LPV design for the control of heterogeneous traffic flow with autonomous vehicles

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Abstract: The paper proposes a strategy to control heterogeneous traffic flow which contains both autonomous and human-driven vehicles. The purpose of the control strategy is to consider differences in the longitudinal driving characteristics of autonomous and human-driven vehicles. In the paper the modeling of the heterogeneous traffic flow based on the results of the VISSIM traffic simulator is presented. The traffic model yielded is in a Linear Parameter-Varying (LPV) form. The control design is based on the Takagi-Sugeno methodology, in which the performances, the constraints of the ramp-controlled interventions and the uncertainties are incorporated. The design task leads to an optimization with Linear Matrix Inequality (LMI) constraints. The result of the method is the optimal intervention of the freeway ramps with which traffic inflow can be controlled.

Keywords: Takagi-Sugeno LPV design, traffic control, autonomous vehicles

1 Introduction and motivation

The growing importance of autonomous functionality in vehicle control systems poses novel challenges in the research of intelligent transportation systems. One of these problems is the modeling and control of heterogeneous traffic flow, which is based on the difference between the speed profiles of conventional human-driven vehicles and autonomous vehicles. The autonomous vehicles can have more information about the forthcoming environments, e.g. road slopes and traffic signs [1], with which their current speed profile is modified. Thus, in heterogeneous traffic the participant vehicles have different motions, which makes traffic modeling and the control problem more complex.

Most of the novel traffic control design methods are based on the state-space representation of traffic flow dynamics. It is incorporated in several relationships [2], e.g. the conservation of vehicles, the equilibrium speed equation, the fundamental equation and the momentum equation. Although it can provide an enhanced description of traffic dynamics, due to the uncertainties, the estimation of model parameters may be difficult. For example, [3] proposed an identification method for traffic model parameters, especially the fundamental diagram, which has an important role in traffic control design. In a mixed traffic scenario the identification problem can be more difficult because the deviation of the measured data is more significant due to the varying speed profiles of the vehicles. The most important modeling approach for mixed traffic was summarized in the survey of [4]. The analysis of the traffic flow in which semi-autonomous and autonomous vehicles were traveling together with conventional vehicles was proposed by [5, 6]. A control law which considers the different speed profiles of the semi-autonomous vehicles was proposed by [7, 8].

In this paper a robust control design based on the Linear Parameter-Varying (LPV) method is presented [9, 10], with which the inflow ramps of the freeway in a heterogeneous traffic flow can be controlled. The proposed method can be used for the control of heterogeneous traffic flow which contains vehicles with the autonomous driving levels from 2 to 5. It means that the acceleration/deceleration functionalities of the controlled vehicles are automated. The control design is based on a control-oriented LPV system, which contains disturbances. The model is based on the simulation results of the high-fidelity VISSIM traffic simulator [11, 12]. In the method the maximization of the traffic flow is formed as an optimal control problem with Linear Matrix Inequality (LMI) constraints [13, 14], by which disturbance rejection and stability are guaranteed. The advantage of the control design based on the proposed Takagi-Sugeno LPV method is that it is able to consider several properties of the control problem, e.g. uncertainties, parameter-variation and constraints. The method is able to guarantee robustness against the uncertainty of the traffic flow model and the constraints on the physical properties on the controlled ramp can be incorporated in the control task.

The paper is organized as follows. Section 2 proposes a novel control-oriented model of the heterogeneous freeway traffic flow in an LPV form. The control design in presented in Section 3, in which the input constraints, the performances, the parameter-varying property and the disturbances are considered. Finally, the method is presented through a simulation example in Section 4 and the paper is concluded, see Section 5.

2 Modeling of heterogeneous traffic flow dynamics

Traffic dynamics represents the traffic network, which is gridded into N number of segments. The traffic flow of each segment is represented by a dynamical equation, which is based on the law of conservation. The relationship contains the sum of inflows and outflows for a given segment i. Thus, traffic density ρ_i $[veh/km]$ is expressed in the form

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i}\left[q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k)\right], \tag{1}$$

where k denotes the index of the discrete time step, T is the discrete sample time, L_i is the length of the segment, q_i $[veh/h]$ and q_{i-1} $[veh/h]$ denote the inflow of the traffic in segments i and $i-1$, r_i $[veh/h]$ is the sum of the controlled ramp inflow, while s_i $[veh/h]$ is the sum of the ramp outflow. The model of the traffic system is illustrated in Figure 1.

