Digital Simulation of Hydraulic Modes of Heat Supply Systems of Variable Technological Structure

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Abstract: The paper presents a new approach to the formation of a system of equations – the nodal pressure model, which is based on the inverse characteristics of passive and active elements of pipeline networks developed during the study, the closing system of equations of stationary isothermal modes of the heat supply system, interpreted in terms of typical regime situations, fixing their state according to the content of the initial data. The solution of which made it possible to create an effective algorithm and computer implementation of simulation modeling of modes of large-size heat supply systems of variable technological structure.

Keywords: heat supply system; digital modeling; hydraulic mode; heat network; modeling method; models

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1 Introduction

One of the main ways to intensify production and increase its efficiency is digitalization based on modern computing capabilities. Digitalization is a set of special measures aimed at providing reliable, comprehensive and timely knowledge in all significant types of casual human activity.

The structure of the life support of modern megacities and its performance is represented by power, heat, gas, water, sewerage, transport and communications, protection of the air and water basins, which are financially expensive and science-intensive man-made systems.

Analysis of existing approaches to the formation of technological structures of large-scale district heating systems of megacities (HSM) and functional and design characteristics of technological equipment shows that centralized heat supply developed in accordance with long-term forecasts of urban infrastructure development and taking into account the peculiarities of the territorial distribution of fuel and energy resources throughout the country. Key trends in the development of district heating and district heating were:

- unit capacity enlargement of heat sources led to an increase in heat transfer radii up from 10 to 30 km and necessitated the construction of many pumping substations;
- use of joint operation of several heat sources for common heat networks potentially contributed to an increase in the reliability and efficiency of heat supply, but, at the same time, led to a significant complication of the development and implementation of thermal and hydraulic modes;
- continuous increase of the share centralized hot water supply inevitably led to significant changes in the thermal and hydraulic modes of heating systems during the day;
- the wide use of open heat supply systems with very difficult to implement thermal and hydraulic modes;
- in the use of simple, but of little use for high-quality automated control of technological schemes for connecting subscriber installations;
- the hydraulic stability decreases of heat networks with minimal capacity reserves.

Thus, for a long period of time, the main technological elements of heating and district heating systems were built and operated mainly without taking into account the requirements of using informatization and automation tools to control their operation modes and the entire system as a whole.

The performed research is devoted to solving urgent problems of digital modeling of technological structures and control processes of operation modes of complex heating networks of district heating systems by developing:

- models of the movement of the coolant in the paths of pipeline sections, active and passive elements of regulation and control of the modes of the thermal network, based on their inverse characteristics;
- classifiers of typical operating situations of closed, open and mixed district heating systems, fixing their condition according to the content of the initial data;
- models of steady-state hydraulic modes of district heating systems of variable technological structure, interpreted in terms of regime situations;
- digital models for calculating flow distribution based on the use of a new approach to the formation of systems of nodal pressure equations using not direct, but inverse characteristics of pipeline network elements.

2 Methods for Modeling Hydraulic Modes of Heat Supply Systems

Currently, the operation and development of complex heating networks of district heating systems becomes impossible:

- without solving the problems of improving the energy efficiency of district heating systems;
- without a systematic approach to fundamental research of physical and technological problems of energy;
- without solving the problems of automated control of the processes of transmission and distribution of thermal energy to the objects of the heat supply system.

The fundamental works that influenced the formation and development of these scientific and technical areas of research and development:

- H. Averfalk [1], A. Billerbeck [2], H. Lund [3], N. Javanshir [4], J. Vivian [5], Z. Ma [6], M. Balboa-Fernández [7], A. Steingrube, [8], M.A. Ancona [9], G.H. Bergsteinsson [10], A. Sandvall [11], V. Stennikov [12], J. Röder [13], M. Abugabbara [14], L. Sánchez-García [15], M. Wissner [16];
- K. Sartor [17], Sam van der Zwan [18], P. Leoni [19], B. Grassi [20], M. Capone [21], S.S. Meibodi [22], Y. Chen [23];
- T. Oppelt [24], G. Barone [25], A. Dénarié [26], A. Menapace [27], Y. Wang [28], Zheng Xuejing [29].

Mathematical and simulation modeling using computer technology of complex heat networks hydraulic modes at district heating systems in operational and emergency conditions requires the use of general methods for calculating the flow distribution in hydraulic circuits [30].

The hydraulic circuit of a thermal network can be represented by an abstract mathematical model of a hydraulic system in which the junctions and divisions of flows are replaced by nodal points, and sections of the pipeline network, including fittings and other sources of local resistance, by branches.

When modeling flow distribution processes in arbitrary hydraulic circuits with lumped and controlled parameters, the algebraic approach has become predominant, according to which the steady flow of liquids and gases along a hydraulic path distributed in space can be described by closed systems of Kirchhoff equations of finite complexity [31].

However, along with the algebraic approach, the theoretical and computational aspects of the extremal approach, based on the physical and mathematical essence of the flow distribution problem, methods for minimizing a special function corresponding to a particular variational principle, non-linear programming problems, and gradient or step-by-step methods of unconditional minimization [32].

Extremal methods of describing flow distribution have an undoubted theoretical significance, however, their practical implementation through the necessary extremum conditions leads to systems of Kirchhoff equations and, therefore, does not give anything new from the point of view of organizing computational processes compared to the algebraic approach.

