zone for Kazakhstan, to obtain X, Y coordinates in meters. This transformation enables accurate distance calculations for supply chain optimization models, including Voronoi clustering and the Center of Gravity method. A summary of the transformed geospatial data is presented in Table 2, showing both the original geographic coordinates and their UTM equivalents.

Table 2
Geospatial and Operational Data for Mining Sites

№	Latitude	Longitude	Waste Generated (kg/day)	Transport Cost ($/km per 5000 kg) [37]	X (UTM meters)	Y (UTM meters)
1	50,338	83,509	19178	0,43	1,105,111.39	5,610,918.54
2	48,329	70,157	15890	0,43	141,086.42	5,364,211.77
3	49,812	84,314	5205	0,67	1,169,603.99	5,559,474.80
4	53,288	69,401	3288	0,83	1,269,239.99	5,918,941.99
5	49,971	82,597	5205	0,82	1,044,503.95	5,563,122.27
6	50,051	72,969	44658	0,53	354,603.52	5,546,277.08
7	46,843	74,973	1644	0,76	497,941.30	5,187,717.86
8	48,315	74,984	146301	0,76	498,813.76	5,351,312.25
9	50,91	71,645	88767	0,43	264,153.39	5,645,178.82
10	41,259	71,069	3288	0,53	170,660.38	4,574,965.94

To ensure consistency and compatibility with the optimization models, the waste generation data, originally provided in millions of tons per year, was converted into kilograms per day. This transformation allows for a more precise estimation of transportation costs and logistical requirements in the reverse supply chain model.

2.5 Optimization Methods for Facility Location

This section presents the mathematical formulation of the three facility location methods – Center of Gravity (CoG), Voronoi Diagrams, and k-Means Clustering – and explains how each method is applied to the Kazakhstan mining waste collection case study. The objective is to identify the optimal location(s) for waste collection centers to minimize transportation costs while ensuring efficient waste allocation from tailing dams to the processing facility.

These methods were selected due to their ability to address different aspects of facility location optimization. The CoG method provides a simple mathematical approach to minimizing transportation cost based on weighted averages, making it effective for centralized networks. The Voronoi method ensures that each mining site is assigned to the closest facility, reducing individual transportation distances while maintaining a static regional partition. k-Means clustering offers a more dynamic allocation, grouping mining sites based on both proximity and waste generation levels. This flexibility makes k-Means particularly suitable for scenarios with geographically dispersed waste sources and varying production rates, which are key characteristics of the study region.

2.6 Problem Definition and Assumptions

Objective Function

The overall goal is to minimize the total transportation cost, defined as:

$$min\, Z = \sum_{i=1}^{n} c_i \times d_i \times w_i \,/ \tag{6}$$

where:

n = number of mining sites (10 tailing dams),
d_i = road distance from mining site i to the collection center (km),
w_i= waste generated at site i (kg),
c_i= transportation cost per km per 5000 kg of waste from site i.

Assumptions for Optimization Models

- Facility location coordinates are optimized based on minimizing transportation cost.
- No external disruptions (e.g., road conditions, weather) are included in calculations.
- Single-facility vs. multi-facility analysis: CoG and Voronoi assume a single optimal, while k-Means allows multiple facilities.

For the CoG method, distances are computed as Euclidean distances, while for k-Means and Voronoi, real-world road distances are considered. This distinction is critical, as CoG assumes straight-line travel, which may not reflect actual transportation conditions.

While the models developed in this study incorporate geographical dispersion and waste generation variability, road network constraints and infrastructure quality were not included in the optimisation process. In Kazakhstan, however, mining sites are often located in remote regions where limited road access and seasonal disruptions significantly affect transportation efficiency. Incorporating these constraints in future models would provide a more realistic approximation of logistical feasibility and could strengthen the applicability of the findings for decision makers.

3 Results

This section presents the results obtained from applying the Center of Gravity (CoG), Voronoi Diagrams, and k-Means Clustering methods to determine the optimal waste collection center locations for Kazakhstan's mining waste management. The results are analyzed in terms of cost efficiency, facility placement accuracy, and transportation cost reduction. The Figure 1 shows the single collection center determined by the CoG method, which places the facility at the weighted average location of all mining sites based on waste generation. Where blue circles are mining sites (tailing dams) and red star is optimal collection center (weighted centroid).

