Effect of Surface Roughness on the Behavior of a Magnetic Fluid-based Squeeze Film between Circular Plates with Porous Matrix of Variable Thickness

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Abstract: An endeavor has been made to investigate the performance of a magnetic fluid based squeeze film between rough circular plates while the upper plate has a porous facing of variable porous matrix thickness. The bearing surfaces are transversely rough. The random roughness of the bearing surfaces is characterized by a random variable with non-zero mean, variance and skewness. The associated Reynolds' equation is stochastically averaged with respect to the random roughness parameter and is solved with appropriate boundary conditions. The results for bearing performance characteristics such as pressure, load carrying capacity and response time for different values of mean, standard deviation and measure of symmetry are numerically computed and presented graphically. First of all it is observed that these performance characteristics increase with the increasing magnetization parameter thereby, suggesting that the performance of the bearing with magnetic fluid lubricant is better than that with the conventional lubricant. It is noticed that the bearing suffers owing to transverse surface roughness. However, the negatively skewed roughness tends to increase the load carrying capacity. Moreover, there is a very significant observation that with a proper selection of the thickness ratio parameter a magnetic fluid based squeeze film bearing with variable porous matrix thickness in the case of negatively skewed roughness can be made to perform considerably better than that of a conventional porous bearing with an uniform porous matrix thickness working with a conventional lubricant. In addition, this article makes it clear that by choosing properly the thickness ratio parameter and the strength of the magnetic field, the adverse impact induced by roughness on the bearing system can be minimized in the case of negatively

*skewed roughness especially when negative variance occurs. Thus, this study makes it
mandatory to account for roughness while designing the bearing system.*

*Keywords: magnetic fluid; squeeze film; transverse roughness; Reynolds' equation;
variable film thickness; load carrying capacity*

Nomenclature:

a	Radius of the circular plate (mm)
h	Uniform film thickness (mm)
H	Porous wall thickness (mm)
$\overline{M}$	Oblique magnetic field (Gauss)
p	Pressure distribution (N/mm^2)
P	The pressure in the porous matrix (N/mm^2)
w	Load carrying capacity (N)
k	As defined in [6] (Thickness ratio parameter)
μ	Dynamic viscosity of the lubricant (N.s/mm^2)
ϕ	Permeability of the porous matrix (Col^{2}kgm/s^2)
θ	Inclination of the magnetic field (Radians)
σ	Standard deviation (mm)
α	Variance (mm)
ε	Skewness (mm^3)
ΔT	Dimensionless squeeze time
μ_0	Permeability of the free space (N/A^2)
H_0	Porous wall thickness at r = 0 (mm)
H_1	Porous wall thickness at r = a (mm)
$\overline{p}$	Dimensionless pressure
W	Load carrying capacity in dimensionless form
$\overline{\mu}$	Magnetic susceptibility (mm^3/kg)
μ^*	Magnetization parameter
$\boldsymbol{\sigma}$	Standard deviation in dimensionless form
$\boldsymbol{\alpha}$	Variance in dimensionless form
$\boldsymbol{\varepsilon}$	Skewness in dimensionless form

1   Introduction

The squeeze film behavior between non porous plates was analyzed by Archibald
(1956). Of course, the porous bearings have been used for a quite long time, the
theoretical analysis of such bearings have various advantages over conventional
non-porous bearings. For instance, porous bearings can run hydrodynamically for
a longer time without maintenance and are more stable than the equivalent
conventional bearings. Murti (1974) considered the squeeze film behavior
between two circular disks, when one of them has a porous facing press fitted into
a solid wall. The analysis of this investigation was much simplified by Prakash
and Vij (1973) by incorporating the Morgan Cameron approximation, when the

porous facing thickness is assumed small. It is well known that the introduction of a sintered porous bush in the bearing results in loss of mechanical strength and reduction of film pressure and, consequently, in load carrying capacity.

All the above analyses investigating the porous bearings, considered the wall thickness to be uniform. However, in reality the porous facing may not be of uniform thickness due to various manufacturing reasons; for instance, the non uniform application of pressure while sintering may lead to a non uniform thickness of porous bush. But the non uniformity of the thickness of porous bush gives rise to an additional degree of freedom for its design. The degree of freedom is nothing but the choice of the value of the porous thickness ratio parameter for the bearing. Therefore, the design consideration of the bearing may also dictate the choice of non uniform wall thickness of the porous housing. The effect of the variable thickness of the porous matrix on the performance of a squeeze film behavior between porous circular plates was analyzed by Prajapati (1995). The above investigations considered conventional lubricant. Verma (1986) and Agrawal (1986) dealt with the application of magnetic fluid as a lubricant. They observed that the bearing system registered an improved performance. Subsequently, Bhat and Deheri (1993) studied the squeeze film between porous annular disks using a magnetic fluid lubricant with the external magnetic field oblique to the lower disk. Also, Bhat and Deheri (1991, 1993) investigated the performance of the magnetic fluid based squeeze film behavior in curved porous circular disks. Prajapati (1995) studied the effect of axial current pinch on squeeze film between circular plates with lubricant inertia adopting momentum integral method. Patel, Deheri and Vadher (2010) discussed the performance of a magnetic fluid based short bearing and established the importance of magnetic fluid lubricant for extending the life period of the bearing system. By now, it is well known that the bearing surfaces particularly, after having some run-in and wear, develop roughness. In order to study and analyze the effect of surface roughness on the performance of squeeze film bearings, various methods have been employed. Several investigators have proposed a stochastic approach to mathematically model the random character of the roughness (Tzeng and Saibel (1967), Christensen and Tonder (1969a, 1969b 1970)). Christensen and Tonder (1969a, 1969b 1970) presented a comprehensive general analysis for both transverse as well as longitudinal surface roughness based on a general probability density function by developing and modifying the approach of Tzeng and Saibel (1967). Subsequently, this approach of Christensen and Tonder (1969a, 1969b, 1970) laid down the basis for investigating the effect of surface roughness in a number of investigations (Ting (1975), Prakash and Tiwari (1983), Prajapati (1992), Guha (1993), Gupta and Deheri (1996)). Andharia, Gupta and Deheri (1997, 1999) discussed the effect of surface roughness on the performance of a squeeze film bearing using a general stochastic analysis [without making use of a specific probability distribution] for describing the random roughness.