That is why in the practice of modeling the steady flow of liquids and gases through pipeline networks of any complexity, mathematical models of the algebraic approach have become widespread and developed: contour and nodal mathematical models that go back to their counterparts from electrical circuits and networks [33].

However, in the problems of machine analysis of complex electrical circuits, the method of nodal potentials has been predominantly developed due to its greater computational efficiency. It is this circumstance that prompted us to turn to the development and implementation of another approach to the formation of a system of equations – nodal pressures, which is based on the inverse characteristics of passive and active elements of pipeline networks that close the systems of equations of the stationary isothermal mode of heat supply systems. Hence, the study carried out within the framework of this study, on the effectiveness of numerical methods for solving large-scale flow distribution problems on systems of contour and nodal equations of hydraulic circuits, has become one of the important tasks, the solution of which made it possible to create effective algorithms and programs for simulating the modes of large-scale heat supply systems of variable technological structure.

3 Digital Modeling of the Coolant Movement in the Hydraulic Path of the Heat Network at the Heat Supply System

The processes of preparation, transport and use of the coolant in the technological elements in the HSM are implemented through organized hydrodynamic and thermal processes, the nature of which depends on the geometric dimensions, shape, surface condition, thermodynamic and transfer properties of the materials of the walls of the hydraulic tracts, water as the predominant coolant, and as well as the temperature of the environment.

In view of this, in practice it is permissible to use simpler models, with the help of which, it is possible to identify the main patterns of motion processes with a sufficient degree of accuracy. When creating such models, we will use the following fair assumptions [34-36]:

- water, as the predominant coolant of the HSM, is considered a single-phase incompressible viscous liquid of the Newtonian type;
- movement of the coolant in the pipes of technological elements is turbulent with a logarithmic velocity profile in the flow core;

- changes in the hydrodynamic and thermodynamic parameters of the coolant can be taken into account along one spatial coordinate coinciding with the axis of the hydraulic path. At the same time, the speed, temperature, pressure and density of water are assumed to be constant in a fixed section of the tract and can only change along the axis of the tract;
- in connection with the turbulence of flows, it is legitimate to use mass-average velocity, temperature, and enthalpy in motion models;
- geometrical dimensions of hydraulic circuits of technological elements are fixed;
- thermal processes in technological elements of HSM are characterized by small temperature changes, and therefore it is possible to use the model of isothermal movement of water with a constant density.

The above assumptions allow us to obtain the following basic patterns of coolant movement in the sections of pipelines of the thermal network, in active and passive elements of regulation and control of HSM modes, which, in turn, allow us to proceed to the next stage of modeling, the stage of developing typical operating situations and digital models of hydraulic modes of the thermal network of the district heating system of variable technological structure.

3.1 Models of Coolant Movement in the Tracts of Pipeline Sections

The main regularities of the coolant movement along the sections of the HSM hydraulic path will be obtained on the basis of the law of conservation of mass and mechanical energy of the flow. When the coolant moves under the conditions of the above assumptions, mass conservation is always performed and takes the form:

$$q_i = f_{io} \cdot v_{io} = f_{i1} \cdot v_{i1}, \tag{1}$$

where q_i– volumetric flow rate of the coolant in the path of the i-th technological element, m^3/s;

$f_{i0}, v_{i0}, f_{i1}, v_{i1}$ – respectively, the cross-sectional area, m^2 and the coolant velocity in the initial and final nodes of the path, m/s.

The conservation of mechanical energy, written for the specific energy of the flow of the i-th technological element, takes the form of the Bernoulli equation [37]:

$$\frac{v_{i0}^2}{2g} + \frac{P_{i0}}{\gamma} + Z_{i0} - \frac{v_{i1}^2}{2g} - \frac{P_{i1}}{\gamma} - Z_{i1} - \Delta h_i = 0, \tag{2}$$

where Δh_i – lost head, i.e. specific energy value, spent on overcoming friction forces and local resistance, m;

P_{i0}, P_{i1} – piezometric pressure in the input and output sections of the technological element, Pa;

Z_{i0}, Z_{i1} – geodetic marks of the area of the input and output sections, m;

γ – specific gravity of the coolant, kg/m^3.

For passive technological elements of sections of the hydraulic tract of the HSM, losses of mechanical energy are characteristic with insignificant losses of internal energy, therefore the Bernoulli equation takes the following form, provided that small changes in dynamic pressure are neglected:

$$\begin{cases} \Delta h_i = h_{i0} - h_{i1} \\ h_{i0} = \frac{P_{i0}}{\gamma} + Z_{i0} \\ h_{i1} = \frac{P_{i1}}{\gamma} + Z_{i1} \end{cases}, \tag{3}$$

where h_{i0}, h_{i1} – total pressure in the inlet and outlet sections of the passive technological element, which determines the specific energy reserve in these sections.

The lost pressure Δh_ias the difference between the total specific energies in the initial and final sections of passive technological elements is always a positive value, i.e., $\Delta h_i > 0$.

The value of the pressure lost during the movement of the coolant in the pipes depends on such factors as the inner diameter of the pipe and its length, the physical properties of the coolant, the average speed of movement in the pipe, the average height of the roughness protrusions on the pipe walls.

For hydraulic pipe paths with different types of local resistances, the lost pressure is expressed by an empirical formula, the form of which is determined by the Fanning, Darcy, Weisbach equations [37]:

$$\phi(q_i) = sign(q_i) r_i \cdot |q_i|^{\alpha_i}, \tag{4}$$

where r_i is the total hydraulic resistance of the technological section for fixed length element, m/(m^3/h);

α_i – empirical constant for the rate of flow.

