Orthorectifying Archive Aerial Photos with Rational Polynomial Camera Model

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Abstract: In this paper we present a workflow, how a set of metadata can be calculated for a raw aerial image, that enables end-users to simply orthorectify the aerial image, using end-user's own digital elevation model. The set of metadata mentioned here are the coefficients of the Rational Polynomial Camera (RPC) model. The Rational Polynomial Camera model is an extension of the rigorous collinearity equations (used for frame camera orthocorrection) with higher power terms of the object space coordinates. We used the fiducial points of the image to estimate the interior orientation parameters and we used Ground Control Points (GCPs) to estimate exterior orientation parameters of the photo. As the GCPs were measured in map projection system, and the RPC standard requires ellipsoidal coordinates as object space coordinates, we also present the methodology, how we calculated the local transformation coefficients between these two systems. All these efforts resulted in the calculation of the RPC coefficients with closed form equations. We generated the orthophoto using the RPC model, and the accuracy of the orthophoto equals to the accuracy obtained using collinearity equations. Besides the collinearity model representing an ideal camera, we also calculated RPC coefficients additionally considering the radial lens distortion. From this set of RPC coefficients we also generated the orthophoto. Comparing these two results, we found, that considering radial distortions did not significantly improve the accuracy of the generated orthophoto. To make our results repeatable we also present the calculated RPC coefficients in the standard form, enabling the reader to generate the orthophoto.

Keywords: Rational Polynomial Camera; archive aerial photography; orthophoto; camera model; open source software

1 Introduction

In Hungary, a significant amount of archive aerial photographs of the various aerial survey campaigns of the past decades were stored in Lechner Knowledge Center (formerly Institute of Geodesy, Cartography and Remote Sensing) and these photos were declassified after 1989. Fortunately, a great number of these have been digitized, and published online, providing great help for researchers [1]. Images can be downloaded from the site for free after registration.

The archived images are not georeferenced, only the approximate image center coordinates (calculated from the flight plan) are aligned to the images.

Rudimentary georeferencing of images can be done by users, simply by selecting three GCPs on the photo and on the adjacent map. After confirmation by the operators of the website, a world file (`*.twf`) representing an affine transformation is attached to the image.

The above mentioned method is not suitable for precise orthorectification of the images. This means that distortion due to topography cannot be eliminated and consequently, the images can be fit to a ground coordinate system with quite low accuracy.

2 Materials

2.1 The Aerial Photo

The aerial survey from which the image was chosen to be processed, was carried out on 11th of April, 1976, and covered the administrative area of Gyöngyös town (Northern Hungary). It was covered by east-west rows and the altitude of flight was about 850 m above the terrain. Each image covers about $1km^2$. The black and white panchromatic photos were taken with a calibrated aerial camera (Wild RC-8) for which the exterior and interior orientation parameters can be determined accurately.

Images were recorded to a 230×230 mm film and the contact prints were digitised with a pixel size of 14 μm (i.e. 1814 dpi resolution). It must be noted that it is not identical to the resolution of the original image. The digitized image was oversampled as the mean resolving power of the analogue image is only 44 line pairs per millimeter.

The approximate scale of the image can be computed from the relative flight altitude (h) and the focal length of the camera (c_k) as:

$$S = \frac{c_k}{h} \approx 1 : 5395 \tag{1}$$

The scenes in the archive are available in `*.tif` format. A `*.tfw` world file is attached to each image, which describes the position of the georeferenced image in the EOV, the Uniform National Projection system (of Hungary) [2], [3]. It must be emphasised that it represents only the approximate position of the image and it can be used only for preliminary value for the further steps [1].

The image presented in this paper has a unique identifier `1976_0029_9095` in the archive, and it covers the urban area of the southern part of Gyöngyös town. The size of the scanned image is 17698 × 16880 pixels, and the uncompressed file requires 59.53 MB memory size.

2.2 The Digital Elevation Model

As only one image was used for the orthophoto, a digital surface model was needed for the process [4]. We used the DDM-10 dataset, which was generated from Warsaw Pact Gauss-Krueger military topographic maps of scale 1:50 000, and the elevation values were obtained by digitising the contour lines and interpolating between them [5]. As a result, it represents the topographic terrain surface and can therefore be considered a terrain model.

The elevation data are arranged in a regular grid of 10×10 m. The projection of the model is EOV, elevation data is above the Baltic sea level, using the EOMA vertical datum, and elevation values are rounded to the nearest metre. There are several margins of error for describing the accuracy of the model depending on the type of the relief, but for our purpose, the values for hilly terrain were relevant [6]. The mean error is less than $2.5\,m$, the maximum error for 90% of the points is less than $5.0\,m$, and the maximum error is less than $7.5\,m$.

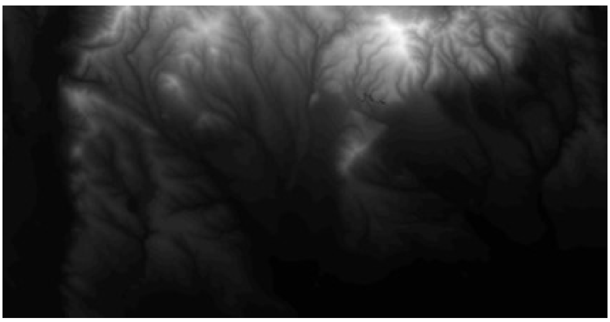

Figure 1
Detail of the DDM-10 elevation model shown as a grayscale image.