Deheri, Patel and Patel (2006) dealt with the behavior of magnetic fluid-based squeeze film between porous circular plates with a porous matrix of variable thickness. Here a significant observation was made that with the proper selection of thickness ratio parameter, a magnetic fluid-based squeeze film bearing with variable porous matrix can be made to perform considerably better than that of a conventional porous bearing with a uniform porous matrix thickness working with a conventional lubricant. It has been sought to discuss the effect of transverse surface roughness on the geometry and configurations of Deheri, Patel and Patel (2006).

2   Analysis

The configuration of the bearing system is presented below, which considers the laminar axisymmetric flow of an incompressible fluid between two parallel circular plates of radius a. The lower one is fixed while the upper plate has a porous facing of variable porous matrix thickness backed by a solid wall.
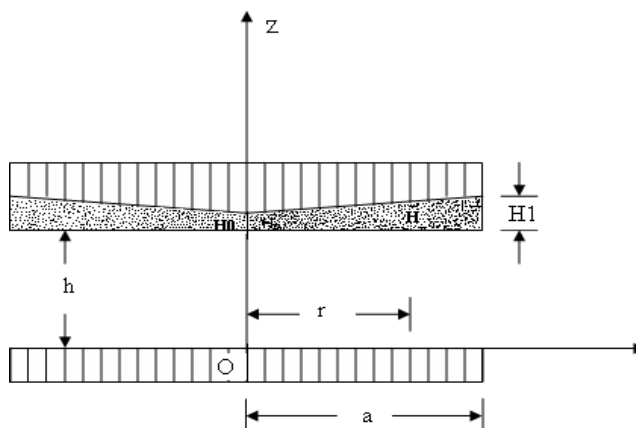
Figure 1
Configuration of the bearing system

The upper plate approaches the lower one normally with velocity $\dot{h} = \dfrac{dh}{dt}$; where h is the uniform film thickness. The porous wall thickness is assumed to vary linearly with its value at r = 0 as H_0 while this value is H_1 at r = a. Consequently, the porous wall thickness H is given by

$$H = H_0 + (H_1 - H_0)\left(\frac{r}{a}\right) \tag{1}$$

The Z – axis is taken normal to the lubricant film.

The bearing surfaces are assumed to be transversely rough. Following the discussions of Christensen and Tonder (1969a, 1969b, 1970) regarding the stochastic modeling of roughness the actual film thickness is given by

$$h = \bar{h} + h_s(\xi)$$

where $\bar{h}$ is the mean film thickness and is given by (Pinkus and Sternlicht (1961))

$$\bar{h} = C_r(1 - E_c \cos\gamma)$$

wherein, C_r is the radial clearance, E_c is the eccentricity ratio e / C_r and e the eccentricity of the bearing. h_s measured from the nominal mean level of the bearing surface and ξ is the random variable characterizing the roughness of the bearing surfaces. In general h_s may not be symmetric in nature. h_s is considered to be stochastic in nature and governed by the probability density function $f(h_s)$, $-c \leq h_s \leq c$ where c is the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε which is the measure of symmetry of the random variable h_s are defined by the relationships

$$\alpha = E(h_s),$$

$$\sigma^2 = E[(h_s - \alpha)^2]$$

and

$$\varepsilon = E[(h_s - \alpha)^3]$$

where E denotes the expected value defined by

$$E(R) = \int_{-c}^{c} R f(h_s) dh_s$$

while,

$$f(h_s) = \begin{cases} \dfrac{35}{32c^7}(c^2 - h_s^2)^3, & \text{if} - c \leq h_s \leq c \\ 0, & \text{elsewhere} \end{cases}$$

Assuming axially symmetric flow of the magnetic fluid between the plates under an oblique magnetic field $\bar{M} = [M(r)\cos\theta, 0, M(r)\sin\theta]$ whose magnitude vanishes at $r = 0$ and $r = a$, following the analysis of Prajapati (1995) the concerned Reynolds' equation governing the film pressure is obtained as [c.f. Bhat and Deheri (1993)].