Another important relation of the traffic dynamics is the fundamental relationship, which creates a connection between the outflow $q_i(k)$, the traffic density $\rho_i(k)$ and the average traffic speed $v_i(k)$, see e.g., [15]. The fundamental relationship is formed as

$$q_i(k) = \rho_i(k)v_i(k). \tag{2}$$

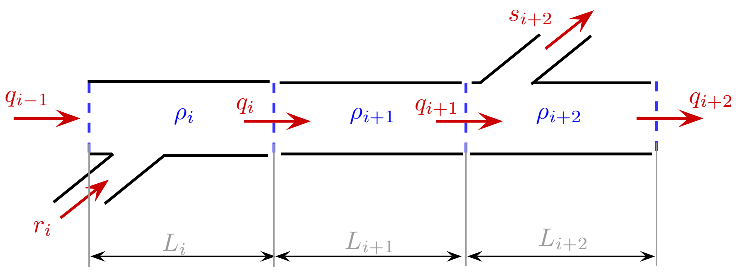


Figure 1: Illustration of the traffic system model

The average traffic speed $v_i(k)$ can be formed in the traffic flow modeling studies as a nonlinear function of traffic density [16], which results in the relationship

$$q_i(k) = \mathcal{F}(\rho_i(k)), \tag{3}$$

where $\mathcal{F}$ is a nonlinear function. Its reason for that is the increase in traffic density leads to the reduction of the distance between the vehicles on the road section. Due to the reduced distance the speed of the vehicles must also be reduced to avoid the risk of collision. Thus, the average traffic speed is also reduced due to the reduced speeds of the individual vehicles. The characteristics of $v_i(k)$ depending on $\rho_i(k)$ are decreasing and nonlinear, causing $\mathcal{F}(\rho_i(k))$ to have nonlinear characteristics as well.

Conventionally, the fundamental relationship is derived from historic measurements, and it also depends on several factors, see [2, 17]. Therefore, the analysis on the mixed traffic flow requires several experiments with various rates of autonomous vehicles κ. Figure 2 shows an example of the result of the analysis, which is performed through the VISSIM traffic simulator. It can be seen that the increase in $\kappa(k)$ has the following effects on the linear section ($\rho_i(k) \in [0; \rho_{i,crit}]$) of the fundamental characteristics, where $\rho_{i,crit}$ is the density value at the maximum of $q_i(k)$.

- Through the increase in $\kappa(k)$ the mean value of the traffic flow characteristics decreases. The decrease has a progressive tendency.
- Similarly, the increase in $\kappa(k)$ leads to the increase in the density in the set of traffic flow values. The increase in density is also nonlinear.

The experiments of the simulations on the linear section of the fundamental diagram are formulated in the following relationship

$$\begin{aligned} q_i(k) &= \big(\alpha_0 - f_\beta(\kappa(k))\big)\rho_i(k) + f_\gamma(\kappa(k))\rho_i\Delta_i \\ &= (\alpha_0 - (\beta_2\kappa^2(k) + \beta_1\kappa(k)))\rho_i(k) + \\ &+ (\gamma_3\kappa^3(k) + \gamma_2\kappa^2(k) + \gamma_1\kappa(k) + \gamma_0)\rho_i\Delta_i, \end{aligned} \tag{4}$$

where f_β, f_γ are $\kappa(k)$-dependent polynomial functions of the model with the parameters $\alpha_0, \beta_2, \beta_1$ and $\gamma_3, \gamma_2, \gamma_1, \gamma_0$. $\Delta_i \in [-1; 1]$ represents the uncertainty in the system, which results in the density in the flow characteristics.

The traffic flow model of a freeway section i is formed through the law of conservation

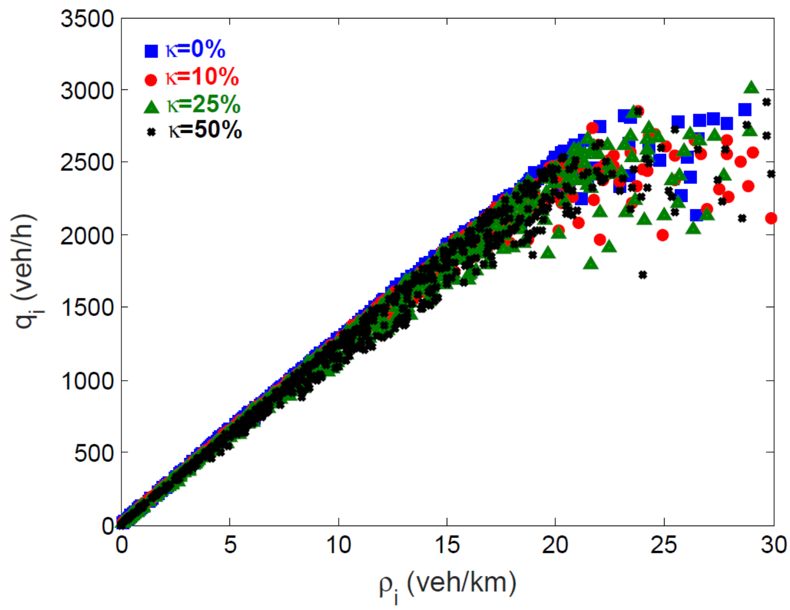


Figure 2: Charactersitics of the fundamental diagram

(1) and the proposed form of the fundamental relationship (4)