The constant α_i for mainline and distribution heating networks takes the value $\alpha_i = 2$, since the experience of hydraulic testing of heating networks has established the independence of the coefficient of hydraulic friction from the flow rate. Since due to the high velocity of the coolant, all the roughnesses protrude from the viscous sublayer of the turbulent flow.

The flow $sign$ in formula (4) determines the direction of the coolant flow in the tract section of the i-th technological element. The flow rate q_i will be considered positive if its direction coincides with the initial flow orientation adopted in the description of the HSM structure, and negative otherwise. That is, the $signq_i$ is used to describe the direction of the coolant flow in the section of the pipeline network and the formation of the head loss sign corresponding to this direction.

The given model of specific hydraulic losses, represented by formula (4), meets all the formal requirements below, ensuring the adequacy of this model to the most general physical properties of a steady flow of fluid in pipeline networks:

- from the condition of positivity of energy losses due to friction, the function must have the property of oddness, i.e.,

  $$\phi(-q_i) = -\phi(q_i) \text{ at } q_i < 0,$$

  $$\phi(q_i) > 0, \text{ at } q_i > 0,$$

  $$\phi(q_i) = 0, \text{ at } q_i = 0,$$

  in formula (4), this property is provided by the $signq_i$;
- from the condition of incompressibility of water, a strict increase in the function $\phi(q_i)$ is necessary, i.e. $\phi(q_i') < \phi(q_i'')$ при $q_i' < q_i''$;
- from the continuity condition, the existence of continuous derivatives is necessary, i.e. $\phi_i'(q_i) > 0$.

The total hydraulic resistance of the i-th technological element is determined by the state of the inner surface of the section, its geometric parameters, the composition and nature of local resistances [37, p. 182]:

$$r_i = \frac{0.7}{10^9} \cdot \frac{k_{ei}^{0.25}}{d_i^{5.25}} \cdot (l_i + l_{ei}), \tag{5}$$

where d_i, l, l_{ei} – respectively, the inner diameter of the pipe, m, the length of its linear part and the equivalent length of local resistances, m;

k_e – equivalent roughness of the inner surface of the pipe of the i-th section of the heat pipeline, mm.

In this case, the equivalent length is determined by the following expression:

$$l_{ei} = \sum \zeta \cdot d_i/\lambda_i, \tag{6}$$

where $\sum \zeta$ – the sum of the coefficients for all local losses occurring in a section of a hydraulic circuit of a fixed length;

λ_i – dimensionless coefficient of hydraulic friction determined by the systems of empirical relations of Shifrinson, Altshul, Nikuradze.

The real water flow modes in pipeline networks made of steel pipes are characterized by the limiting values of the

Reynold’s criterion, therefore, λ_i does not depend on the f low rate, as well as in the linear sections of the tract

The model of the lost pressure represented by formula (4) is traditional in the hydrodynamics of continuum media and originates from the historically established scheme of experimental cause-and-effect relationships between the quantities that determine the medium flow mode.

However, when studying the movement of liquid media in a network of a fixed structure, which are HSM, an effective model from the point of view of computer implementation of flow distribution processes is the inverse (inverse) model of pressure loss, in which the function is the value of the flow rate of the medium in the section of the tract, and the arguments are head and hydraulic resistance.

The inverse model of the lost pressure in hydraulic circuits of a fixed section and length will be obtained from the model (4), and it will take the form:

$$\phi(\Delta h_i) = sign(\Delta h_i) \cdot [|\Delta h_i / r_i|]^{1/\alpha_i}. \tag{7}$$

The formal properties of model (7) also meet the requirements of positivity of energy losses, strict increase and continuity, and therefore the presented inverse model is also adequate to the most general physical properties of a steady flow of a Newtonian type fluid (single-phase incompressible viscous fluid).

3.2 Models of Coolant Movement in the Paths of Passive Elements Regulation and HSM Modes Control

Among the passive elements of the hydraulic circuits of the HSM, a special place is occupied by shut-off and control valves: with a controlled drive and throttle control bodies (DRO) of flow, pressure, temperature, differential pressure regulators, which provide throttle control and control of hydraulic and thermal modes in network technological installations and heat-consuming systems HSM.

The following model of the steady flow of the coolant through the shutter of the throttle control valve was obtained from the Bernoulli equation and empirical laws, confirmed by the experience of adjusting and normalizing the characteristics of valves under factory test conditions:

$$\begin{cases} \Delta h_i = h_{i0} - h_{i1} = \frac{P_{i0}}{\gamma} - \frac{P_{i1}}{\gamma} \\ \phi(q_i) = sign(q_i) r_i \cdot q_i^2 \\ r_i \in [r_i^-, r_i^+] \\ r_i^- = \Delta h_H / k_v^2, \\ r_i^+ = r_i^- / \beta^2 \end{cases}, \tag{8}$$

where r_i^-, r_i^+ – hydraulic resistance of the DRO valve at fully open and closed positions of the shutter, m/(m^3/h)^2;

Δh_H – normative pressure drop on the valve gate during its certification hydraulic tests;

k_v – conditional capacity of the valve, normalized by the standard in its characteristic, m^3/h;

β – normative relative passage of the energy carrier through the closed sh utter (relative leakage), % of k_v.