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rg(h)\frac{d}{dr}(p-0.5\mu_0\bar{\mu}M^2)\right)=12\mu\left[\dot{h}-\frac{\phi}{\mu}\left(\frac{\partial}{\partial z}(p-0.5\mu_0\bar{\mu}M^2)\right)_{z=h}\right] \quad (2)$$

where

$$M^2 = r(a-r),$$

and

$$g(h) = h^3 + 3\sigma^2 h + 3h^2\alpha + 3h\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon$$

μ is the fluid viscosity, $\bar{\mu}$ represents the magnetic susceptibility, μ_0 stands for the permeability of the free space and ϕ is the permeability of the porous housing and $p(r, z)$ is the pressure in the porous matrix governed by the Laplace's equation [c.f. Prajapati (1995), Bhat and Deheri (1993) Prakash and Vij (1973.b)] which assumes the following form in view of the Appendix - B

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(p-0.5\mu_0\bar{\mu}M^2)\right)+\frac{\partial^2}{\partial z^2}(p-0.5\mu_0\bar{\mu}M^2)=0 \quad (3)$$

The associated boundary conditions for solving equations 2 and 3 are

$$p = 0 \text{ at } r = 0, a$$

$$P = 0 \text{ at } r = 0, a$$

$$p = P \text{ at } z = h$$

and

$$\frac{\partial p}{\partial z} = 0 \text{ at } z = h + H \quad (4)$$

The physical meaning and the related significance of these boundary conditions are explained in Prajapati (1994). The system 2-4 appears to be coupled both in terms of differential equation as well as boundary conditions. To obtain its solution, we first uncouple the system by making use of the simplifying assumption that H may be considered so small that a Taylor's series representations may be used. This results in the uncoupled modified Reynolds equation as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}(p-0.5\mu_0\bar{\mu}M^2)\right)=\frac{12\mu\dot{h}}{(h^3 + 3\sigma^2 h + 3h^2\alpha + 3h\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon + 12\phi H)}$$

$$(5)$$

Introducing the dimensionless quantities

$$\mu^* = -\frac{\mu_O \, \overline{\mu} \overline{h}^3}{\mu \dot{h}}, \quad \overline{r} = \frac{r}{a}, \quad \overline{p} = -\frac{h^3 p}{\mu \dot{h} a^2}, \quad \overline{H} = \frac{H}{H_0} = 1 + k\overline{r}, \quad \psi = \frac{\psi_0}{\overline{h}^3},$$

$$\psi_0 = \frac{\phi H_0}{h_0^3}, \quad \overline{h} = \frac{h}{h_0}, \quad \sigma = \frac{\sigma}{h} \quad \alpha = \frac{\alpha}{h} \quad \varepsilon = \frac{\varepsilon}{h^3} \tag{6}$$

where

$$k = \frac{H_1}{H_0} - 1$$

and h_0 is the initial film thickness; one obtains the differential equation for the pressure in dimensionless form

$$\frac{1}{\overline{r}} \frac{d}{d\overline{r}} \left(\overline{r} \frac{d}{d\overline{r}} (\overline{p} - 0.5\mu^* \overline{r}(1 - \overline{r})) \right) = \frac{-12}{G(h) + 12\psi(k\overline{r} + 1)} \tag{7}$$

where

$$G(h) = 1 + 3\sigma^2 + 3\alpha + 3\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon$$

Owing to the boundary conditions

$$\frac{d\overline{p}}{d\overline{r}} = 0 \text{ at } \overline{r} = 0$$

and

$$\overline{p} = 0 \text{ at } \overline{r} = 1$$

the solution of equation (7) is given by

$$\overline{p} = 0.5 \, \mu^* \overline{r}(1 - \overline{r}) - \frac{1}{\psi k} \int_1^{\overline{r}} \left[1 - \frac{\ln(b\overline{r} + 1)}{b\overline{r}} \right] d\overline{r} \tag{8}$$

where

$$b = \frac{12\psi k}{G(h) + 12\psi}$$

Next, the load carrying capacity of the bearing

$$w = 2\pi \int_0^a rp(r)dr$$

in dimensionless form is found to be

$$W = -\frac{wh^3}{\mu \dot{h} a^4} = \frac{\pi \mu^*}{12} + \frac{\pi}{\psi k}\left[\frac{1}{3} - \frac{1}{2b}\left\{\ln(b+1)\left(1-\frac{1}{b^2}\right) - \frac{1}{2} + \frac{1}{b}\right\}\right] \tag{9}$$

If the time taken for the plate to move from the film thickness $h = h_0$ to $h = h_1$ then the non dimensional squeeze time ΔT is derived from equation (9) as

$$\Delta T = \frac{wh_0^2}{\mu a^4}\int_0^{t_1} dt = -\int_1^{\bar{h}0} \frac{\bar{W}}{\bar{h}^3}d\bar{h} \tag{10}$$

3 Results and Discussion

It is observed from equations 8, 9 and 10 that the dimensionless pressure, load carrying capacity and squeeze time depend on the magnetization parameter μ^*, roughness parameters σ, α, ϵ and the thickness ratio parameter k. The results for uniform porous matrix thickness are obtained by taking thickness ratio parameter k to be zero (Appendix A: Case – 2). When ψ is assumed to be zero, the results for the conventional non porous magnetic fluid-based squeeze film bearings are obtained (Appendix A: Case – 1). Furthermore, the corresponding non magnetic case is obtained by setting μ^* to be zero. From equation 9 it is evident that the load carrying capacity increases by $\pi\mu^*/12$.