2.3 Ground Control Points

On the area covered by the aerial photograph, 12 Ground Control Points (GCPs) were selected taking into account the criterion of identifiability on the photo. The ground coordinates were measured with RTK GNSS receiver, and their pixel coordinates on the digital image were specified in `QGIS` software.

As the used GNSS receiver determined coordinates in ETRS89 system, the horizontal coordinates had to be transformed into the EOV system, and heights above ellipsoid had to be transformed into EOMA heights. For these transformations, horizontal and vertical grid shift type correction were used [7].

Figure 2
The orthophoto generated from the raw image. The 12 GCPs used for the computations are marked with white circles.

3 Methods

3.1 Interior Orientation

Collinearity equations describe the transformation between object space and image space coordinates. As we use a digitized image, we can measure only pixel coordinates on them, but we need image space coordinates for the further computations.

Table 1
Coordinates of GCPs

X (Eastings)	Y (Northings)	Z (height)	u	v
715800.552	269485.589	147.297	464.9	-10659.0
715158.090	269844.880	145.997	4611.9	-2056.9
715696.346	269859.321	150.967	5148.9	-9119.7
715832.619	269722.758	149.977	3490.7	-10954.6
715871.711	270342.277	155.387	11584.1	-11158.1
715274.340	270409.330	151.777	12165.3	-3161.3
715302.550	270693.090	155.417	16063.5	-3318.3
716245.202	270290.139	162.067	11112.6	-16095.8
715444.307	269891.193	148.737	5399.2	-5802.0
715846.318	269927.167	151.687	6122.9	-11032.2
715910.530	270335.100	155.727	11512.0	-11675.9
715578.256	270648.775	156.667	15579.7	-7081.7

Table 2
Image- and pixel coordinates of the fiducial marks

	ξ	η	u	v
1	-106.000	-106.000	910	-838
2	-105.995	106.002	16040	-904
3	105.994	105.996	15974	-16034
4	106.002	-105.998	845	-15969

Fiducial marks of the photogrammetric camera are used to determine the position of the image space coordinate system on the image. Image space coordinates of the fiducial marks were recorded in the calibration report and their pixel coordinates can be measured directly from the digitized image (see Table 2).

The relationship between pixel coordinates and image space coordinates can be described by a 2D affine transformation, as we must consider that film position in the scanner varies at every single image [8]. Furthermore, the relation between the two system is also affected by the deformation of the film and the errors of the scanner.

As the image space coordinates of the 4 fiducial marks were known, we could use them to determine the 6 parameters of the affine transformation, using least-squares adjustment.

$$\begin{aligned} \xi &= a_0 + a_1 u + a_2 v \\ \eta &= b_0 + b_1 u + b_2 v \end{aligned} \tag{2}$$

where u and v are digital image column and line coordinates, ξ and η are camera (image space) coordinates and $a_0 \ldots a_2, b_0 \ldots b_2$ are the coefficients of the affine transformation, stored in Table 3.

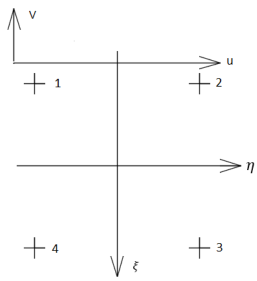

Figure 3
The image space coordinate system (ξ, η) and the pixel coordinate system (u, v) of the image. The image was scanned rotated 90 degrees clockwise, to be oriented in N-S direction, however camera calibration report uses refers to the image space coordinate system. Fiducial marks are numbered.

Table 3
Affine transformation parameters used for interior orientation of the image

a_0	-117.6877331557
a_1	-0.0000604159
a_2	-0.0140104304
b_0	-118.8057265671
b_1	0.0140116577
b_2	-0.0000606816

3.2 Determining Exterior Orientation Parameters Using Collinearity Equations

In this section we shortly present the methodology used for the calculation of the camera (projection centre) position (X_0, Y_0, Z_0) and attitude angle $(\omega, \varphi, \kappa)$ determination by space resection (see figure 4). A least squares adjustment was used for obtaining these exterior orientation parameters using GCPs.

In the firs step, preliminary parameters of exterior orientation ($X_0^{(0)}$, $Y_0^{(0)}$, $Z_0^{(0)}$, $\omega^{(0)}$, $\varphi^{(0)}$, $\kappa^{(0)}$) were estimated for the image. Preliminary horizontal coordinates ($X_0^{(0)}, Y_0^{(0)}$) were the approximate image center coordinates, while $Z_0^{(0)}$ was taken from the flight plan. We used the approximate image orientation to estimate the $\kappa_0^{(0)}$ (yaw) angle and as the image was near vertical, we choose $\omega_0^{(0)}$ and $\varphi_0^{(0)}$ as zero.

Using these preliminary angles, we calculated the preliminary **R** rotation matrix that represent the rotation of the image space coordinate system with respect to the object space. **R** can be obtained as:

$$\mathbf{R} = \mathbf{R}_\omega \cdot \mathbf{R}_\varphi \cdot \mathbf{R}_\kappa \tag{3}$$

where we used the preliminary values of the exterior orientation angles.