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i}\Big[q_{i-1}(k) + r_i(k) - s_i(k) - \\ - \big((\alpha_0 - f_\beta(\kappa(k)))\rho_i(k) + f_\gamma(\kappa(k))\rho_i\Delta_i\big)\Big] \tag{5}$$

The equation can be transformed into a state-space representation as

$$x(k+1) = A(\kappa)x(k) + B_1 w(k) + B_2 u(k), \tag{6}$$

where $x(k) = \rho_i(k)$ is the state of the system, $u(k) = r_i(k)$ is the control input and $w(k) = \begin{bmatrix} q_{i-1}(k) & s_i(k) & \Delta_i(k) \end{bmatrix}^T$ is the disturbance vector. The matrix of the system is represented by A, and simultaneously B_1 is the matrix of the disturbances and B_2 is the matrix of the control input, such as

$$A(\kappa) = \left(1 - \frac{T}{L_i}\left[\alpha_0 - f_\beta(\kappa(k))\right]\right), \tag{7a}$$

$$B_1 = \begin{bmatrix} \frac{T}{L_i} & -\frac{T}{L_i} & -\frac{T}{L_i} f_\gamma(\kappa(k))\rho_i \end{bmatrix}, \tag{7b}$$

$$B_2 = \begin{bmatrix} \frac{T}{L_i} \end{bmatrix}. \tag{7c}$$

3 Design of LPV control for the traffic flow

The purpose of this section is to find a control method by which the controlled inflow of the freeway ramp $r_i(k)$ is set, while the control input is limited and the impact of disturbances must be eliminated. Moreover, a challenge in the control design is that the dynamics of the traffic is modeled in an LPV form, see (6).

The aim of the control is to guarantee the maximum outflow $q_i(k)$ of the traffic network. However, the outflow can be improved by increasing $\rho_i(k)$ until reaching the critical density $\rho_{i,crit}$. Since $q_i(k)$ has a maximum at $\rho_{i,crit}$ [18], it must be guaranteed through the coordination of the system inputs:

$$z_1 = \rho_{i,crit} - \rho_i(k), \qquad |z_1| \to min, \tag{8}$$

Although at a low number of inflow vehicles $\rho_{i,crit}$ cannot be achieved, but increasing $\rho_i(k)$ through z_1 results in the maximization of $q_i(k)$. The value of critical density is selected through the previous analysis of the traffic network.

The design of the control requires several steps to find an appropriate controller on the existing complex problem. Thus, the following steps must be performed in the method.

1. The constraint on the control input is considered in the design method.
2. The traffic model through the consideration of (8) is reformulated.
3. The LPV system is described in a Takagi-Sugeno form, which leads to a linear control design problem.
4. The control synthesis is formed as an optimization problem, in which the impact of the disturbances on the performance is reduced.

3.1 Consideration of the input constraints

In the traffic system the value of control input $r_i(k)$ must be non-negative. Moreover, the inflow on the ramps can have a maximum capacity due to the physical limits of the road. Thus, it is necessary to design a control strategy, by which the following constraint is handled

$$0 \leq r_i(k) \leq r_{i,max}, \tag{9}$$

where $r_{i,max}$ represents the maximum capacity of the inflow ramps. The criterion is considered as a soft constraint during the control actuation in the following way.

It is necessary to consider that the control intervention has importance at high ρ_i values, which are close to $\rho_{i,crit}$. If ρ_i is significantly smaller than $\rho_{i,crit}$, $r_i = r_{i,max}$ is selected. This intervention is operated in the range of $0 \leq \rho_i(k) \leq \rho_{i,des}$, where $\rho_{i,des} < \rho_{i,crit}$ is a design parameter.

However, if $\rho_i > \rho_{i,des}$, the value of r_i must be reduced to avoid the saturation of the traffic network. In this case the dynamic control must be actuated, whose design is based on the control-oriented traffic model. The model (6) in the range of $\rho_i > \rho_{i,des}$ must be reformulated to eliminate the static density value of $\rho_{i,des}$ in the control design. The model (6) at $x(k+1) = x(k) = \rho_{i,des}$ is formed as

$$\rho_{i,des} = A(\kappa)\rho_{i,des} + B_1 w_{st} + B_2 u_{st}, \tag{10}$$

where w_{st} is considered to be an average disturbance at $\rho_{i,des}$ and u_{st} is the related control input, which is computed as

$$u_{st} = B_2^{-1}\big[(1 - A(\kappa))\rho_{i,des} - B_1 w_{st}\big]. \tag{11}$$