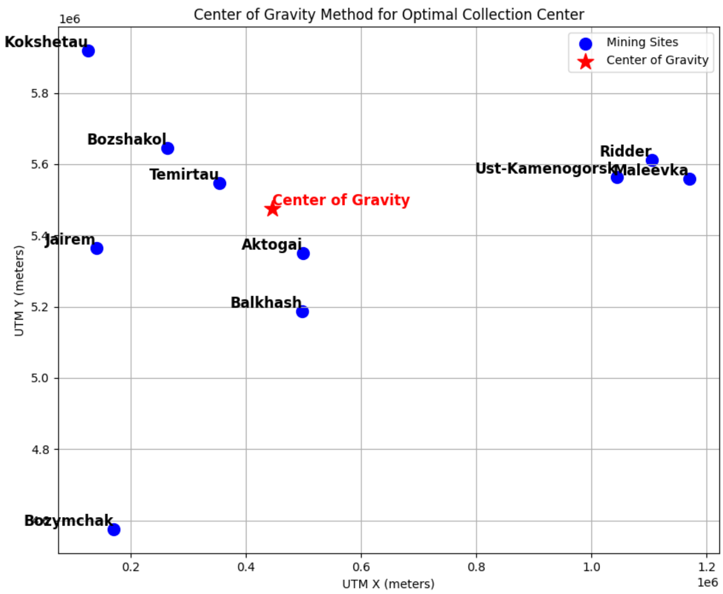

Figure 1

Center of Gravity (CoG) Facility Placement and Assigned Mining Sites

The method minimizes the total transportation cost assuming straight-line distances (Euclidean distances). All mining sites are assigned to this single facility, requiring all transportation routes to converge at this point. The method works well for supply chain systems where demand points are centralized and distances between sites are not extreme. While the CoG method provides a quick estimation of an "optimal" central location, its inability to consider real-world logistics, road networks, and facility capacity constraints makes it less suitable for large, geographically dispersed mining operations like those in Kazakhstan. In contrast, Figure 2 presents the Voronoi-based facility allocation, demonstrating a clear reduction in transportation distances through strategic partitioning.

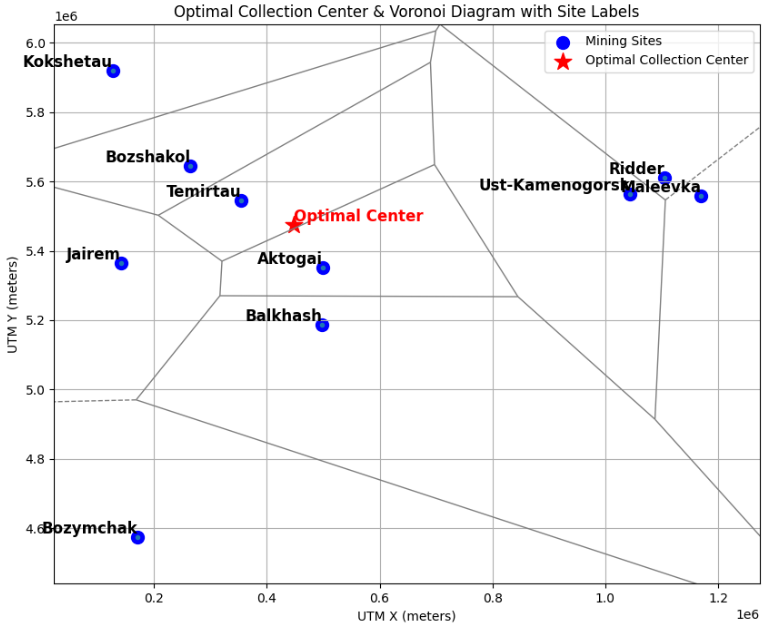

Figure 2

Voronoi-Based Waste Collection Regions and Optimal Facility Placement

A single collection center is placed, and each mining site is assigned to its nearest collection facility based on straight-line distance. Where gray lines present boundaries of supply areas for each site. The Voronoi diagram partitions the region into distinct service areas, ensuring each mining site is linked to the closest collection center. Compared to CoG the Voronoi method present significant reduction in transportation distances. The Voronoi method provides a more practical and cost-effective alternative to CoG by reducing distances and improving efficiency. However, its rigid assignment of service regions can lead to imbalanced facility loads, making it less adaptable to real-world fluctuations in waste generation.

Finally, Figure 5 displays the k-Means clustering results, where two optimized collection centers were identified, ensuring a more balanced distribution of waste among facilities.