Knowing the rotation matrix and having preliminary coordinates for camera position, the collinearity equations were used to calculate the camera coordinates of the GCPs'.

$$\begin{aligned}\xi_i^{calc} &= -c_k \frac{r_{11}(X_i - X_0^{(0)}) + r_{21}(Y_i - Y_0^{(0)}) + r_{31}(Z_i - Z_0^{(0)})}{r_{13}(X_i - X_0^{(0)}) + r_{23}(Y_i - Y_0^{(0)}) + r_{33}(Z_i - Z_0)^{(0)}} + \xi_0 \\ \eta_i^{calc} &= -c_k \frac{r_{12}(X_i - X_0^{(0)}) + r_{22}(Y_i - Y_0^{(0)}) + r_{32}(Z_i - Z_0)^{(0)}}{r_{13}(X_i - X_0^{(0)}) + r_{23}(Y_i - Y_0^{(0)}) + r_{33}(Z_i - Z_0^{(0)})} + \eta_0\end{aligned} \quad (4)$$

where $\xi_i^{calc}, \eta_i^{calc}$ are the calculated image space coordinates of the *i*-th GCP's, X_i, Y_i and Z_i are the object space coordinates of the *i*-th GCP, r_{ij}-s are the elements of the **R** rotation matrix, ξ_0, η_0 are the image coordinates of the principal point, taken from the camera calibration report $\xi_0 = 0.000\ mm$ and $\eta_0 = -0.003\ mm$. The focal length (c_k) is $152.340\ mm$. $X_0^{(0)}, Y_0^{(0)}$ and $Z_0^{(0)}$ are the preliminary object space coordinates of the focal point of the camera.

The condition to be fulfilled is the weighted sum of the squares of the differences of the measured and calculated image space coordinates of the GCPs' should be minimal:

$$\mathbf{v}^T \mathbf{P}\ \mathbf{v} := min \quad (5)$$

where **v** is the residual vector composed from differences of the measured and calculated ξ and η coordinates of the GCPs' and **P** is the weight matrix. In our case we assumed equal weights, so we applied an identity matrix as weight matrix ($\mathbf{P} = \mathbf{I}$).

To fulfil this condition we applied the nonlinear least squares method.

For this we had to calculate the differences of the measured and the calculated image space coordinates for the GCPs, and arrange them into one single vector **l**.

We also needed the Jacobians of the equation 4 with respect to the six exterior orientation parameters: the three coordinates of the focal point (X_0, Y_0, Z_0) and the three angles (ω, φ, κ) that describe the attitude of the camera [8].

We can form **A** matrix as:

$$\mathbf{A} = \begin{pmatrix} \vdots & & & & & \\ \frac{\partial \xi_i}{\partial X_0} & \frac{\partial \xi_i}{\partial Y_0} & \frac{\partial \xi_i}{\partial Z_0} & \frac{\partial \xi_i}{\partial \omega} & \frac{\partial \xi_i}{\partial \varphi} & \frac{\partial \xi_i}{\partial \kappa} \\ \frac{\partial \eta_i}{\partial X_0} & \frac{\partial \eta_i}{\partial Y_0} & \frac{\partial \eta_i}{\partial Z_0} & \frac{\partial \eta_i}{\partial \omega} & \frac{\partial \eta_i}{\partial \varphi} & \frac{\partial \eta_i}{\partial \kappa} \\ \vdots & & & & & \end{pmatrix} \quad (6)$$

We obtained the **A** matrix by numeric derivation of equation 4.

With **A** matrix and **l** vector we could calculate the corrections for the preliminary parameters:

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{l} \quad (7)$$

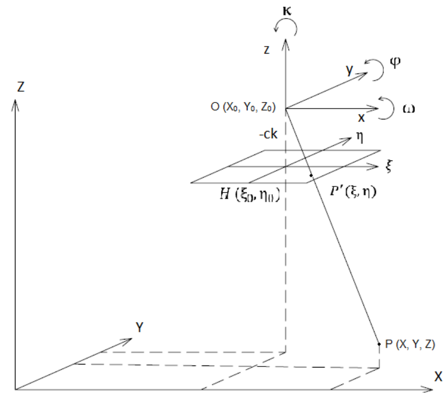

Figure 4
Explanation of the collinearity equations. The $\vec{OP}$ vector is rotated and reduced to the $\vec{OP'}$ vector, representing the ξ, η coordinates, so exterior orientation parameters provide a relationship between and object space (ground) coordinates and image space (camera) coordinates. Exterior orientation parameters were obtained using adjustment using GCPs [8]
.

Table 4
Exterior orientation parameters

X_0	715636.701
Y_0	270130.443
Z_0	977.371
ω	1.08778
φ	1.34381
κ	2.80681

where **x** vector consists of the correction of the parameters:

$$\mathbf{x} = \begin{pmatrix} \Delta X_0 & \Delta Y_0 & \Delta Z_0 & \Delta\omega & \Delta\varphi & \Delta\kappa \end{pmatrix}^T \tag{8}$$

Adding these corrections to the preliminary parameters, and repeating the calculation starting with equation 3, the differences of the measured and calculated image space coordinates of the GCPs decrease. After some iteration step, the corrections of the parameter are almost zeros. We accepted the corrected parameters as estimates of the exterior orientation of the image. Actual values of the exterior orientation are in Table 4, where angles are in degrees.