Moreover, the avoidance of the saturation requires a dynamic actuation $u_{dyn}(k)$, which guarantees the performance (8) and reduces the impact of $w_{dyn}(k)$ on the performance. $w_{dyn}(k) > 0$ is considered to be the difference between $w(k)$ and w_{st}. Thus, the control-oriented traffic model (6) for the control design on the range of $\rho_{i,des} \leq \rho_i(k) \leq \rho_{i,crit}$ is reformulated as

$$x_{dyn}(k+1) = A(\kappa)x_{dyn}(k) + B_1 w_{dyn}(k) + B_2 u_{dyn}(k), \tag{12}$$

where $x_{dyn}(k) = x(k) - \rho_{des,i}$. Simultaneously, the overall control actuation is

$$r_i(k) = u(k) = u_{st} + u_{dyn}(k), \tag{13}$$

from which the constraints of $u_{dyn}(k)$ is

$$-u_{st} \leq u_{dyn}(k) \leq r_{i,max} - u_{st}. \tag{14}$$

The main result of the reformulation is that the dynamic control input u_{dyn} can have both positive and negative values, see (14), with which the complexity of the control design can be significantly reduced. However, the overall control input on the traffic system $u(k)$ is always non-negative.

3.2 Performance-driven reformulation of the traffic model

The goal of the control design is to guarantee the defined performance (8). Thus, it is necessary to minimize the difference between the current traffic density and the critical density value. Due to the partition of the system into static and dynamic parts, performance z_1 is modified to

$$e(k) = \rho_{ref} - x_{dyn}(k), \qquad |e(k)| \rightarrow min, \tag{15}$$

where $\rho_{ref} = \rho_{i,crit} - \rho_{i,des}$ is a constant value.

The error for $k+1$ is derived as $e(k+1) = \rho_{ref} - x_{dyn}(k+1)$. Using the relationship (12), the dynamics of the error is

$$\begin{aligned} e(k+1) &= \rho_{ref} - A(\kappa)x_{dyn}(k) - B_1 w_{dyn}(k) - B_2 u_{dyn}(k) = \\ &= \rho_{ref} - A(\kappa)(\rho_{ref} - e(k)) - B_1 w_{dyn}(k) - B_2 u_{dyn}(k) = \\ &= A(\kappa)e(k) + (1 - A(\kappa))\rho_{ref} - B_1 w_{dyn}(k) - B_2 u_{dyn}(k). \end{aligned} \tag{16}$$

The controller of the traffic system is considered to be full-state feedback, whose input is $e(k)$. Thus, the control law is $u_{dyn} = Ke(k)$, where K represents the controller. Using the relationship between u_{dyn} and $e(k)$, the error dynamics is formed as

$$e(k+1) = \big(A(\kappa) - B_2 K\big)e(k) + W(k) = A_{cl}(\kappa, K)e(k) + Wd(k), \tag{17}$$

where $Wd(k)$ is an upper-bound approximation of the overall disturbance $(1 - A(\kappa))\rho_{ref} - B_1 w_{dyn}(k)$, in which $||d(k)|| \leq 1$ is a noise and W represents its scaling. The formulated system on the tracking error (17) is an LPV system, in which K must be selected to stabilize the system, guarantee the performances and reduce the impact of $d(k)$ on $e(k)$.

3.3 Takagi-Sugeno description of the system

In the following the LPV system (17) is reformulated to the sum of Linear Time Invariant (LTI) systems using the Takagi-Sugeno description, see [10]. The advantage of the method is that the control design becomes simpler due to the linear formulation.

The scheduling variable κ has lower $\overline{\kappa}$ and upper $\underline{\kappa}$ limits. Similarly, the limits determine the lower and upper limits of $A_{cl}(\kappa, K)$, such as $\underline{A_{cl}} = A_{cl}(\underline{\kappa}, K)$ and $\overline{A_{cl}} = A_{cl}(\overline{\kappa}, K)$. Thus, $A_{cl}(\kappa, K)$ can be reformulated as

$$A_{cl}(\kappa, K) = \frac{A_{cl} - \underline{A_{cl}}}{\overline{A_{cl}} - \underline{A_{cl}}}\overline{A_{cl}} + \frac{\overline{A_{cl}} - A_{cl}}{\overline{A_{cl}} - \underline{A_{cl}}}\underline{A_{cl}} = \mu_1 \overline{A_{cl}} + \mu_2 \underline{A_{cl}}, \tag{18}$$

where $0 \leq \mu_1, \mu_2 \leq 1$ are the multipliers of the matrices $\underline{A_{cl}}, \overline{A_{cl}}$. Similarly, the system (17) can be reformulated using (18) as

$$e(k+1) = \mu_1(\overline{A_{cl}} e(k) + W d(k)) + \mu_2(\underline{A_{cl}} e(k) + W d(k)), \tag{19}$$

which means that the original LPV system can be reformulated as a sum of two linear systems, which represent the convex hull of the κ-dependent LPV system.

