Priority Decision Rules with a Fuzzy MCDM Approach for Solving Flexible Job Shop Problem: A Real Case Study of Optimizing Manufacturing

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Abstract: The Flexible Job Shop Problem (FJSP) represents a significant challenge in the field of planning due to its complexity and the constant need for problem-solving in the market. It should be emphasized that FJSP is one of the most difficult NP problems in combinatorial optimization. One of the key factors in maintaining the competitiveness of small and medium enterprises in the market is increasing productivity while minimizing costs and manufacturing time. This paper proposes a multi-criteria approach, whose main role is job prioritization and FJSP optimization using a metaheuristic algorithm in a specific case study of a furniture manufacturing company. To determine the weight coefficients, two methods, the fuzzy Analytic Hierarchy Process (FAHP) and the fuzzy Full Consistency Method (FFUCOM), were integrated, while the fuzzy Weighted Aggregated Sum Product Assessment (FWASPAS) method was used for job ranking. As the next step in the study, the NSGA II algorithm was applied to optimize FJSP. Based on the conducted case study and production optimization, experimental results demonstrated the success of the proposed methodology and improved the organization of production resources after job prioritization.

Keywords: priority decision; fuzzy MCDM; NSGA II algorithm; flexible job shop problem; real case study; manufacturing optimization

1 Introduction

Resource planning and scheduling (PIRP) as a basic term represents a daily activity and plays an important role in the life of every individual, and is as such essential to all fields of research. *PIRP* is one of the primary stages of system management where fundamental planning goals, management strategies, and work methodologies are established to ensure the efficiency of the system. The PIRP stage achieves the initial criteria in the form of company objectives. Resource planning consists of several phases: short-term planning, medium-term planning, and long-term planning [1]. During the phases of *PIRP*, the company assesses its resources and determines key and prioritized activities that can impact the implementation of projects [2]. The activity plan represents the organization and management of activities based on which the future state and behavior of the system are predicted soon. To approach strategic resource planning and establish the company's primary goals, it is necessary to evaluate activities in terms of market demand and potential opportunities that can result in company profit [1]. During the creation of activity schedules, project leaders examine when the planned portion of the project needs to achieve specific objectives, by creating key control points for such trials. Resource planning and scheduling are two interdependent decision making processes in the planning world that are used daily in production activities and industries. Within the general *PIRP* process, one of the key challenges is the flexible scheduling of tasks. Flexible Job Shop Problem (FJSP) is one of the models used in the *PIRP* domain. In this study, *FJSP* was chosen due to its applicability to the observed production, which led to improvements in productivity and resource efficiency. The specific conditions in the production system require a high-level of flexibility and efficiency, which is precisely what *FJSP* provides. Therefore, *FJSP* offers a solution that not only meets the company's needs but also improves productivity and rational resource utilization.