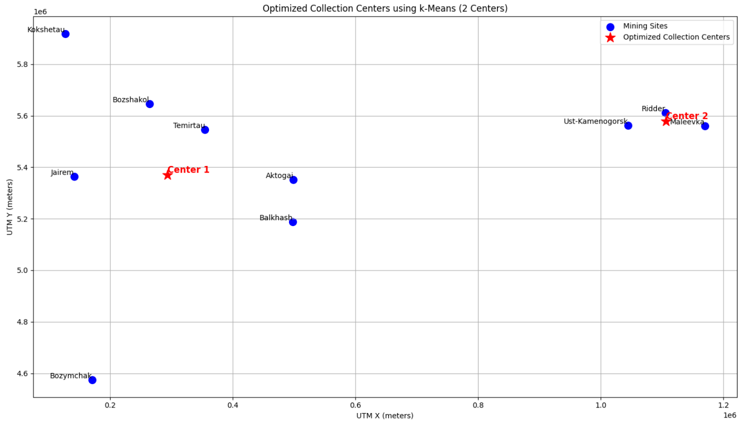

Figure 3

k-Means Clustering for Optimized Multi-Facility Waste Collection

The algorithm adjusts facility placement dynamically based on both geographic proximity and waste generation.

The k-Means clustering method achieved the lowest total transportation distance among all optimization approaches, reducing the total travel requirement to 2,630 km. Unlike single-facility methods, k-Means effectively balances waste distribution across multiple collection centers, ensuring that no facility is overloaded while maintaining efficient logistics. By considering geographic spread, the method optimally minimizes intra-cluster distances, making it particularly suitable for dispersed mining networks where waste sources are widely distributed. Additionally, k-Means offers flexible clustering, meaning that if waste production fluctuates over time, the cluster assignments dynamically adjust to maintain efficiency – an advantage over the Voronoi method, which operates with fixed service regions and cannot adapt to changes in waste generation patterns. The k-Means clustering approach is the most effective optimization method in this study. It successfully reduces transportation distances and balances waste loads while maintaining flexibility for future adjustments. However, its computational complexity and dependence on choosing the right k-value must be considered.

4 Discussion

The results of this study highlight the varying effectiveness of different facility location optimization methods for mining waste collection in Kazakhstan. The k-Means clustering method emerged as the most effective approach, achieving the lowest total transportation distance (2,630 km) while ensuring a balanced waste distribution across multiple collection centers. Unlike single-facility models, such as the Center of Gravity (CoG) method, which assumes a centralized location and results in longer travel distances, or the Voronoi method, which divides the region into static service areas, k-Means offers a flexible and adaptive clustering approach. By dynamically adjusting facility assignments based on both geographic spread and waste generation rates, k-Means minimizes intra-cluster transportation distances while maintaining operational efficiency. These findings suggest that a multi-facility clustering strategy is preferable over rigid, single-facility methods for optimizing reverse supply chains in the mining sector.

Table 3

Comparison of Optimization Methods for Waste Transportation

Method	Total cost, $	Total distances, km	Number of Facilities	Reduction Compared to Direct Transportation (%)
Direct transportation to processing facility	$36904.88	11000	Processing facility	Baseline (0%)
Center of Gravity	$8715.89	4606	1	76.4% (cost), 58.14% (distance)
Voronoi method	$5215.40	2820	1	85.87% (cost), 74.36% (distance)
k-mean clustering	$8608.52	2630	2	76.68% (cost), 76.09% (distance)

The findings of this study demonstrate the varying effectiveness of different facility location optimization methods for waste collection in Kazakhstan's mining sector. The Center of Gravity (CoG) method, despite its simplicity and ease of implementation, proved to be the least efficient approach for this case. As a single-facility placement method, CoG does not adequately account for the geographic dispersion of mining sites, leading to increased transportation distances and higher overall costs. This method is well suited for supply chain networks with a relatively centralized demand structure; however, in cases where facilities are widely distributed, such as in mining operations, its applicability is limited. The results indicate that the reliance on a single collection center results in suboptimal logistical efficiency, suggesting that alternative approaches should be considered for cost-effective and sustainable waste management.