3.4 Synthesis of the optimal control

During the control synthesis it is necessary to guarantee the stability of the system and the improvement of the performances. In addition, the impact of the disturbances on the tracking must be reduced. These criteria are formed in the following way.

- **Stability**: For the stability of the set of LTI systems (19) it is necessary to guarantee that all trajectories of the systems converge to zero as $t \to \infty$ [14]. Thus, it is necessary to guarantee the stability criterion

$$\Delta V(e(k)) < 0, \tag{20}$$

where $V(e(k)) > 0$ is the Lyapunov function. $\Delta V(e(k))$ is selected in a quadratic form, such as $V(e(k)) = e(k)^T P e(k)$, $P > 0$ and P is a symmetric matrix. The stability criterion for the system $\overline{A}_{cl} e(k) + W d(k)$ is derived as

$$\begin{aligned}
&\Delta V(e(k)) = V(e(k+1)) - V(e(k)) = \\
&= (\overline{A}_{cl} e(k) + W d(k))^T P (\overline{A}_{cl} e(k) + W d(k)) - e(k)^T P e(k) = \\
&= e^T(k)(\overline{A}_{cl}^T P \overline{A}_{cl} - P)e(k) + e^T(k)\overline{A}_{cl}^T P W d(k) + W^T d^T(k) P \overline{A}_{cl} e(k) + \\
&+ W^T d^T(k) P W d(k) = \begin{bmatrix} e(k) \\ d(k) \end{bmatrix}^T \begin{bmatrix} \overline{A}_{cl}^T P \overline{A}_{cl} - P & \overline{A}_{cl}^T P W \\ W^T P^T \overline{A}_{cl} & W^T P W \end{bmatrix} \begin{bmatrix} e(k) \\ d(k) \end{bmatrix} < 0.
\end{aligned} \tag{21}$$

The result of the derivation can be formed as a Linear Matrix Inequality (LMI) condition on system $\overline{A}_{cl} e(k) + W d(k)$, such as

$$\begin{bmatrix} \overline{A}_{cl}^T P \overline{A}_{cl} - P & \overline{A}_{cl}^T P W \\ W^T P^T \overline{A}_{cl} & W^T P W \end{bmatrix} \prec 0. \tag{22}$$

Similarly, the LMI condition for system $\underline{A_{cl}}e(k) + Wd(k)$ is

$$\begin{bmatrix} \underline{A}_{cl}^T P \underline{A}_{cl} - P & \underline{A}_{cl}^T PW \\ W^T P^T \underline{A}_{cl} & W^T PW \end{bmatrix} \prec 0. \tag{23}$$

Since $\underline{A}_{cl}, \overline{A}_{cl}$ depend on the controller K, it is necessary to select K and $P > 0$, by which the previous conditions are guaranteed.

- **Performance**: The tracking capability of the system (8) can be improved through the selection of K. Since the control input is defined as $u_{dyn}(k) = Ke(k)$, the tracking can be improved by increasing the gain K.
- **Disturbance**: The reduction of the impact of $d(k)$ on $e(k)$ requires that the $\mathcal{H}_\infty$ norm of the transfer function $T_{d,e}$ from $d(k)$ to $e(k)$ be reduced. In the system $\overline{A}_{cl}e(k) + Wd(k)$ the transfer function is computed as

$$T_{d,e} = \left(zI - \overline{A}_{cl}\right)^{-1} W, \tag{24}$$

where I is an identity matrix. Thus, the condition is

$$||T_{d,e}||_\infty = \left|\left|\left(zI - \overline{A}_{cl}\right)^{-1} W\right|\right|_\infty < \gamma, \tag{25}$$

where $\gamma > 0$ is a predefined upper bound of the norm. If $\gamma < 1$ is selected then the robustness of the system can be guaranteed, see [19]. Similarly, the criterion on the system $\underline{A_{cl}}e(k) + Wd(k)$ is

$$\left|\left|\left(zI - \underline{A}_{cl}\right)^{-1} W\right|\right|_\infty < \gamma. \tag{26}$$