The value of the conditional throughput of the i-th shutoff valve is determined by the degree of its opening:

$$k_{iV} = \frac{5.04 \cdot F_{iY}}{\xi_i}, \tag{9}$$

where ξ_i – hydraulic resistance coefficient of valves,

$\xi_i = \phi(N_{iO})$, where N_{iO} – relative number of revolutions of the drive flywheel, $N_{iO} = N_i / N_{iMAX}$, where N_{iMAX}, N_i – values that determine the degree of opening of the shutter, respectively, the maximum possible and set number of revolutions of the flywheel;

$F_{iY} = \frac{\pi}{4} \cdot \frac{D_{iY}^2}{100} \cdot k_P$– cross-sectional area of the passage along the nominal diameter, m;

k_P– inlet section correction factor;

D_{iY} – nominal diameter, mm.

3.3 Models of Coolant Motion in the Ducts of Active Elements

All pumping units used in HSM technological processes provide an increase in the specific energy of the coolant flow due to the energy supplied from outside, therefore they are called active elements.

The model of the steady motion of the coolant in the path of the pumping unit is completely exhausted by the Bernoulli equation written for the inlet and outlet pipes of the pump, and the pressure-flow characteristic determined during factory or industrial tests:

$$\begin{cases} H_i = -(h_{i0} - h_{i1}) = -\left(\frac{P_{i0}}{\gamma} - \frac{P_{i1}}{\gamma}\right) \\ \phi(q_i) = r_{0i} + r_{1i} \cdot q_i + r_{2i} \cdot q_i^2 \end{cases}, \tag{10}$$

where H_i – head developed by the pump in the i-th mode, m;

h_{i0}, h_{i1} – total head in the inlet and outlet pipes of the pump, m;

q_i – pump delivery in i-th mode, m^3/h;

r_{0i}, r_{1i}, r_{2i} – hydrodynamic constants obtained in the process of approximation of the working area $q_i^- \leq q_i \leq q_i^+$ of normal pressure-flow characteristics of the i-th pump at the rated speed at the nominal diameter of the impeller. The dimension of the constant s, respectively, is equal to m, m/(m^3/h), m/(m^3/h)^2.

For almost the entire range of pumping units used in the HSM, the head-flow characteristics are determined in tabular form and are monotonically decreasing, and therefore their approximation is performed using a second-degree polynomial by the least squares method with a maximum error in the working area of 0.01% - 0.07% for ten points of the passport tables.

For the entire working area of the polynomial pressure-flow characteristic, the property of monotonic decrease is preserved, i.e. $\frac{d\phi(q_i)}{dq_i} \leq 0$ due to the increase in flow energy losses in the hydrodynamic path with an increase in the load of the pumping unit. However, the pressure-flow characteristics of network pumping stations consist of a fixed number of parallel-connected pumping units of the same type and have a fairly wide operating range in which the operating pressure of the station does not depend on the load, i.e. $H_i = const$ and $\frac{d\phi(q_i)}{dq_i} = 0$, with $q^{'} \leq q_i \leq q^{''}$, where q', q'' – respectively; range of flow rates in which the working head of the pumping station is independent of the flow rate. Pressure-flow characteristic type $H_i = const$ is a characteristic of an "ideal" pumping unit or pumping station, and we will make extensive use of this when simulating modes.

In the nodal models of the steady motion of the coolant in pipeline networks, we will use a polynomial model of the flow-pressure characteristic of the type:

$$q_i = \psi_i(H_i) = R_{0i} + R_{1i} \cdot H_i + R_{2i} \cdot H_i^2, \tag{11}$$

where H_i – head developed by the i-th pumping unit in operating mode;

R_{0i}, R_{1i}, R_{2i} – hydrodynamic constants obtained in the process of approximating the working area $H_i^- \leq H_i \leq H_i^+$ of the inverse normal pressure-flow characteristic of the i-th pump at the nominal speed at the nominal diameter of the impeller. The dimension of the constants, respectively m^3/h, (m^3/h)/m, (m^3/h)/m^2.

The presented mathematical models adequately reflect the main physical processes of the steady motion of the coolant, however, the accuracy of the quantitative prediction of the mode parameters depends on the degree of compliance of real processes with the system of assumptions and assumptions adopted above.

In existing heat supply systems, technological elements are subject to aging and loss of their original properties due to the intense impact of the environment and consumer loads, and therefore the parameters of technological elements $\{r_i, \alpha_i, r_{0i}, r_{1i}, r_{2i}, r_i^-, r_i^+\}$ may undergo changes in relation to their nominal values at the time of their commissioning. In this regard, the use of any empirical models requires periodic identification of parameters based on direct measurements of pressure losses and flow rates and processing of these measurements by methods of nonlinear estimation and identification.

That is why mandatory scheduled hydraulic tests have been established in the field of HSM operation, which allow to maintain the necessary degree of adequacy of the parameters of steady-state motion models used in the tasks of developing and implementing hydraulic modes.

4 Digital Modeling of Stationary Hydraulic Modes of Large-Scale Heat Supply Systems of Variable Technological Structure

The incomparability and uniqueness of complex systems, including HSM, is caused to a large extent by a clear manifestation of one of the forms of the dialectical principle of the transition of quantitative changes into qualitative ones, called the principle of functional integrity and introduced into the problems of complex systems.