When organizing production resources, the process of arranging production activities is divided into two fundamental activities: the process of planning production resources and the process of achieving planning goals through resource scheduling in production [2]. The primary goal of *FJSP* is to increase productivity while reducing the required time to achieve the main production objectives within the objective function. The concept of supporting *PIRP* processes in large companies involves the use of software planners and information systems for activity management and resource planning PIRP aims to enhance the organization of all business activities. Implementing information systems in the planning process is a crucial step for any enterprise, with the main objective being to improve business operations and enhance the overall system. Today, in the world of planning, the survival of companies in the market requires continuous progress and the implementation of various software solutions. The process of improving production services, product innovation, and enhancing organization and employee conditions are just some of the factors that contribute to enhancing business activities and advancing the company. Information systems are present in nearly all major companies, commonly referred to as Enterprise Resource Planning systems (ERP) [3]. Based on various case studies in the scientific literature, the implementation and utilization of *ERP* systems in Small and Medium Enterprises (*SME*) face various obstacles due to high maintenance costs, implementation costs, and the limited flexibility of the software in accommodating production processes [4], [5], [6]. When it comes to *SME*, different programming techniques are commonly used in practice to increase productivity [7], [8], [9], [10], [11]. Based on a review of the literature and quantitative and qualitative analysis of scientific papers, the research in this study is based on the concept of providing adequate support for resource management in *SME.* The main research motivation is rooted in the belief that there is still room for improvement in production practices today. In the observed company, which is engaged in furniture manufacturing, the main challenge in their production system is the lack of a structured and optimized job scheduling process. The current *PIRP* process largely depends on the workers' experience, where task priorities are determined using traditional methods. In this study, the *FJSP* model was introduced in combination with Fuzzy Multiple Criteria Decision Making Methods (FMCDM) to prioritize tasks and optimize the production schedule. The main idea of the study is to provide support for *SME* and improve the overall *PIRP* system by applying various programming techniques. To solve the *FJSP* problem in the observed company, a new scientifically grounded planning system is proposed. The implementation of fuzzy numbers within the framework of *FMCDM* methods, combined with metaheuristics, poses a significant challenge in the field of planning. Defining criteria and alternatives (jobs) is also a crucial prerequisite for an optimal sequential job schedule and for defining input data for production optimization. The proposed multicriteria methods used in the study for job ranking and prioritization include Fuzzy Analytic Hierarchy Process (FAHP), Fuzzy Full Consistency Method (FFUCOM), and Fuzzy Weighted Aggregated Sum Product Assessment (FWASPAS) [12], [13], [14], [15], [16], [17]. Analyzing the production state and defining criteria based on the analysis of direct and indirect external influences in production, along with job prioritization and the combination of artificial intelligence and implementation of the organization into the planning model, represent a new approach, a significant challenge, and an innovation in the study. The academic contribution of this paper lies in the innovative integration of *MCDM* techniques with the *FJSP* model to address *PIRP* problems in *SMEs*. While *FJSP* has been studied extensively in larger production systems, its application in *SMEs*, particularly in combination with decision making techniques for job prioritization, has not been explored. Our approach offers a novel solution for optimizing production processes in *SMEs*, as demonstrated in the case study, where significant improvements in organization and productivity were achieved compared to existing methods. The following sections of the paper present the work through several key components, as will be seen below. In the introduction section of the paper, Section 1 presents the main objective of the study and the primary motivation behind the development of the idea. Section 2 showcases the mathematical formulation and integration of two independent methods, fuzzy *MCDM* and *NSGA II* algorithm, through several key stages in solving the *FJSP*. Section 3 describes the applied methods and proposes a fuzzy approach using *MCDM* methods for job prioritization. Section 4 presents a case study, providing an overview of all input parameters, robust job ranking, and the optimization of the *FJSP* problem. The final part of the paper, presents the experimental research results and the benefits for the company when using the proposed methodology.

2 Mathematical Formulation and Description of FJSP

When discussing the flexible job scheduling model, it is important to note that the literature mentions two types within this model: partial and complete flexibility [18]. Complete flexibility refers to the availability of all machines to perform operations within a job, while partial flexibility implies restrictions on certain machines for job execution, which is the case in this paper. The main objective of the proposed model is to assign priorities to each job based on ranked alternatives (jobs) using fuzzy *MCDM* methods and to optimize the *FJSP* using the *NSGA II* algorithm [19], [20], [21]. The problem of flexible job scheduling involves a set of jobs, where each job represents a finished product.. Job priority and methods for determining job priorities play an important role in within a production system. Therefore, this work presents a new scientifically grounded approach to determining job priorities. The task is to schedule n jobs $J = \{J_1, J_2, ..., J_n\}$ where each job has a sequential order of operations $O = (O_{1,j}, O_{2,j}, ... O_{ij})$, and the set of all operations forms a complete product. It is necessary to schedule all operations on a set of machines $M = \{M_1, M_2, ... ,M_k\}$ to achieve maximum productivity in minimal time based on the objective function C_{max} [22], [23]. Additionally, within the mathematical model and input characteristics, it is necessary to define the input parameters for optimization [24], [25]. After ranking the jobs using the *fuzzy MCDM* method, the next step is assigning priorities to the jobs as input parameters for optimization. The job priority is defined by ranking the jobs and assigning weights $W= \{W_1, W_2, ..., W_n\}$ to each job individually based on their ranking, as shown in Table 3. Figure 1 depicts the graphical diagram of *FJSP*.