The conditions of (25) and (26) can be composed with the criteria (22) and (23) through the dissipativity of the system, the construction of the supply function and the Schur lemma, see [14]. Thus, the LMI conditions which incorporate the stability and the distrubance rejection criteria are

$$\begin{bmatrix} P & 0 & \underline{A}_{cl}^T P & I \\ 0 & \gamma^2 I & W^T P & 0 \\ P\underline{A}_{cl} & PW & P & 0 \\ I & 0 & 0 & I \end{bmatrix} \succeq 0, \tag{27a}$$

$$\begin{bmatrix} P & 0 & \overline{A}_{cl}^T P & I \\ 0 & \gamma^2 I & W^T P & 0 \\ P\overline{A}_{cl} & PW & P & 0 \\ I & 0 & 0 & I \end{bmatrix} \succeq 0. \tag{27b}$$

During the control synthesis it is necessary to minimize γ, while K is maximized. Therefore, during the optimization $\frac{1}{\gamma}$ in a cost function

$$J = K + \alpha\gamma \tag{28}$$

is maximized, where α is a scaling parameter. Thus, the resulting optimal control problem is

$$\max_{K,\gamma} K + \alpha\gamma \tag{29a}$$

$$\text{subject to} \quad P \succ 0, \tag{29b}$$

where

$$\begin{bmatrix} P & 0 & \underline{A}_{cl}^T P & I \\ 0 & \gamma^2 I & W^T P & 0 \\ P\underline{A}_{cl} & PW & P & 0 \\ I & 0 & 0 & I \end{bmatrix} \succeq 0, \tag{30a}$$

$$\begin{bmatrix} P & 0 & \overline{A}_{cl}^T P & I \\ 0 & \gamma^2 I & W^T P & 0 \\ P\overline{A}_{cl} & PW & P & 0 \\ I & 0 & 0 & I \end{bmatrix} \succeq 0. \tag{30b}$$

The resulting optimal controller K is used to compute u_{dyn}, which is applied as a control input to the system together with the u_{st}.

4 Simulation example

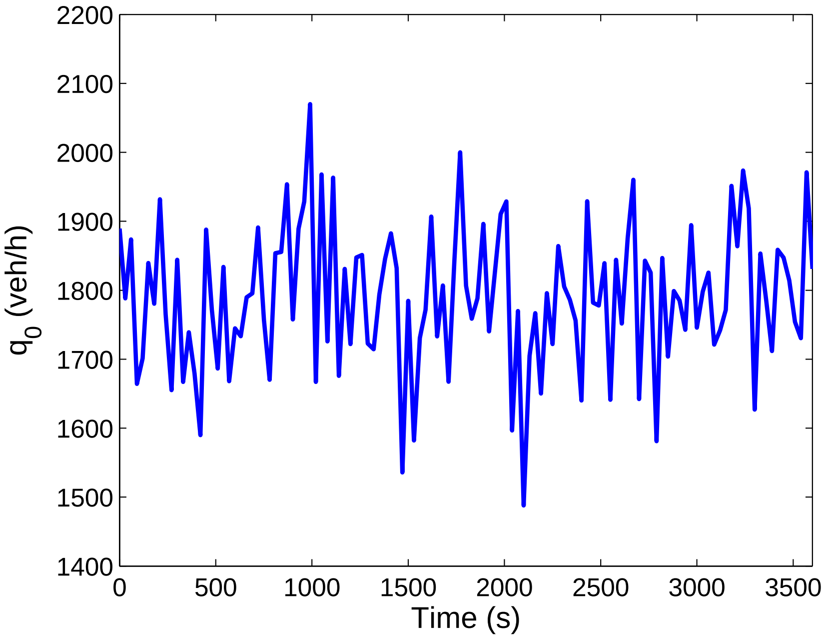
In the following a simulation example on the robust LPV control is presented. In the simulation a 1.5-km-long section of the Hungarian M1 freeway between Tatabánya and Budapest with two lanes is examined. Previously, several simulations have been performed in the VISSIM traffic simulator to generate the fundamental characteristics of the freeway. During these simulations several traffic scenarios with various κ and q_0 values were performed. Some preliminary results can be found in [20]. During the simulations it was observed that ρ_{crit} was around $25\ veh/km$, which resulted in the setting of $\rho_{des} = 22\ veh/km$.

In the simulation the freeway section has two inflows. First, q_0 is the uncontrolled inflow from the previous highway section. Second, the traffic system has one controlled ramp with inflow u. The controlled gate is located at the beginning of the freeway section. Moreover, the vehicles can leave the freeway section on an outflow ramp s_1 and they can also transfer to the next freeway section with the flow q_1. During the simulation the ratio of the autonomous vehicles κ continuously varies. In the traffic model the freeway section is handled as one segment, thus $i \equiv 1$.