The variation of load carrying capacity with respect to magnetization parameter μ^* for different values of porosity ψ, roughness parameters σ, α, ϵ and the thickness ratio k is presented in Figures 2-6 respectively. From these figures it is noticed that the performance of the present bearing system is relatively better than that with a conventional lubricant as the load carrying capacity increases with increasing magnetization parameter μ^*. The load carrying capacity decreases with increasing values of ψ (Figure 2) and also with increasing values of σ, α, ϵ (Figures 3-5). It can be easily observed from Figure 6 that the load carrying capacity decreases with the increasing values of the thickness ratio parameter k.
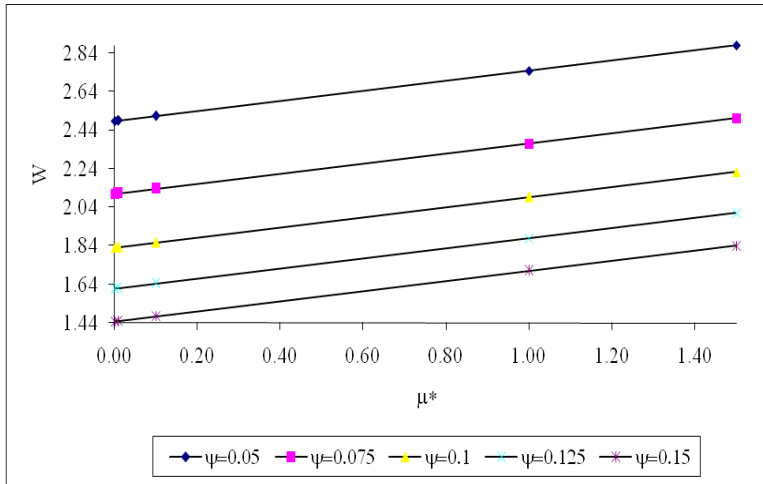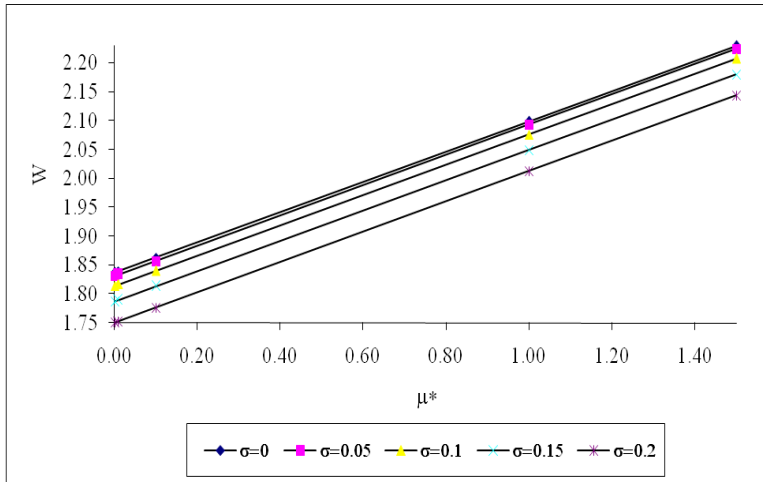
Figure 2

Variation of load carrying capacity with respect to μ* and ψ

Figure 3

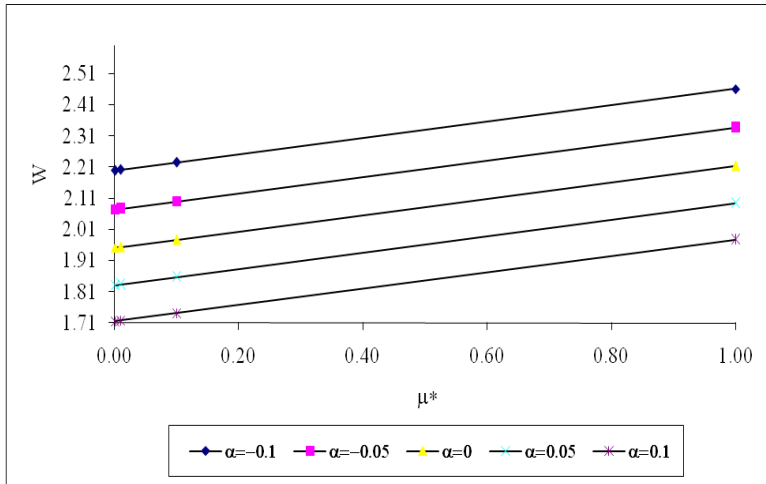Variation of load carrying capacity with respect to μ* and σ

Figure 4

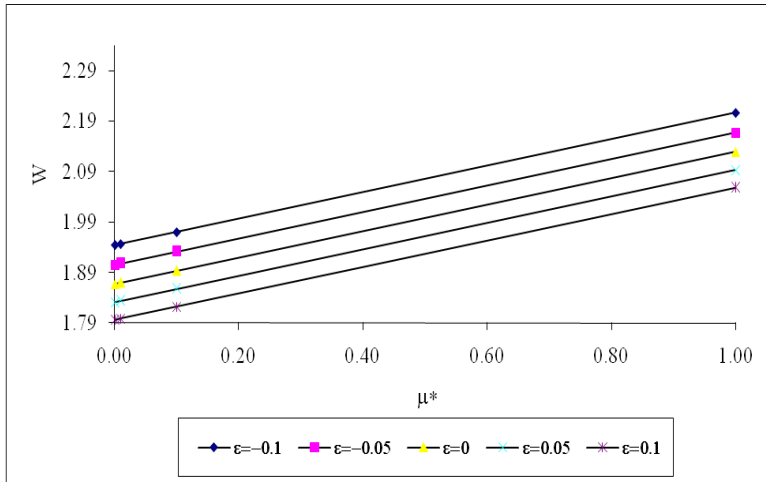Variation of load carrying capacity with respect to μ* and α