3.3 The RPC Camera Model

The RPC model was developed for Very High Resolution (VHR) satellite imagery at the end of 1990s. Image providers supplied raw images with a set of RPC coef-

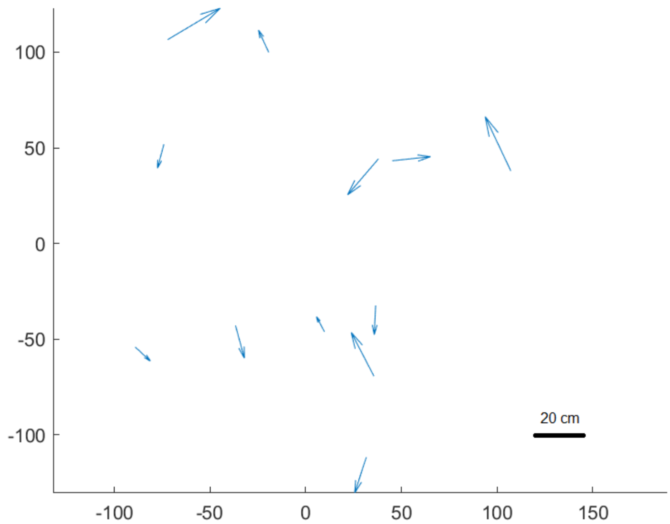

Figure 5
Residual errors between measured and calculated image space coordinates of the GCPs. Error values in image space coordinates are multiplied by the approximate image scale (see equation 1) to get error values in object space coordinates. Coordinate axes are in mm. (ξ, η)

ficients, hiding the actual imaging geometry. With this set of parameters end-users could orthorectify the image, with simple open-source software using end-user's own digital elevation model.

The Rational Polynomial Camera model is an extension of the rigorous collinearity equations (used for frame camera orthocorrection) with terms of higher power of object space coordinates.

The camera model uses rational polynomial functions, hence it also referenced as RPFs, (Rational Polynomial Functions). It is a more generic camera model, that allow for photogrammetric processing without requiring a physical camera model.

This model describes relation between ground ellipsoidal coordinates (latitude, longitude and height) and the image coordinates (line and sample) as ratios of cubic polynomials.

The transformation is represented by the coefficients of the ground (object space) coordinates (they are the RPCs – Rational Polynomial Coefficients). In order to avoid rounding errors, each input data types are required to be re-scaled [9]. The normalized ground- and pixel coordinates can be obtained as:

$$\begin{aligned} l &= \frac{v - v_0}{v_{SCALE}} \\ s &= \frac{u - u_0}{u_{SCALE}} \\ P &= \frac{\phi - \phi_0}{\phi_{SCALE}} \\ L &= \frac{\lambda - \lambda_0}{\lambda_{SCALE}} \\ H &= \frac{Z - Z_0}{Z_{SCALE}} \end{aligned} \tag{9}$$

where l (line) and s (sample) are the normalized pixel coordinates; P, L and H are the normalized ellipsoid latitude, longitude and height. Offset of the certain parameters are marked with $_0$ subscript, scale factors are marked with $_{SCALE}$ index.

Based on RPCs, normalized pixel coordinates can be obtained as:

$$l = \frac{N_l(L,P,H)}{D_l(L,P,H)} \qquad s = \frac{N_s(L,P,H)}{D_s(L,P,H)} \tag{10}$$

where numerators (N) and denominators (D) are both cubic polynomials of normalized ground coordinates. The order of the terms is the following [10]:

$$\begin{array}{cccc} c_1 & c_6LH & c_{11}PLH & c_{16}P^3 \\ c_2L & c_7PH & c_{12}L^3 & c_{17}PH^2 \\ c_3P & c_8L^2 & c_{13}LP^2 & c_{18}L^2H \\ c_4H & c_9P^2 & c_{14}LH^2 & c_{19}P^2H \\ c_5LP & c_{10}H^2 & c_{15}L^2P & c_{20}H^3 \end{array} \tag{11}$$

Here, $c_1, ..., c_{20}$ are the coefficients related to the respective polynomials, so (nominators and denominators for line and sample) are described by $4 \cdot 20 = 80$ coefficients. With the offsets and scale factors (of latitude, longitude, height, line and sample), the RPC model consists of 90 coefficients.

3.4 Coordinate Transformation Between EOV and WGS84 System

As RPC model can only handle ellipsoidal coordinates, we need to transform EOV coordinates of GCPs to ETRS89 system (it is the European realization of the WGS84 system, and it is fixed to the European continental plate [11]).

Because of the relatively small area covered by the photograph, the use of accurate projection equations is not necessary. Instead, we determined the local transformation parameters between the two system and substitute EOV coordinates to equation

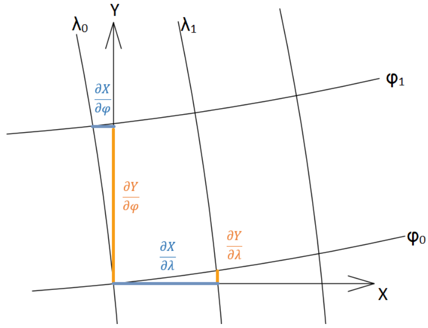

Figure 6
Relationship between the object space reference system and the ellipsoidal coordinates used in RPC model. The local transformation parameters obtained from numeric derivation. For this, coordinates were transformed by a `cs2cs` command utilizing correction grid to ensure high accuracy.

4 as functions of ETRS89 coordinates.