According to this principle, the properties of a complex system cannot be discovered and inferred from the known and observable properties of its elements. Indeed, if a large number of technological elements of HSM are combined into a complex structure, then new relationships arise between the state variables of these elements, generating new properties that cannot be predicted using models of isolated elements.

What fundamental physical regularities and mathematical structures can be used to formalize these relations? One of the answers to this question was first given by Gustav Kirchhoff when he solved the problem of the distribution of electric currents in branched electrical circuits and formulated his two famous laws for steady currents and voltages. Later, analogies between structured complex systems of various physical nature were studied and it was found that the analogues of the electric potential are pressure (pressure), temperature, speed, and the analogues of the electric current are the flow of liquids, gases, heat flow, mechanical force.

It is these analogies that made it possible to expand the scope of Kirchhoff's laws to physical systems of any nature in Gabriel Krohn's diacoptics and Khasilev-Merenkov's theory of hydraulic circuits [2, p. 7].

The statements of the theory of hydraulic circuits apply to structured complex systems of any nature, regardless of the physical properties of the transferred substance along the branches of the network, as long as these properties fit into the model of a continuous medium.

At the same time, in network systems there is something in common that unites them on a variety of classes of systems, but there is also something specific that fixes individual classes and gives systems from these classes a functionally specific integrity, requiring specific formalization methods based on general methods of the theory hydraulic circuits.

Complex heat supply systems differ from complex water supply, ventilation and gas networks, primarily in the specificity of the technological structure, which consists in the presence of technological feedback between heat consumers and heat supply sources through a return pipeline network, as well as in the specificity of mass transfer with the environment in closed and open HSM.

Three classes of specific technological structures of complex heat supply systems are widely used in the infrastructure of modern cities: closed, open and mixed systems.

Let us unite into classes all the variety of problem statements for analyzing modes in open, closed, and mixed HSM, using as a classification feature the content of the initial data that fix the initial state of the HSM in a typical mode situation.

To designate typical mode situations, we will use the construction $R_i, i = 1, 2, ...$. Then, the typical mode situations of closed HSM, in accordance with the selected classification feature, will take the form presented in Table 1.

Table 1

Classification of typical mode situations closed by HSM

Situation	Content of initial data
R_1	– consumer loads, – parameters of active and passive technological elements of the system, – geodetic terrain marks for loaded and unloaded nodes.
R_2	– hydraulic resistance of consumers, – parameters of active and passive technological elements of the system, – geodetic terrain marks for loaded and unloaded nodes.
R_3	– hydraulic resistance of consumers, – parameters of active and passive technological elements of the system, – leakage costs from fixed units of supply and return pipelines, – make-up expenses in make-up nodes of heat supply sources, – geodetic terrain marks for loaded and unloaded nodes.
R_4	– hydraulic resistance of consumers, – parameters of active and passive technological elements of the system, – hydraulic resistances of fictitious leakage and make-up branches connecting the given nodes of the pipeline network with a fictitious reference node, – geodetic terrain marks for loaded and unloaded nodes.

Typical initial mode situations of open HSM are presented in Table 2.

Table 2

Classification of typical mode situations open HSM

Situation	Content of initial data
R_5	– hot water supply (HWS) loads at consumer connection points (open loads), – loads of heating, ventilation, air conditioning, process heat consumption (closed loads), – loads for charging storage tanks at their connection points, – loads for charging storage tanks at their connection points, – make-up costs to the make-up nodes of heat supply sources and to the connection nodes of storage tanks operating in the make-up mode,

	– geodetic terrain marks for loaded and unloaded units.
R_6	– HWS loads at consumer connection points, – loads for charging storage tanks, – hydraulic resistance of consumers in closed load, – parameters of active and passive elements of the system, – make-up costs to the make-up nodes of heat supply sources and to the connection nodes of storage tanks operating in the make-up mode, – geodetic terrain marks for loaded and unloaded nodes
R_7	– hydraulic resistance of consumers in closed load, – parameters of active and passive elements of the system, – parameters of active and passive elements of the system, – heads of fictitious pumps connecting the fictitious reference node with the given recharge nodes, – geodetic terrain marks for loaded and unloaded nodes
R_8	– HWS loads at consumer connection points, – loads for charging storage tanks, – hydraulic resistance of consumers in closed load, – parameters of active and passive elements of the system, – make-up costs to the make-up nodes of heat supply sources and to the connection nodes of storage tanks operating in the make-up mode, – geodetic terrain marks for loaded and unloaded nodes

In Figure 1, to demonstrate a visual representation of fragments of design schemes of technological structures of HSM, a graph of an open HSM with specified resistances and loads of consumers in a mode situation R_5 $Q_{zj} \neq 0, \forall j = \overline{1, n_p}, Q_{gj} \neq 0, \forall j = \overline{1, n_p}, Q_{pj} \neq 0, \forall j = \overline{1, n_p}$.