Figure 5

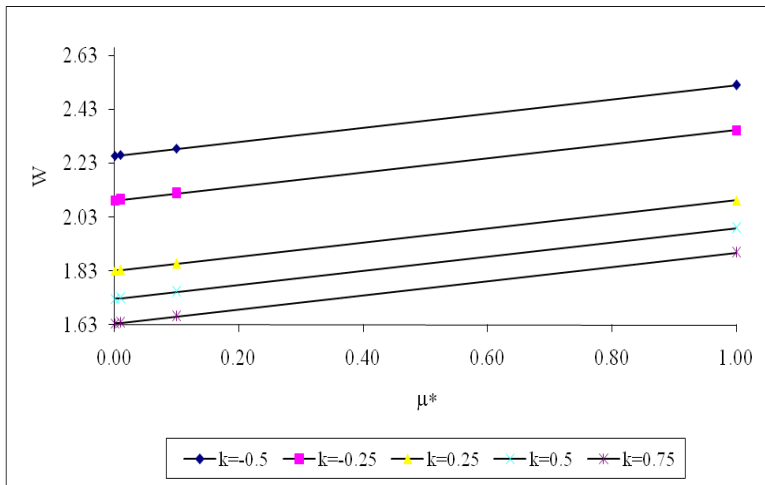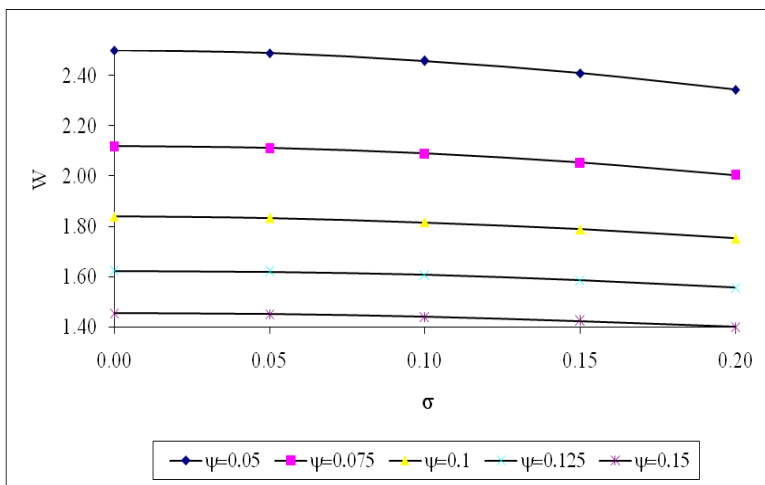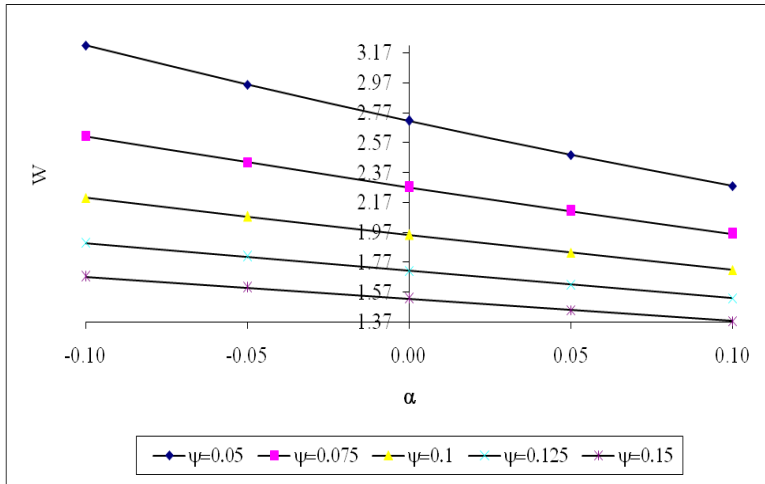Variation of load carrying capacity with respect to μ* and ε

Figure 6

Variation of load carrying capacity with respect to μ^* and k

We have the distribution of load carrying capacity with respect to ψ for various values of σ, α, ε and the thickness ratio k presented in Figures 7-10. It is observed that the load carrying capacity increases with decreasing values of the roughness parameters. The decrease in the load carrying capacity due to porosity ψ and skewness ε (+ve) gets further decreased by the variance α (+ve). Here, it is noted that the role of positive ε in decreasing the load carrying capacity is equally sharp.

Figure 7

Variation of load carrying capacity with respect to ψ and σ

Figure 8

Variation of load carrying capacity with respect to ψ and **α**
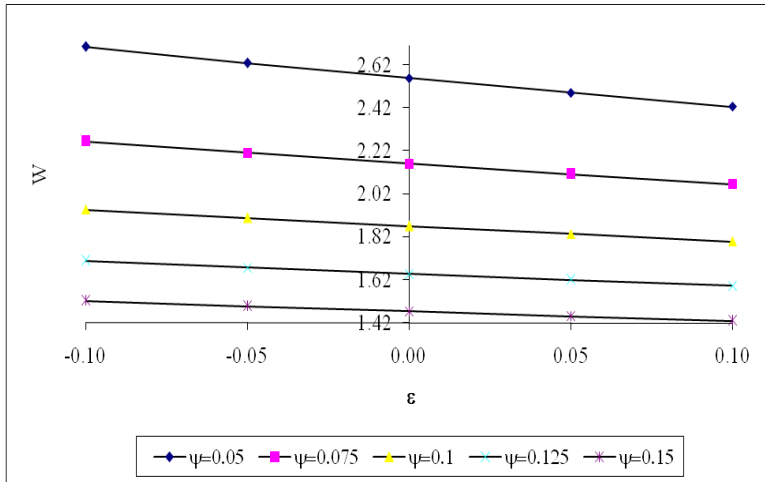