The local transformation parameters can be obtained using numeric derivation. It means that we determine the change of EOV coordinates of a point, while changing the ETRS89 coordinates of this point with a small $+\Delta\phi$ and $+\Delta\lambda$ value. We can obtain more precious result when we change these coordinates with $+\Delta\phi$, $-\Delta\phi$, $+\Delta\lambda$, $-\Delta\lambda$ values too, and compute their mean, respectively (two-sided numerical derivates). The meaning of these derivatives can be seen on figure 6.

$$\begin{aligned} \frac{\partial X}{\partial \phi} &= \frac{X(\phi+\Delta\phi,\lambda)-X(\phi-\Delta\phi,\lambda)}{2\cdot\Delta\phi} & \frac{\partial X}{\partial \lambda} &= \frac{X(\phi,\lambda+\Delta\lambda)-X(\phi,\lambda+\Delta\lambda)}{2\cdot\Delta\lambda} \\ \frac{\partial Y}{\partial \phi} &= \frac{Y(\phi+\Delta\phi,\lambda)-Y(\phi-\Delta\phi,\lambda)}{2\cdot\Delta\phi} & \frac{\partial Y}{\partial \lambda} &= \frac{Y(\phi,\lambda+\Delta\lambda)-Y(\phi,\lambda+\Delta\lambda)}{2\cdot\Delta\lambda} \end{aligned} \tag{12}$$

We used the horizontal object space coordinates of the focal point of the camera for this computation, because it is located in the middle of the assessed area that can minimize errors of the method.

For these computations we needed to accurately transform coordinates between the two system. Takács and Siki [7] have developed a precise solution for open-source `proj` library using a 2×2 km resolution correction grid. This solution can be used in various open-source softwares (`cs2cs, ogr2ogr, QGIS`, etc.). We used it in `cs2cs` function in this form:

From EOV to ETRS89:

```
”C:\Program Files\QGIS 3.28.4\bin\cs2cs” -f ”%%.10f” +proj=
    somerc +lat_0=47.14439372222222 +lon_0=19.04857177777778
    +k_0=0.99993 +x_0=650000 +y_0=200000 +ellps=GRS67 +
    nadgrids=etrs2eov_notowgs.gsb +geoidgrids=geoid_eht2014.
    gtx +to +init=epsg:4326 +no_defs <eov.txt >etrs89.txt
```

From ETRS89 to EOV:

```
"C:\Program Files\QGIS 3.28.4\bin\cs2cs" -f "%%.10f" +init=
    epsg:4326 +to +proj=somerc +lat_0=47.14439372222222 +
    lon_0=19.04857177777778 +k_0=0.99993 +x_0=650000 +y_0
    =200000 +ellps=GRS67 +nadgrids=etrs2eov_notowgs.gsb +
    geoidgrids=geoid_eht2014.gtx +units=m +no_defs <etrs89.
    txt >eov.txt
```

The horizontal (`etrs2eov_notowgs.gsb`) and vertical (`geoid_eht2014.gtx`) grid shift files can be downloaded from [7].

3.5 Computing RPC Parameters from Rigorous Sensor Model

It is possible to compute RPC parameters only with the use of GCPs (its advantage that it does not require to know the physical camera model) [12]. In this case, equations 10 should be applied for the coordinates of the GCPs and RPC parameters can be determined with least-squares adjustment. The disadvantage of this solution that it totally disregards the camera intrinsics.

In our case, as we know the interior and exterior orientation parameters with high precision, it is more practical to determine RPC parameters as functions of camera intrinsics and extrinsics [13]. Pixel coordinates should be computed only from the 1st order terms, that appears in collinearity equations, coefficients and terms containing the 2nd or 3rd power of ellipsoidal latitude, longitude and height should be omitted.

In this case, the RPC equations (equation 10) for l (line) and s (sample) have only first order terms:

$$l = \frac{n_{l_2} L + n_{l_3} P + n_{l_4} H}{d_{l_2} L + d_{l_3} P + d_{l_4} H}$$

$$s = \frac{n_{s_2} L + n_{s_3} P + n_{s_4} H}{d_{s_2} L + d_{s_3} P + d_{s_4} H} \qquad (13)$$

In case of collinearity equations, the denominators are equivalent.

Substituting equation 4 (collinearity equation) and 2 (defining the affine transformation), into equation 10, we get:

$$
\begin{aligned}
l = b_0 + b_1\Big(\xi_0 - c_k\frac{r_{11}(X-X_0)+r_{12}(Y-Y_0)+r_{13}(Z-Z_0)}{r_{31}(X-X_0)+r_{32}(Y-Y_0)+r_{33}(Z-Z_0)}\Big)+ \\
+b_2\Big(\eta_0 - c_k\frac{r_{21}(X-X_0)+r_{22}(Y-Y_0)+r_{23}(Z-Z_0)}{r_{31}(X-X_0)+r_{32}(Y-Y_0)+r_{33}(Z-Z_0)}\Big) \\
s = a_0 + a_1\Big(\xi_0 - c_k\frac{r_{11}(X-X_0)+r_{12}(Y-Y_0)+r_{13}(Z-Z_0)}{r_{31}(X-X_0)+r_{32}(Y-Y_0)+r_{33}(Z-Z_0)}\Big)+ \\
+a_2\Big(\eta_0 - c_k\frac{r_{21}(X-X_0)+r_{22}(Y-Y_0)+r_{23}(Z-Z_0)}{r_{31}(X-X_0)+r_{32}(Y-Y_0)+r_{33}(Z-Z_0)}\Big)
\end{aligned}
\tag{14}
$$

We might further substitute into equation 14 the expression of object space coordinates as functions of ellipsoidal coordinates (equation 12):

$$
\begin{aligned}
X - X_0 = \frac{\partial X}{\partial \phi}(\phi-\phi_0) + \frac{\partial X}{\partial \lambda}(\lambda-\lambda_0) \\
Y - Y_0 = \frac{\partial Y}{\partial \phi}(\phi-\phi_0) + \frac{\partial Y}{\partial \lambda}(\lambda-\lambda_0)
\end{aligned}
\tag{15}
$$

Substituting these back into equation 14, and highlighting the polynomials to the coefficients of $(\phi-\phi_0)$, $(\lambda-\lambda_0)$ and $(Z-Z_0)$ (i.e. P, L and H), we obtain the RPC coefficients (since they are also coefficients of P, L and H). Due to RPCs computed from collinearity equations, ground coordinates are interpreted relative to the focal point (ϕ_0, λ_0) of the camera and pixel coordinates relative to the principal point (ξ_0, η_0) of the image. Therefore, offset values of the ground coordinates must be the latitude, longitude and height of the focal point, and offset of the pixel values are the pixel coordinates of the principal point.