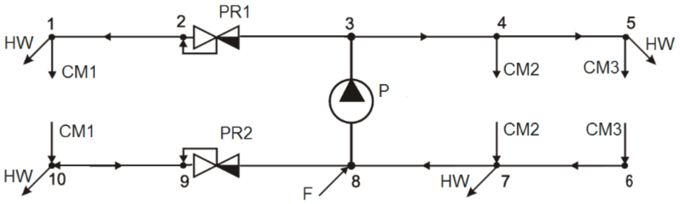


Figure 1

Design schemes of hydraulic circuits of HSM (HW – Open Hot Water Supply, PR – Pressure Regulator, F – Feeding, CM – Consumer Modeling, P – Pump)

Having classified the general technological structures of large-scale heat supply systems that have become the most widespread, and highlighting typical operating situations, using:

- graphical representation of technological structures;
- real vector space associated with HSM graphs;

- models of isolated technological elements associated with branches and nodes of the graph;
- Kirchhoff's network laws,

we obtain a model and an algorithm for simulating the steady-state hydraulic regime of HSM (Figure 2).

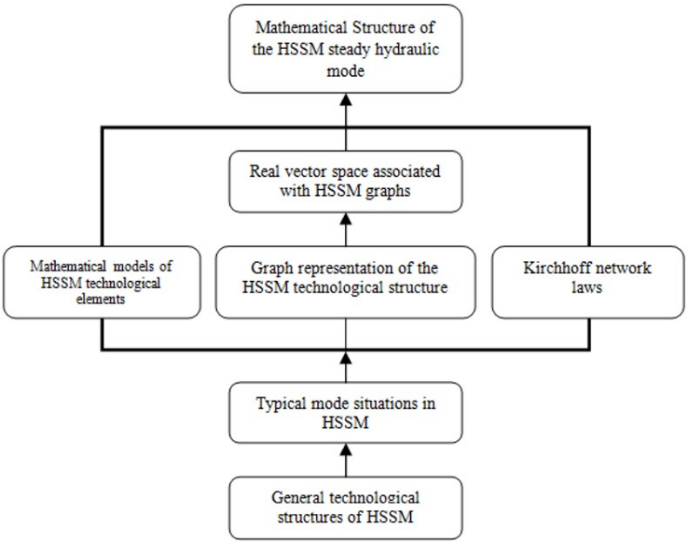


Figure 2

Simulation modeling structure of the steady-state of the HSM hydraulic mode

Algorithm for Building a General Mathematical Model for Simulation of the HSM Steady Hydraulic Mode:

- the computational scheme of the HSM of a fixed technological structure can be represented by a connected-oriented graph G(M,N), which contains m nodes, n branches, and p connectivity components;
- select a spanning tree, for example, of minimum hydraulic resistance, thus fixing Kchords and, accordingly, K linearly independent contours;
- build matrices of $A_C$connections and Bcontours;
- let us define the algebraic structure of a finite set of objects with the help of a finite-dimensional V_g vector space.

In theory of electrical and hydraulic circuits, a formal approach has been developed to represent all subgraphs of a linear graph G(M,N) into elements of V_g vector space of n dimension. The coordinate system in this space is determined by elements, each of which represents one branch of the G graph, and the rows of matrices and become vectors in this space.

Mapping a connected graph G(M,N) onto a V_g linear vector n-dimensional space using matrices A_c and B allows using the linear algebra apparatus to automatically generate Kirchhoff equations for each fixed HSM technological structure.

In V_g space, define vectors $(q, h, \Delta h, P, Q)$ formally representing the simulated state of HSM [9]:

- $q = (q_1, q_2, \dots, q_n)^T - n$-dimensional vector volume of expenses in the branches in the graph G;
- $h = (h_1, h_2, \dots, h_m)^T - m$-dimensional vector of total heads at the nodes of the graph G;
- $\Delta h = (\Delta h_1, \Delta h_2, \dots, \Delta h_m)^T - m$-dimensional vector of differences in total heads at the initial and final nodes of the branches of the graph G;
- $P = (P_1, P_2, \dots, P_m)^T - m$-dimensional vector of piezometric heads at the nodes of the graph G;
- $Q = (Q_1, Q_2, \dots, Q_m)^T - m$-dimensional vector of the nodal volume flow.

Being abstract objects, the vectors $(q, h, \Delta h, P)$ quantitatively reflect the internal state of the HSM in the process of the steady isothermal movement of the coolant along the branches of the hydraulic tract, and the Q vector displays the direction and intensity of mass transfer with the environment.

Put three models of the steady-state hydraulic mode, interpreted in terms of mode situations $R_1 - R_7$, into correspondence with the selected typical structures of the HSM (Tables 1, 2).

Model M_1. A model of this type is appropriate for simulation modeling of design, operational and prospective modes with given consumer loads, both closed HSM in a mode situation R_1 and open HSM in a mode situationR_5.

Model M_2. A model of this type is appropriate for modeling emergency or operational modes associated with the withdrawal of network or station process equipment for repair, when it is necessary to estimate changes in the coolant flow through the hydraulic circuits of closed heat-consuming consumer systems at a fixed hot water supply load. In closed HSM R_2, R_3 mode situations are modeled, and in open ones the R_6 situation is modeled.

Model M_3. A model of this type is appropriate for modeling emergency or operating conditions associated with the withdrawal of technological equipment for repair, when it is necessary to evaluate changes in the coolant flow both through the hydraulic paths of closed heat supply systems of consumers, and is through the hydraulic paths of hot water supply systems for consumers of open HSMs, or through fictitious hydraulic paths coolant leaks in simulated emergency situations (in closed HSM), as well as flow rates through the hydraulic circuits of the make-up systems. In closed HSM, the mode situation R_4 is modeled, and in open ones, the situation R_7 is modeled.

The generated M_1, M_2, M_3 general mathematical models (Table 3) contain a complete set of formal tools that connect by algebraic relations the parameters of steady-state modes of open and closed HSM in typical mode situations, which have real meaning in the areas of operation and design.