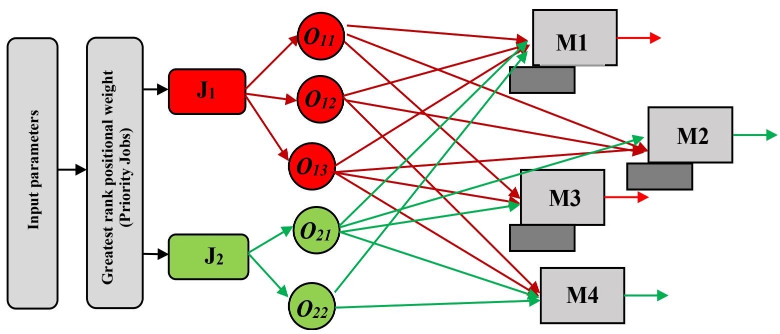

Figure 1
FJSP description and greatest rank positional weight with priority jobs in optimization

Accordingly, the following sections of the paper provide a detailed explanation of the processes involved in applying and integrating these methods into a comprehensive planning system, with an emphasis on their interconnection and impact on improving the efficiency of the production process.

3 Methodology: Integration of FMCDM and NSGA II

The integration of the *NSGA II* algorithm and fuzzy *MCDM* methods for determining job priorities aims to improve the current state of production based on input parameters and criteria that directly or indirectly influence the objective function. Additionally, the stability and organization of all resources in the production system represent a significant responsibility when scheduling jobs on machines. In the realm of planning, one of the fundamental reasons for successful operations is the organization of business activities within the planning system. In the continuation, Figure 2 provides a graphical representation of the concept and implementation of the algorithms used to solve the *FJSP*, along with the various phases of the approach as part of the job planning model in the observed company.