Sign of the line offset changes to its opposite because in `QGIS` software (where GCPs were specified) line pixel values interpreted as negative values, but `gdalwarp` uses the other convention. As we did not rescale values, all of the scale factors were set to 1. The RPC parameters obtained with this method are in Table 5.

3.6 Orthorectification Using `gdalwarp` Program

In this section the procedure of orthophoto generation is discussed.

The user has to specify the input image, the elevation model, the properties of the output file and the RPC parameters used for the orthocorrection.

The `gdalwarp` command can orthorectify raster data if its metadata is stored in headar of the `*.tif` file. Unfortunately `*.tif` header manipulation is not properly works in `gdal_translate` environment, so an alternative method was used. For the `*.tif` image, a virtual raster header file (`*.vrt`) was defined, and RPC metadata section was added to this file. The `*.vrt` contains metadata of the image in XML format, including statistical indicators, georeferencing information, interpretation of bands of the image, grouped into so-called ”metadata domains”. We could also add new types of metadata (in this case the RPC parameters, called `Metadata domain="RPC"` (see figure 7). Names, meanings, and the values of the parameters are in Table 5 [14].

For further processing, it is important that the image file and its `.vrt` file must have the same name.

The used DDM-10 elevation model was clipped and reprojected to WGS84 (practically ETRS89) system using `GDAL`-command:

```
gdalwarp -s_srs EPSG:23700 -t_srs EPSG:4326 DDM-10.tif dem.
    tif
```

(because `gdalwarp` command can only handle elevation models with ellipsoidal coordinates.) Alternatively, one can use SRTM global elevation model dateset [15].

The orthophoto was generated using the following GDAL command:

```
gdalwarp -rpc -to "RPC_DEM=dem.tif" -t_srs EPSG:23700 input.
    vrt output.tif
```

The `-rpc` option means that the transformation is to be performed on the basis of RPC coefficients. The `RPC_DEM` switch defines the elevation model to be used and after the `-t_srs` switch the coordinate system (map projection) of the orthophoto can be specified. Finally the input `.vrt` file of the raw image, and the name and extension of the result file should be defined. In this case, the generated orthophoto was a `.tif` file in EOV projection (EPSG:23700).

3.7 Determining RPC coefficients considering radial distortion of the image

To take into account the physical camera model, that slightly differs from the mathematical model representet by the collinearity equations, we implemented a new mathematical model that considers the effects of radial lens distortions.

The camera calibration report contains information about the image radial distortion: there are distortion curves in principal directions and mean distortion curve, and distortion values are also given in numerical form.

In the workflow, exterior orientation parameters determined as described in chapter 3.2, but image space coordinates are corrected with the radial distortion values. Based on these parameters, a grid of virtual GCPs was created and their image coordinates were calculated. These image coordinates were modified according to the radial distortion. As a final step a second order RPC model coefficients were calculated from these virtual GCPs, using a least-squares adjustment.

For the mathematical approach of the radial distortion, we determined the parame-

Table 5
Parameters of RPC model acquired from rigorous sensor model

ERR_BIAS	0
ERR_RAND	0
LINE_OFF	8436.41431219385
SAMP_OFF	8442.52647010182
LAT_OFF	47.7716366637
LON_OFF	19.9231569233
HEIGHT_OFF	977.371008176101
LINE_SCALE	1
SAMP_SCALE	1
LAT_SCALE	1
LON_SCALE	1
HEIGHT_SCALE	1
LINE_NUM_COEFF	0 -813498905.019398 67515173.6469024 -254.103780291664
LINE_DEN_COEFF	0 -1668.69145923504 2263.81159488546 0.999544803422536
SAMP_NUM_COEFF	0 -45861299.107409 -1206708817.92528 207.447253599898
SAMP_DEN_COEFF	0 -1668.69145923504 2263.81159488546 0.999544803422536

ters of the polynomial that can be used for approximate the distortion curve. This polynomial contains a constant, first, third and fifth power of distance (d) measured from the principal point of symmetry.

$$\Delta = A_0 + A_1 d + A_2 d^3 + A_3 d^5 \tag{16}$$

where Δ is the radial distortion at a certain point of the image; A_0, A_1, A_2, A_3 are the parameters of the approximation curve.