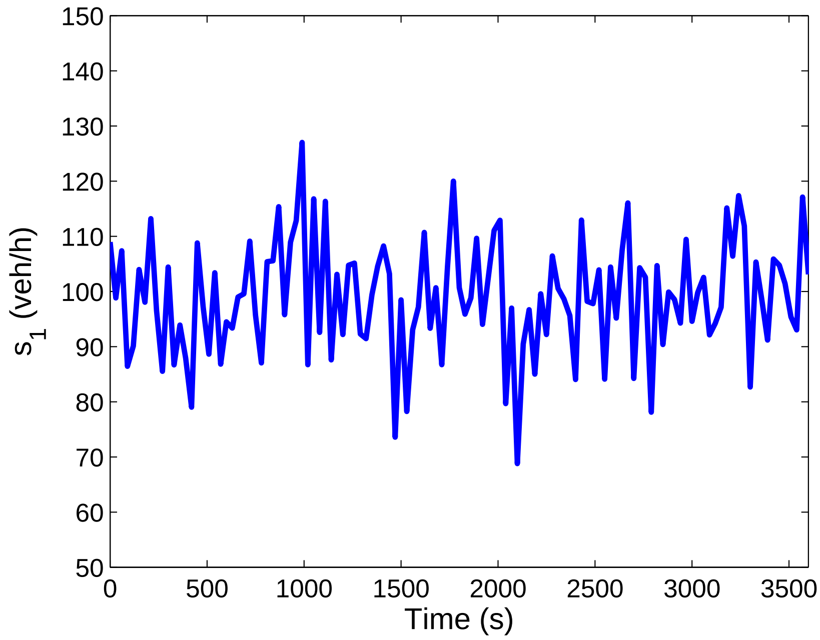
Figure 3 illustrates the disturbances of the system, which are q_0 and s_1. These signals cannot be influenced through the designed control, but the role of the control strategy is to guarantee the maximum outflow and reduce the impact of disturbances on it. It can be seen that the current q_0 oscillates around $1800\ veh/h$, which can result in a high value for ρ_1 together with r_1, see e.g. at $t = 1000\ s$ $q_0 = 2100\ veh/km$, which yields $\rho_1 = 28\ veh/km$. Thus, it is necessary to limit the inflow of the vehicles on the inflow ramp. Moreover, s_1 has a small value, which means that most of the vehicles along the freeway section are driven.

The ratio of the autonomous vehicles in the heterogeneous traffic is shown in Figure 4(a). During the simulation it varies between $10\% \ldots 40\%$, which is a significant variation. Moreover, the resulting density ρ_1 can be seen in Figure 4(b). The results show that the required $\rho_{crit} = 25\ veh/km$ is tracked by the controlled system with low error, which provides maximum outflow.

Finally, the control input and its components are illustrated in Figure 5. The control input u_{st} has a higher value, with which the constraint on r_1 is guaranteed. Moreover, u_{dyn} guarantees low error in the tracking. The efficiency of the control can be illustrated at time $t = 1000\ s$. In this case the freeway section has a high load on q_0, which can lead to a congestion. Thus, the control input u is significantly reduced with components u_{st} and u_{dyn}, see Figure 5. Throughout simulation the overall control input $u = u_{st} + u_{dyn}$ has