Figure 9

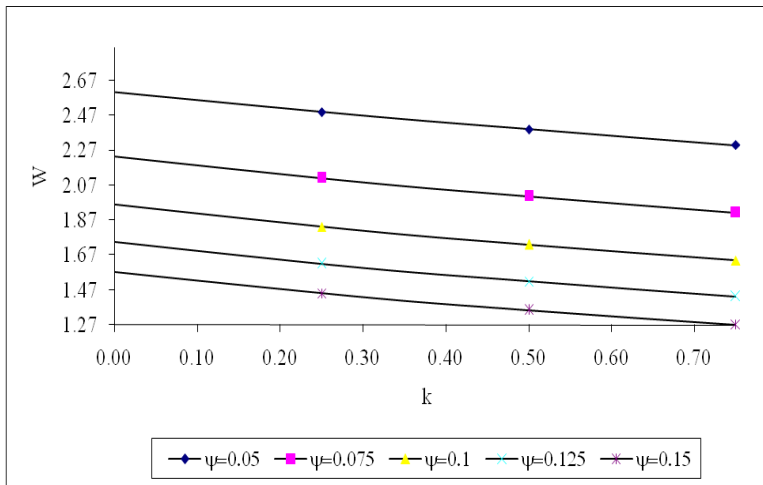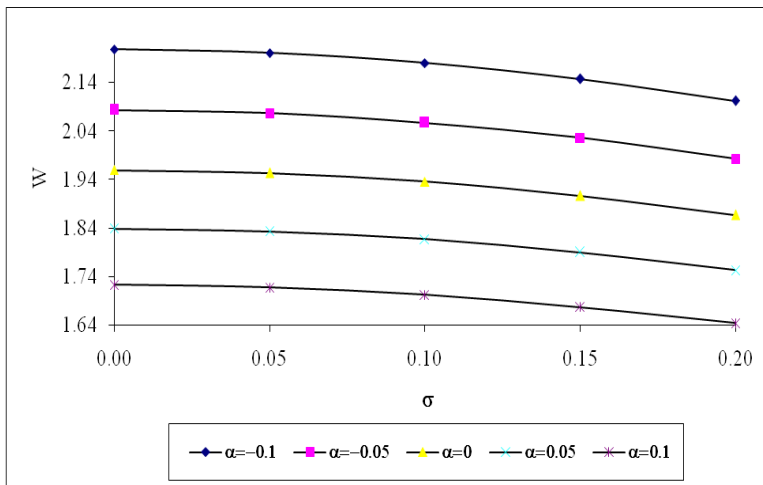Variation of load carrying capacity with respect to ψ and **ε**

Figure 10

Variation of load carrying capacity with respect to ψ and k

The distribution of load carrying capacity with respect to σ for different values of α, ε and the thickness ratio k is given in Figures 11-13. From these figures it is seen that the load carrying capacity decreases significantly with increasing values of α, ε and k wherein even the thickness ratio plays a crucial role in the reduction of load carrying capacity.

Figure 11
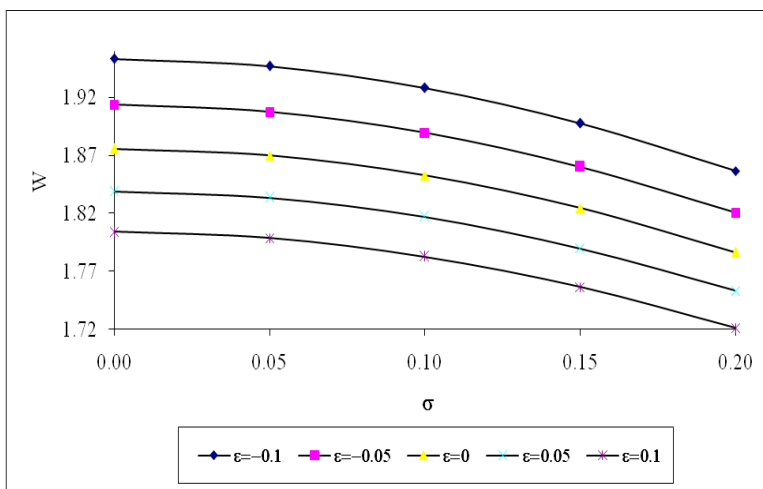
Variation of load carrying capacity with respect to σ and α