The image space coordinates of the GCPs were determined using the rigorous sensor model determined in equation 4. In this case, the effects of radial distortion were added too:

$$\xi = -c_k \frac{r_{11}(X - X_0) + r_{12}(Y - Y_0) + r_{13}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} + \xi_0 + \Delta\xi$$

$$\eta = -c_k \frac{r_{21}(X - X_0) + r_{22}(Y - Y_0) + r_{23}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} + \eta_0 + \Delta\eta \tag{17}$$

$\Delta\xi$ and $\Delta\eta$ can be obtained from Δ determined at equation 16 and α, the direction angle of the distortion vector:

$$\alpha = \arctan\left(\frac{P_\eta - S_\eta}{P_\xi - S_\xi}\right) \tag{18}$$

where P_ξ, P_η and S_ξ, S_η are the image coordinates of the certain point of the image plane and the principal point of symmetry. The ξ and η component of the vector $(\Delta\xi, \Delta\eta)$ can be obtained as:

$$\begin{aligned} \Delta\xi &= d \cdot \cos\alpha \\ \Delta\eta &= d \cdot \sin\alpha \end{aligned} \tag{19}$$

For virtual GCPs, we determined the area covered by the aerial photo, and a 20×20 m point grid was created to this area. In vertical direction, the distance of two horizontal grid was 100 m. The image coordinates of these virtual GCPs computed using equation 17.

Then pixel coordinates of these points were determined using the inverse transformation of interior orientation described in equation 2:

$$\begin{aligned} \xi &= c_0 + c_1 u + c_2 v \\ \eta &= d_0 + d_1 u + d_2 v \end{aligned} \tag{20}$$

Projected object space (EOV) coordinates can be transformed into ellipsoidal coordinates using the method discussed in 3.4 .

As a final step, these coordinates can be substituted into equations 10 and RPC parameters can be computed using least-squares adjustment. As the lens distortion was taken into consideration, we need even the terms of higher degrees.

In this case, it was needed to rescale the ground coordinates as discussed in equation 9. Changing the sign of `LINE_OFF` parameter is also needed, similarly to Section 3.5. The RPC coefficients of line and sample coordinates are adjusted separately, but in the same way. The Jacobian (**A**) matrix of the adjustment is:

$$\mathbf{A} = \begin{pmatrix} 0 & L_1 & P_1 & H_1 & L_1P_1 & L_1H_1 & ... & H_1^3 \\ \vdots & & & & & & & \\ 0 & L_n & P_n & H_n & L_nP_n & L_nH_n & ... & H_n^3 \end{pmatrix} \tag{21}$$

while the (column) vectors of unknowns are:

$$\begin{aligned} \mathbf{x}_{line} &= \begin{pmatrix} c_1 & c_2 & c_3 & ... & c_{39} \end{pmatrix}^T \\ \mathbf{x}_{sample} &= \begin{pmatrix} c_{40} & c_{40} & c_{41} & ... & c_{78} \end{pmatrix}^T \end{aligned} \tag{22}$$

Adjustment has only 78 parameter instead of 80 (the total number of RPC coefficients) because the first coefficient of the denominators were set to 1 to avoid the instability of the adjustment.

Using all of these coefficients can cause artifacts on the orthorectified image, therefore it is advised to use only coefficients up to the first and second power of L, P and H, and third power coefficients should be zeros.

4 Results

Table 6 shows the residual errors of the adjustment calculated with collinearity equations and the residual errors calculated with the RPC model. To express the errors in object space coordinates, the obtained error of the adjustment in image space coordinates had to be multiplied by the approximate scale of the image (described at equation 1).

Table 6 also shows the Root Mean Square (RMS) error of GCPs' derived from the orthophoto. The object space coordinates of GCPs on the orthophoto were measured in `QGIS`, and were compared with the RTK GPS measurements.

Table 6 proves that the accuracy of the transformation based on RPCs does not differ significantly from that computed from collinearity equations.

Our results can be replicated by (1) downloading the raw image as shown in Chapter 2.1, (2) copying content of Table 5 into a Virtual raster header file attached to the raw image, and (3) applying the command presented in Chapter 3.6. Following this procedure, the reader can (4) reproduce the orthophoto shown in figure 2.

Figure 2 shows the orthocorrected image produced using the presented method. Comparing with the results of the orthocorrection without radial lens distortion corrections, the differences are mostly less than the size of one pixel, that makes it difficult to precisely determine the error vectors.

Conclusions

The mathematical methods we used allow us to determine the relationship between pixel and ground coordinates in the images with sufficient accuracy. As Table 6

Table 6
Accuracy of the collinearity equations, RPC camera model and the orthophoto

Root Mean Square (RMS) error of GCPs'	
measured vs. calculated image coordinates using collinearity equations and multiplied by image scale	0.113 m
measured vs. calculated image coordinates using RPC coefficients and multiplied by image scale	0.173 m
measured by RTK GPS vs. measured on the orthophoto	0.960 m

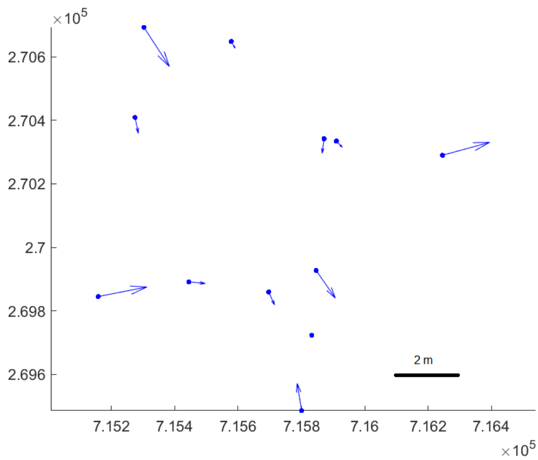

Figure 7
Error vectors of the orthophoto. Vectors represent the difference in coordinates of GCPs measured with RTK GNSS on the field and measured on the orthophoto. The mean error of rectification is 0.96 m that make it suitable for utilizing in GIS applications. Map axes are object space coordinates (EOV projection).