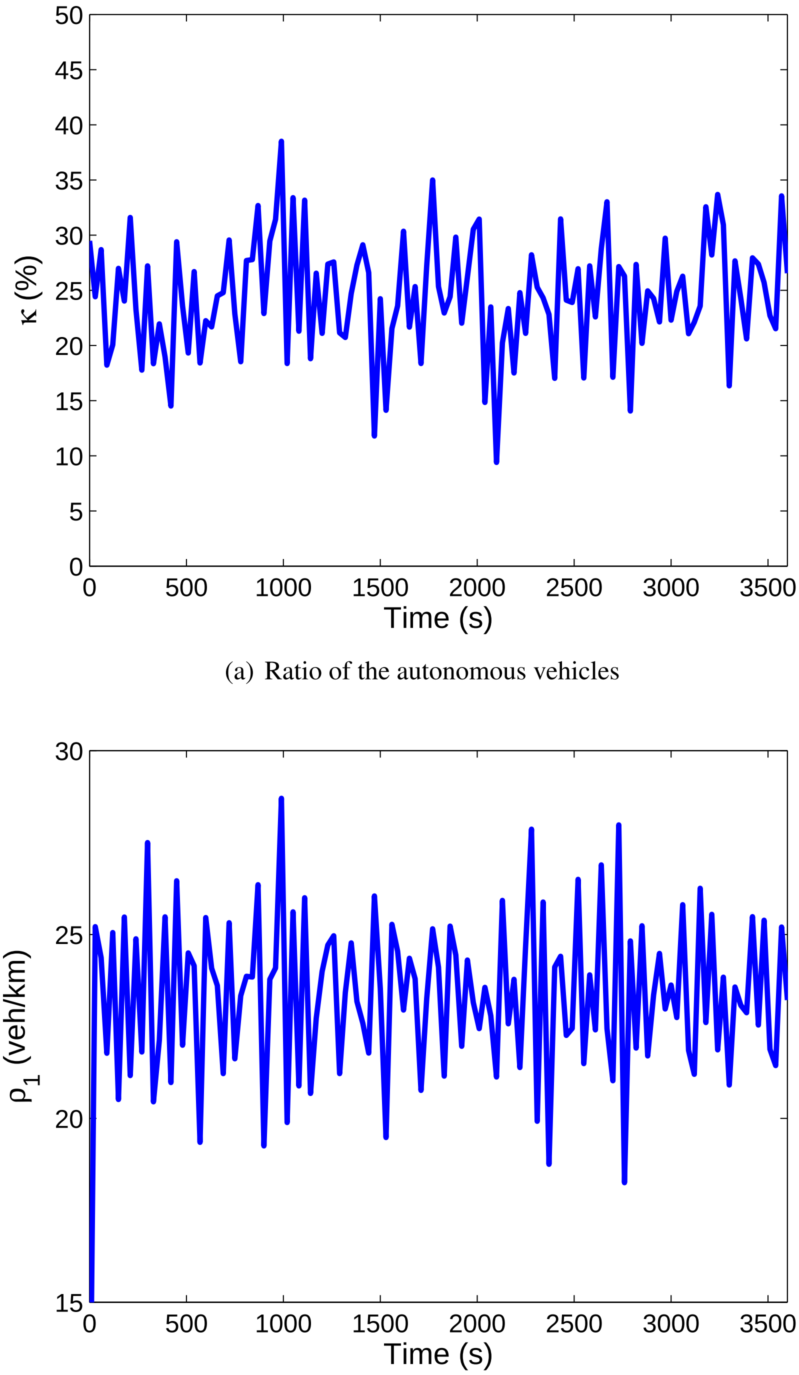
(a) Uncontrolled inflow on the freeway



(b) Outflow on the ramps

Figure 3: Disturbances in the simulation

a value between $0 \ldots 2450\ veh/h$, whose mean is $1100\ veh/h$. As a result the mean of the entire inflow of the section $q_0 + r_1$ is $2900\ veh/h$. In spite of the high inflow and the varying κ performance is guaranteed, which proves that the LPV-based control strategy is suitable for the solution of the traffic control problem.



(a) Ratio of the autonomous vehicles

(b) Traffic density on the section

Figure 4: Simulation results

5 Conclusions

The paper has presented a control strategy for the optimization of heterogeneous traffic flow, which contains conventional human-driven and autonomous vehicles. It has been derived

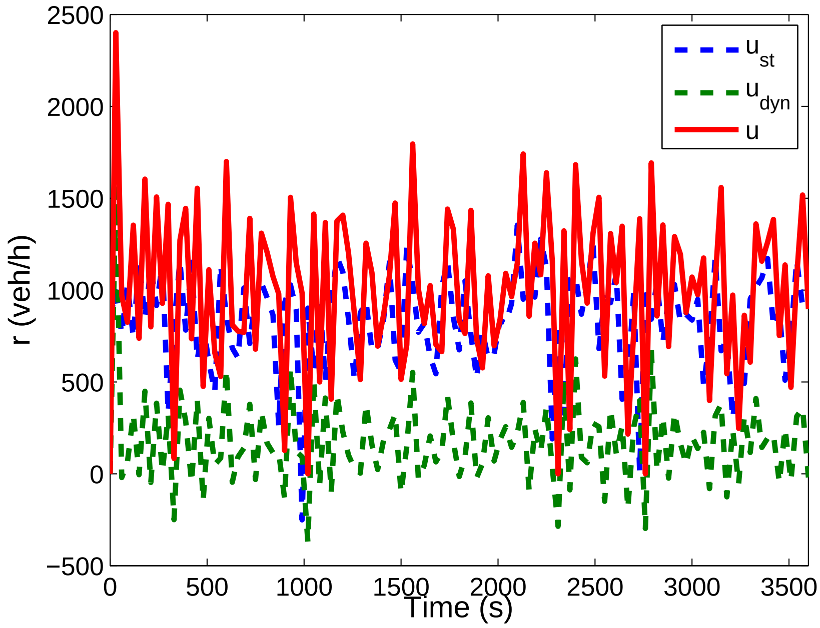


Figure 5: Control input in the simulation

from a LPV model for the heterogeneous traffic flow, in which the ratio of the autonomous vehicles is incorporated in a scheduling variable. The robust control design is based on a maximization criterion, which incorporates LMI conditions. Moreover, the control strategy handles the constraints on the control input. The simulation example has illustrated that the proposed robust LPV control is able to guarantee the performance specification of the system, which results in the required traffic density.

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