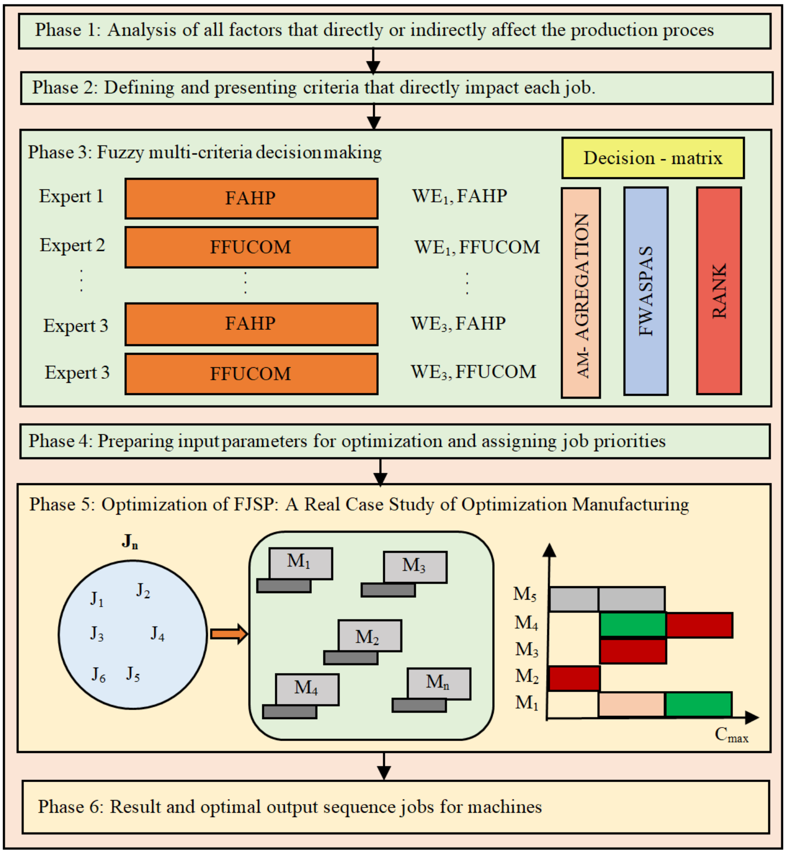

Figure 2

Phase and fuzzy MCDM methods for jobs priority weight and FJSP optimization

Phase 1: In the first phase, a detailed analysis of the entire production process is conducted, starting from factors that directly or indirectly affect the production process. This initial phase is one of the key stages in problem formulation and the creation of a list of activities that result in a detailed plan and understanding of all activities during the production process. Data collection and database formation are carried out to create a risk list. Based on the generated risk list, preventive and corrective actions are taken to eliminate or mitigate any external factors that impact the production process.

Phase 2: In the second phase, a systematic approach is presented, focusing on defining criteria that directly influence each job individually. The main goal in defining these criteria is to analyze and understand the characteristics of each job. After analysis, it is determined that each job possesses certain characteristics that differentiate it in various ways. This means that each job has a different execution time on a particular machine, a different number of operations, a different deadline for completion or start, varying levels of flexibility and suitability for different machines. These are just some of the job characteristics that directly impact the prioritization process. These criteria are: C_1: Processing Times, C_2: Number of operations, C_3: Due Date, C_4: Setup time, C_5: Release dates, C_6: Flexibility, C_7: Recirculation. The importance of these criteria should be emphasized as a key factor in determining job priorities. By selecting these key criteria, each job can be ranked independently of others.

Phase 3: In the third phase, a multi-criteria analysis using fuzzy *MCDM* methods is applied. The main characteristic of this phase is the ranking of criteria and obtaining the arithmetic mean of the weighted coefficients using the methods described in the paper, namely *FAHP*, and *FFUCOM*. In the continuation of phase 3, the goal is to use the obtained results and rank the jobs using the *FWASPAS* method.

Phase 4: The main characteristic of this phase is the formulation of a mathematical model within *FJSP* and the preparation of input parameters for optimization. Additionally, the job priorities are assigned based on the results obtained in phase 3.

Phase 5: is one of the most critical phases in this work, characterized by the implementation of the formulated mathematical model on a specific example, considering all constraints, to ensure the proper functioning of the production process. The *PIRP* within *FJSP* represents one of the most challenging models in combinatorial optimization. Formulating initial conditions and parameters within the *NSGA II* algorithm is just one step in this phase. Additionally, another step in this phase involves creating input files, as illustrated in Table 1, which is directly linked to the observed problem and optimization process. The formation of the mathematical model and the constraints of the mathematical model for job execution on machines, based on the results obtained from phases 3 and 4, will be further examined in Section 3. The goal of phase 5 is to implement the previous phases and the obtained results, based on fuzzy *MCDM* methods, into the *PIRP* process within *FJSP*.

Phase 6*:* The final step of this methodology is the presentation of the obtained results based on the formulated mathematical model and all the preceding phases that build upon each other. The outcome is the optimization of production processes and the representation of optimal output sequences, either in the form of job scheduling on machines or in the graphical form through Gantt charts. The output of the optimization provides maximum productivity in the value of the objective function C_{max}, as well as the sequential order of job execution on machines with assigned priorities.

By applying the *fuzzy MCDM* methodology for determining job priorities and the *NSGA II* algorithm, companies can expect significant benefits in terms of meeting predefined deadlines, ensuring accurate delivery times to customers, increasing productivity, and ultimately increasing company profits.