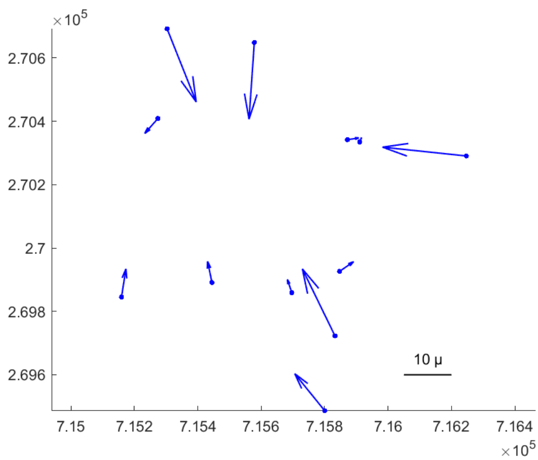

Figure 8
Differences between the orthophotos obtained without and with radial lens distorsion. The direction of the difference vectors are in good accordance with the real direction of radial distortion

shows, the accuracy of the transformation based on RPCs does not differ significantly from that computed from collinearity equations.

The precision of the orthophoto is much lower, but it is also in the sub-meter domain. The main reason is the low resolution and high errors of the applied digital eleva-

Table 7
The `1976_0029_9095.vrt` file. The file contains the `Metadata_domain="RPC"` tag, and the set of coefficients for the orthocorrection of the image.


```
<VRTDataset rasterXSize="17698" rasterYSize="16880">
  <Metadata>
    <MDI key="TIFFTAG_DOCUMENTNAME">{1C6E48BE-1F08-4CD8-BFD2-6EA3C4C8BE4D}</MDI>
    <MDI key="TIFFTAG_MAXSAMPLEVALUE">255</MDI>
    <MDI key="TIFFTAG_MINSAMPLEVALUE">0</MDI>
    <MDI key="TIFFTAG_RESOLUTIONUNIT">3 (pixels/cm)</MDI>
    <MDI key="TIFFTAG_XRESOLUTION">713.91351</MDI>
    <MDI key="TIFFTAG_YRESOLUTION">713.96979</MDI>
  </Metadata>
<Metadata domain="RPC">
<MDI key="ERR_BIAS">0.000000</MDI>
<MDI key="ERR_RAND">0.000000</MDI>
<MDI key="LINE_OFF">8515.0266247209765424</MDI>
<MDI key="SAMP_OFF">8369.5809091688770422</MDI>
<MDI key="LAT_OFF">47.7714271548151714</MDI>
<MDI key="LONG_OFF">19.9235105776815509</MDI>
<MDI key="HEIGHT_OFF">155.3782095766828490</MDI>
<MDI key="LINE_SCALE">7565.2938644475707406</MDI>
<MDI key="SAMP_SCALE">7558.1298023673216449</MDI>
<MDI key="LAT_SCALE">0.0054564478500012</MDI>
<MDI key="LONG_SCALE">0.0080940103499998</MDI>
<MDI key="HEIGHT_SCALE">200.0000000000000000</MDI>
<MDI key="LINE_NUM_COEFF">0.0045283096301385  1.0579858145327732 -0.0599550689146902
0.0113874192019469 -0.2189647341673704  0.1632511150451613 -0.0112486099496269
0.0673072441759231  0.0120116016535110  0.0016429795902539 0 0 0 0 0 0 0 0 0 0</MDI>
<MDI key="LINE_DEN_COEFF">1.0000000000000000  0.0800381719663870 -0.2183302928775431
-0.0893428120224111 -0.0042771585552811 -0.0129383791664858  0.0471198767165540
0.0008490499043254  0.0029198666931829 -0.0373466367198778  0 0 0 0 0 0 0 0 0 0</MDI>
<MDI key="SAMP_NUM_COEFF">-0.0009784500033390  0.0597926099817746  1.0588908126748307
-0.0091367321685560  0.0595410916984312  0.0072842863675571  0.1412244087940583
0.0040512507558638 -0.2019676081091925 -0.0011862196128534  0 0 0 0 0 0 0 0 0 0</MDI>
<MDI key="SAMP_DEN_COEFF">1.0000000000000000  0.0834315148282613 -0.2057272425610065
-0.1112738769064664 -0.0041479735297961 -0.0141239758845238  0.0443738986533749
0.0009993027245646  0.0026669535991317 -0.0320168844078930  0 0 0 0 0 0 0 0 0 0</MDI>
</Metadata>
  <Metadata domain="IMAGE_STRUCTURE">
    <MDI key="COMPRESSION">JPEG</MDI>
    <MDI key="INTERLEAVE">BAND</MDI>
  </Metadata>
  <VRTRasterBand dataType="Byte" band="1" blockYSize="16">
    <ColorInterp>Gray</ColorInterp>
    <SimpleSource>
      <SourceFilename relativeToVRT="1">1976_0029_9095.tif</SourceFilename>
      <SourceBand>1</SourceBand>
      <SourceProperties RasterXSize="17698" RasterYSize="16880" DataType="Byte"
      BlockXSize="17698" BlockYSize="16" />
      <SrcRect xOff="0" yOff="0" xSize="17698" ySize="16880" />
      <DstRect xOff="0" yOff="0" xSize="17698" ySize="16880" />
    </SimpleSource>
  </VRTRasterBand>
</VRTDataset>
```

tion model, discussed in chapter 2.2. The spatial distribution of errors (the control points near the edge of the image mostly have larger displacement) also proves this assumption. Nevertheless, the error of the method is not negligible; it fulfills the requirements of most GIS applications.

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