Figure 12
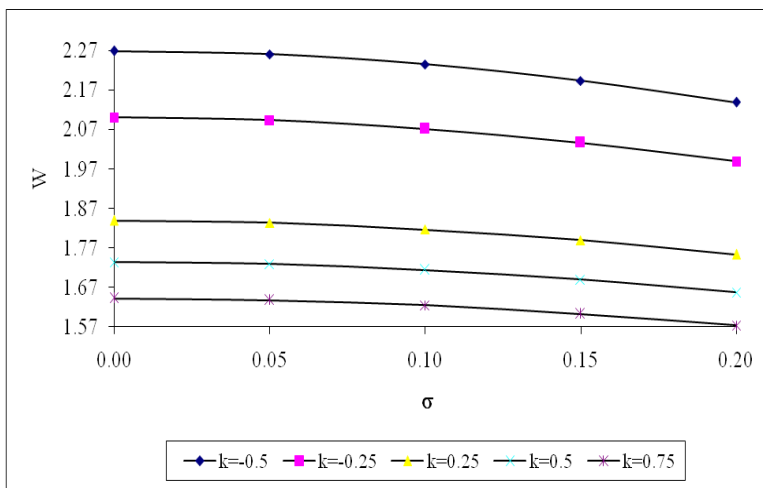Variation of load carrying capacity with respect to σ and ε

Figure 13
Variation of load carrying capacity with respect to σ and k

Lastly, the combined effect of ε and α on the distribution of load carrying capacity is presented in Figures 14-16. It is noticed that negatively skewed roughness tends to enhance the performance of the bearing system which, in turn, gets further increased when negative variance is involved.

It is clearly seen from equation 10 that the squeeze time almost follows the trends of the load carrying capacity.

A close look at some of these figures reveals that by suitably choosing the thickness ratio parameter k, the porosity ψ and standard deviation σ induced negative effect can be neutralized substantially by the positive effect introduced by the magnetization parameter μ^* in the case of negatively skewed roughness.
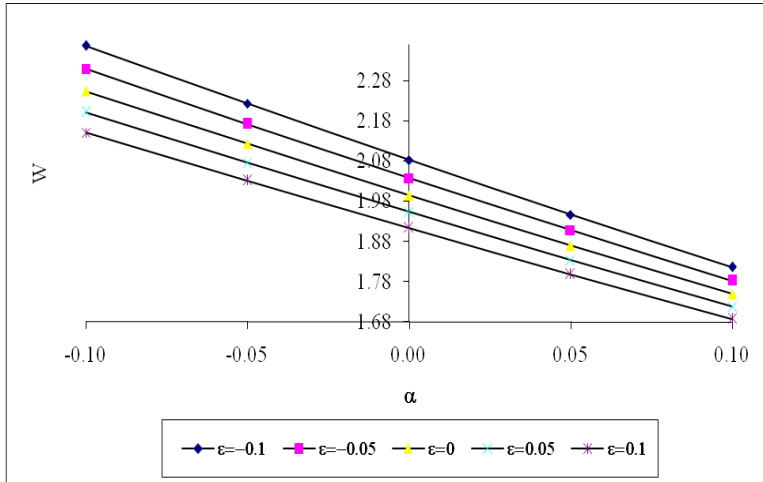

Figure 14
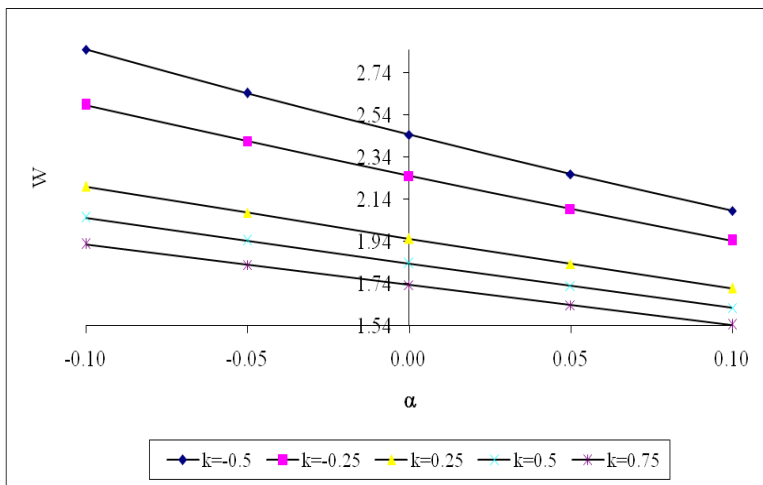Variation of load carrying capacity with respect to α and ε

Figure 15
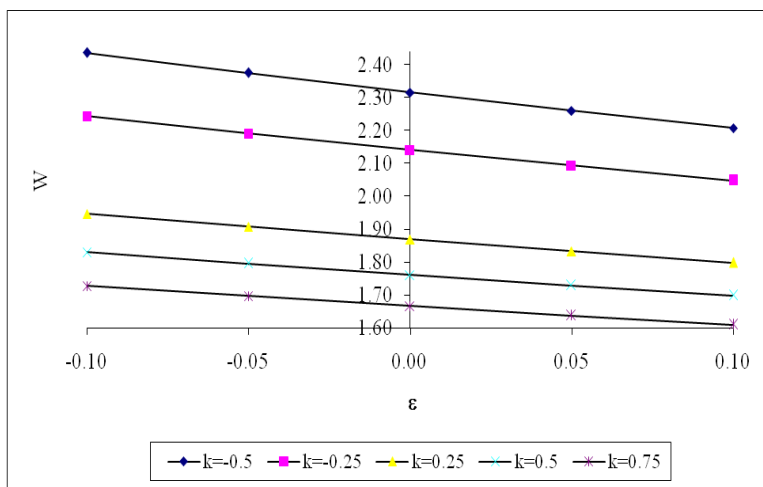Variation of load carrying capacity with respect to α and k

Figure 16
Variation of load carrying capacity with respect to ε and k

Conclusions

It is found that the bearing system can support a load even when there is no flow. In addition, a very significant observation is that, with a proper selection of the thickness ratio, a magnetic fluid based squeeze film bearing with variable porous matrix thickness can be made to perform considerably better than a porous bearing working with a conventional lubricant with a uniform porous matrix thickness. Although, the thickness ratio parameter provides an additional degree of freedom from a design point of view, the roughness must be duly accounted for while designing the bearing system from the bearing's life period point of view.