Table 3

General mathematical models of HSM hydraulic modes

Model	Simulation		Representation of heat consumption systems in the model		
	HSM modes	HSM mode situations	closed	open	leakage/ makeup
M_1	estimated operational perspective	R_1, R_5	load	load	consumptio n
M_2	emergency operational	R_2, R_3, R_6	hydraulic resistance	load	consumptio n
M_3	emergency operational	R_4, R_7	hydraulic resistance	hydraulic resistance	hydraulic resistance

In the theory of electrical and hydraulic circuits, a formal approach has been developed to represent all subgraphs of a linear graph *G(M,N)* into elements of $V_g$vector space of dimension n. The coordinate system in this space is determined by elements, each of which represents one branch of the graph *G*, and the rows of matrices A_C and *B* become vectors in this space.

5 Methods and Digital Models for Describing and Solving Problems of Analyzing the Steady-State Hydraulic Mode of Heat Supply Systems with Variable Technological Structure

However, these models M_1, M_2, M_3 are not yet formally posed tasks for the analysis of the steady-state hydraulic modes of the HSM and, moreover, do not determine the procedure to calculate the components of the vectors $(q, h, \Delta h, P, Q)$ that determine the desired hydraulic state of the HSM.

The mutual connection of the mentioned state vectors necessitates the selection of the primary desired variables, i.e. mode parameters, which will be determined primarily on the basis of the initial data in a specific mode situation.

If assigned the components of the vector q as the primary desired variables, then on the basis of any of the models M_1, M_2, M_3 it can be obtained the so-called *contour model* (contour roots method – CRM) of the analysis of the steady hydraulic mode, which based on the concept of contour flow rates.

If assigned the components of the vector h as the primary desired variables, then after appropriate transformations of the models it can be obtained a *nodal model* (nodal pressure method – NPM), which based on the concept of nodal heads (pressures).

The contour and nodal models are equivalent in terms of the solutions obtained through $(q^*, h^*, \Delta h^*, P^*, Q^*)$, however, from the point of view of computational efficiency, each of them has its own scope of application on the set of the HSM technological structures.

In the problems of mode analysis in pipeline systems, contour-type models have received the greatest development [2, p. 40]. However, in the mode analysis of electric power systems and large electronic circuits of modern circuitry, nodal models have received the greatest development [9, p. 85]. The latter circumstance stimulated work on software implementation and the study of the computational efficiency of contour and nodal models in the problems of modeling large heat supply systems, in which the areas of expedient application of both classes of models were determined.

Presented above the digital models and methods of flow distribution in hydraulic circuits described above made it possible to set and solve the problems of analyzing the operational and post-emergency modes of operation of large heat supply systems of a variable technological structure with any degree of detail of the design scheme. However, in order to answer the question of preference for one of them according to the criterion of computational efficiency, which is very important at the stage of software implementation of an automated solver of flow distribution problems in hydraulic circuits of a variable technological structure, it becomes necessary to experimentally study the computational efficiency indicators on real hydraulic circuits of large heat supply systems.

Research on the efficiency of computational processes was carried out using the TGID-07 Software package [38] on real hydraulic circuits of heat supply systems of the cities of Kazakhstan Shymkent, Kostanay and Almaty, the main structural characteristics of which are given in Table 4.

Hydraulic circuit No.1 corresponds to the design scheme of the fifth district of operation of the Heating grid of JSC "3-Energoortalyk" and has one source of heat supply − CHP-3. Hydraulic circuit No. 2 corresponds to the design scheme of the fifth district of operation of the Heating system of the State Enterprise "Kostanay Thermal Power Company" and has two heat supply sources of the CHP and Boiler Room No. 2 operating on the same network. Hydraulic circuit No. 3 corresponds to the combined design scheme of the first, second and sixth districts of operation of the Almaty Thermal Power Company.

In all investigated hydraulic circuits, the pressure regulators were turned off, and the control valves of the regulators were presented as branches with a fixed hydraulic resistance. Such switching of the regulator operation mode made it possible to exclude external iterations for linking their mode and measure the net operating time of the Flow Distribution Problem Solver.

Table 4

Quantitative characteristics of the investigated circuits

Hydraulic circuit number	Power system name	Branches of the hydraulic circuit graph				Hydraulic circuit graph nodes	Hydraulic circuit graph contours
		Heat pipe sections	Consumers	Pump stations	Total number of branches		
1	3-Energo-ortalyk	1138	412	10	1560	1107	454
2	Kostanay Thermal Power Company	1168	300	7	1475	1155	321
3	Almaty Thermal Power Company	1903	530	16	2449	1776	673

Studies of the efficiency of computational processes were carried out with flow distribution problem solvers implemented on the basis of the methods of contour and nodal models. In this case, two typical operating modes of the system were used:

- operational mode is a mode with fixed real heat loads of consumers in the current heating season and real capacity of pipeline networks;
- emergency mode is a mode with fixed hydraulic resistances of consumers and real throughput of pipeline networks.

During the calculation, the following characteristics were fixed:

- t_k – execution time of all operations that control the integrity of the database, including the connectivity in the graph of the calculation scheme, seconds;
- t_p − time to calculate the mode parameters for all objects of the design scheme of the investigated hydraulic circuit, seconds.

The results of measuring the time characteristics of computing processes are displayed in Table 5.