The Voronoi diagram method provided improved performance by dividing the study region into distinct service areas, ensuring that each mining site was allocated to the nearest collection center. This method effectively reduced overall transportation distances compared to CoG by distributing facilities strategically across the region. However, one notable limitation of the Voronoi approach is its static spatial partitioning, which does not account for fluctuations in waste generation across different mining sites. As a result, some facilities may experience disproportionate waste loads, leading to inefficiencies in collection and processing. While Voronoi-based facility placement offers significant improvements in cost efficiency and logistical feasibility, its rigid assignment of service areas may not be suitable for dynamic supply chain environments where demand varies over time.

A key limitation of this study is the exclusion of climatic and infrastructural constraints. In Kazakhstan, severe winters, seasonal flooding, and the uneven quality of regional road networks may alter the efficiency of facility location strategies derived from geometric optimisation methods. Future research should incorporate these variables to assess the robustness of the proposed models under realistic operating conditions.

Among the three optimization methods, k-Means clustering yielded the most effective results in terms of both cost reduction and transportation efficiency. This approach allowed for a flexible and data-driven allocation of collection centers, ensuring that facilities were positioned in a manner that minimized intra-cluster transportation distances. Unlike Voronoi, which relies on predefined boundaries, k-Means clustering dynamically adjusts cluster assignments, allowing for a more balanced distribution of waste loads among collection centers. This characteristic makes k-Means particularly suitable for industrial applications where logistical requirements may fluctuate. However, the method's reliance on iterative computations increases its computational complexity, making its implementation more resource-intensive compared to CoG or Voronoi-based approaches. Despite this, the significant improvements in transportation cost efficiency justify its use in large-scale facility location problems.

In Kazakhstan, harsh continental climate conditions and limited year-round road accessibility can significantly influence the feasibility of facility location decisions. For example, seasonal road closures and extreme winter temperatures may reduce the applicability of routes that are optimal in purely geometric terms. While these factors were not explicitly incorporated into the present model, acknowledging them highlights an important area for further research. Integrating climate and road conditions could provide a more realistic foundation for reverse supply chain optimisation in mining regions.

From a policy perspective, the adoption of clustering-based optimisation methods has the potential to support national strategies for sustainable industrial development. By reducing transportation distances and costs, optimised RSC networks can incentivise investment into secondary processing facilities and promote regional infrastructure upgrades. Moreover, the integration of such optimisation approaches into waste management policy would align Kazakhstan’s mining sector with international sustainability commitments, bridging the gap between economic performance and environmental governance.

The comparative analysis of these optimization methods suggests that a multi-facility approach is superior to a single-facility strategy for mining waste collection. The CoG method, while simple, fails to accommodate the geographic complexity of mining operations, leading to higher transportation costs and inefficiencies. The Voronoi approach improves upon CoG by establishing multiple service regions, yet its limitations in demand balancing may restrict its applicability in dynamic waste management systems. The k-Means method, by contrast, provides the most optimal facility placement due to its ability to cluster mining sites based on both proximity and waste generation patterns. As a result, this study recommends the adoption of multi-facility clustering techniques in designing cost-effective and operationally efficient reverse supply chain networks for mining waste management.

Conclusion

This study provides a comparative analysis of three facility location optimization methods – CoG, Voronoi, and k-Means Clustering – applied to mining waste collection in Kazakhstan. The results demonstrate that single-facility models, such as CoG, are inadequate for geographically dispersed mining operations, as they lead to longer transportation distances and higher costs. The Voronoi method improves efficiency by assigning mining sites to the nearest collection center, but its static partitioning fails to account for fluctuations in waste generation. In contrast, k-Means Clustering emerges as the most effective approach, dynamically adjusting facility placements based on proximity and waste distribution, resulting in a 76.7% cost reduction compared to direct transportation and the shortest total travel distance (2,630 km).

Despite these advantages, k-Means requires computational resources and careful selection of the number of clusters (k) to achieve optimal results. The results demonstrate that data-driven optimisation approaches can substantially improve both economic and environmental outcomes of industrial waste management. In particular, multi-facility clustering strategies not only reduce costs but also provide a framework for policy makers to align mining logistics with sustainability objectives. Future research should explore the integration of road network constraints and real-time waste generation variations into facility location models. Additionally, investigating hybrid approaches that combine clustering with network-based optimization techniques could further improve efficiency.

These findings provide a strong foundation for developing cost-effective and sustainable waste collection strategies in Kazakhstan's mining industry. They also offer insights applicable to other industrial sectors requiring reverse supply chain optimisation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

The data that has been used is confidential.

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