Acknowledgements

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Appendix: A

Case: 1 When $\psi = 0$ the concerned Reynolds' equation becomes

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d}{d\bar{r}}(\bar{p}-0.5\mu^*\bar{r}(1-\bar{r}))\right) = -\frac{12}{G(h)}$$

Solution of this equation with the boundary conditions

$$\frac{d\bar{p}}{d\bar{r}} = 0 \text{ at } \bar{r} = 0$$

and

$$\bar{p} = 0 \text{ at } \bar{r} = 1$$

yields the pressure in non-dimensional form as

$$\bar{p} = 0.5\,\mu^*\bar{r}(1-\bar{r}) + \frac{3(1-\bar{r}^2)}{G(h)}$$

Notice that the pressure is independent of k.

Then the load carrying capacity of the bearing

$$w = 2\pi \int_0^a rp(r)dr$$

in dimensionless form is obtained as

$$W = -\frac{wh^3}{\mu \dot{h}a^4} = \pi\left[\frac{\mu *}{12} + \frac{3}{2(G(h))}\right]$$

Case: 2 When k = 0 the associated Reynolds' equation turns out to be

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\frac{d}{d\bar{r}}(\bar{p} - 0.5\mu * \bar{r}(1 - \bar{r}))\right) = \frac{-12}{G(h) + 12\psi}$$

Solution of this equation with the boundary conditions

$$\frac{d\bar{p}}{d\bar{r}} = 0 \text{ at } \bar{r} = 0$$

and

$$\bar{p} = 0 \text{ at } \bar{r} = 1$$

turns in the pressure in non-dimensional form as

$$\bar{p} = 0.5 \, \mu * \bar{r}(1 - \bar{r}) + \frac{3(1 - \bar{r}^2)}{G(h) + 12\psi}$$

Then the load carrying capacity of the bearing

$$w = 2\pi \int_0^a rp(r)dr$$

in dimensionless form is calculated as

$$W = -\frac{wh^3}{\mu \dot{h}a^4} = \pi\left[\frac{\mu *}{12} + \frac{3}{2(G(h) + 12\psi2)}\right]$$

From these two cases it is also, observed that for $\psi < 0.0416$ a squeeze film with uniform film thickness performs better than that of a non porous squeeze film with variable film thickness.

Appendix: B

Neuringer and Rosenweig (1964) proposed a simple model to describe the steady flow of magnetic fluid in then presence of slowly changing external magnetic fields. The model consists of the following equations:

$$\rho\left(\overline{q}\bullet\nabla\right)\overline{q}=-\nabla p+\eta\nabla^2\overline{q}+\mu_0\left(\overline{M}\bullet\nabla\right)\overline{H} \tag{A_1}$$

$$\nabla\bullet\overline{q}=0 \tag{A_2}$$

$$\nabla\times\overline{H}=0 \tag{A_3}$$

$$\overline{M}=\overline{\mu}\overline{H} \tag{A_4}$$

$$\nabla\bullet\left(\overline{H}+\overline{M}\right)=0 \tag{A_5}$$

where ρ is the fluid density, $\overline{q}=\left(u,v,w\right)$ is the fluid velocity in the film region, p is the film pressure, η is the fluid viscosity, μ_0 is the permeability of free space, $\overline{M}$ is the magnetization vector, $\overline{H}$ is the external magnetic field and $\overline{\mu}$ is the magnetic susceptibility of the magnetic particles.

Using equations (A_3) – (A_4), equation (A_1) becomes

$$\rho\left(\overline{q}\bullet\nabla\right)\overline{q}=-\nabla\left(p-\frac{\mu_0\overline{\mu}}{2}H^2\right)+\eta\nabla^2\overline{q}$$

This shows that an extra pressure $\frac{1}{2}\mu_0\overline{\mu}H^2$ is introduced into the Navier-Stokes equations when magnetic field is used as lubricant. Thus, the modified Reynolds equation in this case is obtained as

$$\frac{\partial}{\partial x}\left[h^3\frac{\partial}{\partial x}\left(p-\frac{1}{2}\mu_0\overline{\mu}H^2\right)\right]+\frac{\partial}{\partial y}\left[h^3\frac{\partial}{\partial y}\left(p-\frac{1}{2}\mu_0\overline{\mu}H^2\right)\right]=6\eta U\frac{\partial h}{\partial x}+12\eta W_h \tag{A_6}$$

But the modified Reynolds equation for non-porous bearings in cylindrical polar co-ordinates is derived by Bhat (2003) is expressed as

$$\frac{1}{r}\frac{d}{dr}\left(rh^3\frac{dp}{dr}\right)=12\eta\overset{\bullet}{h}_0$$

Thus, the modified Reynolds equation associated with magnetic fluid in view of (A_6) turns out to be

$$\frac{1}{r}\frac{d}{dr}\left[rh^3\frac{d}{dr}\left(p-\frac{\mu_0\overline{\mu}H^2}{2}\right)\right]=12\eta\overset{\bullet}{h}_0$$