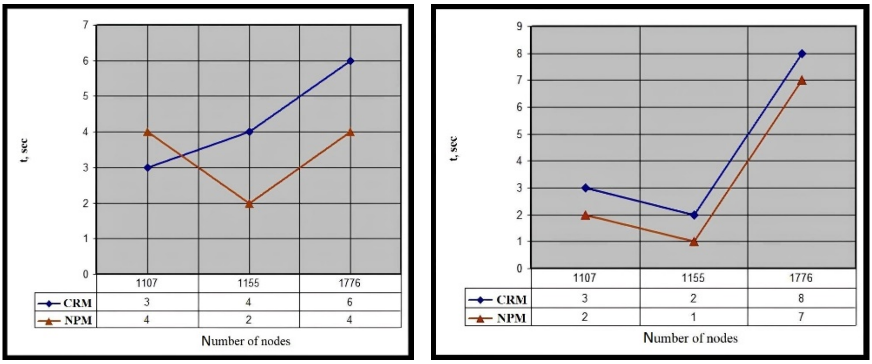
Comparative Efficiency of the Nodal Pressure Method (NPM) with Existing Approaches reveals that during the simulation of emergency modes, the NPM demonstrates a slight increase in computational speed due to the elimination of labor-intensive processes for identifying and processing linearly independent contours. For modeling highly complex pipeline systems with more than 5000 nodes, the NPM proves to be more efficient, while for medium and large-scale systems with up to 5000 nodes, both the NPM and the Contour Root Method (CRM) exhibit comparable performance characteristics.

Table 5

Characteristics of the computing processes efficiency

Hydraulic circuit number	Contour root method				Nodal pressure method			
	Operating mode		Emergency mode		Operating mode		Emergency mode	
	t_k, s	t_p, s	t_k, s	t_p, s	t_k, s	t_p, s	t_k, s	t_p, s
1	3	3	4	8	4	2	5	6
2	4	2	6	10	2	1	3	8
3	6	8	8	12	4	7	5	9

A analysis of the temporal characteristics of the flow distribution processes in real hydraulic circuits confirms the practical coincidence of the performance characteristics of the methods of contour and nodal models (Figures 3-4).



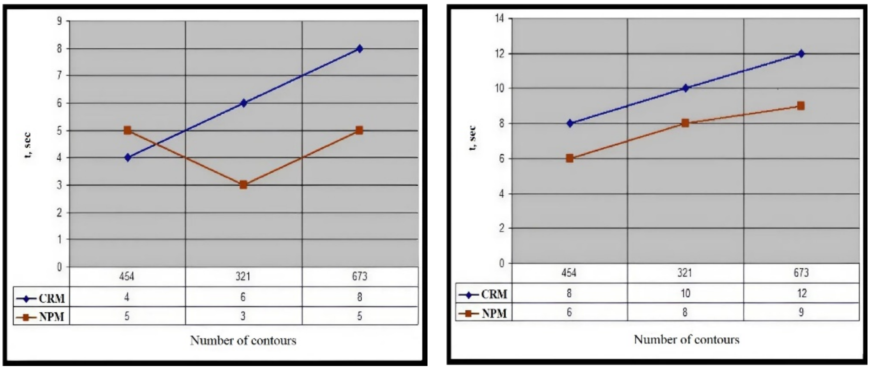
a) b)

a) – the duration of operations that control the integrity of the database, including the connectivity of the graph of the calculation scheme; b) – the duration of the calculation of the mode parameters

Figure 3

The duration of operational computing processes using CRM and NPM methods

The key advantages of the NPM include the use of inverse characteristics of network elements, simplifying the mathematical representation of problems and enhancing the numerical stability of solutions. Furthermore, the simplification of contour processing is particularly beneficial in emergency scenarios. The practical application of the proposed method, implemented in the TGID-07 software package has been validated through real-world calculations of district heating systems in cities across Kazakhstan, confirming its effectiveness and applicability.



a) b)

a) – the duration of operations that control the integrity of the database, including the connectivity of the graph of the calculation scheme; b) – the duration of the calculation of the mode parameters

Figure 5

The duration of the computing processes of the emergency mode using CRM and NPM methods

Traditional approaches, such as the CRM, require substantial computational resources as network size increases, limiting their usability for modeling large-scale and complex systems. In contrast, the proposed method demonstrates consistent performance and superior efficiency under high computational loads.

Conclusions

The results obtained in the course of the conducted research:

- the main patterns of coolant movement along the sections of the hydraulic tract of the fuel and lubricants based on the law of conservation of mass and mechanical energy of the flow and the developed mathematical models of coolant movement in active and passive technological elements of the fuel and lubricants, based on their inverse characteristics;
- classifier of typical regime situations of closed, open and mixed vehicles, fixing their condition according to the content of the source data;
- mathematical models of calculated, operational and prospective steady-state hydraulic modes of the fuel and energy complex, interpreted in terms of operational situationsю

It allowed us to develop a new approach to the formation of a system of equations – nodal pressures, which is based on the inverse characteristics of passive and active elements of pipeline networks that close the systems of equations of the stationary isothermal regime of heat supply systems. The solution of which, in turn, made it possible to create an effective algorithm and computer implementation of simulation of modes of large-size heat supply systems of variable technological structure, in which the function is the amount of medium flow in the tract section, and the arguments are pressure and hydraulic resistance.

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