3.1 Fuzzy Multiple Criteria Decision Making Methods

Fuzzy logic is a mathematical theory used to model uncertainty. In conventional logic, statements are typically either true or false, while fuzzy logic allows for statements that can be partially true [26]. Fuzzy logic is often employed in multi-criteria decision making, where multiple factors are considered in decision making [27], [28]. These fuzzy sets are then combined with fuzzy logic to make a decision that takes into account all the factors and their interrelationships [29]. The theory of fuzzy sets was proposed by Zadeh [30] in an attempt to generalize the understanding of sets. The idea was to enable the handling of uncertain ty as a computational framework for systems involving human language, behavior, emotions, and decision making. In conventional set theory, the membership of elements in a set is based on two-valued Boolean logic [12]. The foundation of fuzzy logic is the concept of a "fuzzy set". Instead of elements belonging or not belonging to a set, as in classical logic, fuzzy sets allow elements to have a degree of membership between 0 and 1. In classical mathematics, numbers are typically considered precise and accurate. For example, the number 5 represents a precise value that is either true or false. However, in the real world, we often encounter situations where we do not have complete or exact information but only partial or uncertain data [12].

Fuzzy numbers consist of three components: a central value, a lower bound, and an upper bound. The central value represents the numerical value around which the values are clustered. The lower bound represents the minimum value the number can have, while the upper bound represents the maximum value. Values between the lower and upper bounds represent the degree of membership to the number. Triangular fuzzy numbers (TFNs) are commonly used in multi-criteria decision making [16], [31], [32]. Triangular fuzzy numbers (r_1, r_2, r_3) represent a range of possible values for a specific parameter or variable. The value r_2 is considered the most probable or central value, while r_1 and r_3 represent the earliest and latest bounds, as can be seen in Figure 3. The triangular shape of membership reflects the uncertainty or vagueness associated with that variable.

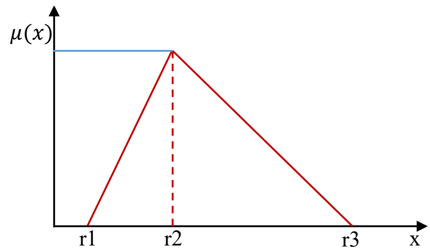

Figure 3
The membership function of the triangular fuzzy number

The basic algebraic operations for two alternatives *TFNs* A_1 (r_1, r_2, r_3) i A_2 (r_1, r_2, r_3), are presented in detail in the work of Petrović et al. [12]. The triangular shape of the membership function reflects the uncertainty or vagueness associated with that variable. The formula for the membership function for *x* is as follows:

$$\mu(x) = \begin{cases} 0, & x \le r1 \\ \frac{x-r1}{r2-r1}, & r1 < x \le r2 \\ \frac{r3-x}{r3-r2}, & r2 < x < r3 \\ 0, & x \ge r3 \end{cases} \tag{1}$$

The fundamental step in implementing fuzzy numbers in all *FMCDM* methods is the fuzzy matrix, which is represented in the following form:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ldots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} = \begin{bmatrix} (x_{11}^{r1} x_{11}^{r2} x_{11}^{r3}) & \cdots & (x_{1n}^{r1} x_{1n}^{r2} x_{1n}^{r3}) \\ \vdots & \ldots & \vdots \\ (x_{m1}^{r1} x_{m1}^{r2} x_{m1}^{r3}) & \cdots & (x_{mn}^{r1} x_{mn}^{r2} x_{mn}^{r3}) \end{bmatrix} \tag{2}$$

In this expression, *m* represents the number of alternative solutions, and n represents the number of evaluation criteria, reflecting the aggregated performance of each alternative i concerning criterion *j*. The values are assigned to each alternative based on the recommendations provided in Table 1, [12], [28].

Table 1
Input parameters of the FJSP problem

Rank	Triangular fuzzy number	Attribute grade
Very Low	(0.00, 0.00, 0.25)	1
Low	(0.00, 0.25, 0.50)	2
Medium	(0.25, 0.50, 0.75)	3
High	(0.50, 0.75, 1.00)	4
Very High	(0.75, 1.00, 1.00)	5

The subsequent sections of the paper provide a brief description of the FMCDM (FAHP, FFUCOM, FWASPAS) methods that were used. In this study, we use the *FAHP* method and *FFUCOM* method to obtain criterion weights based on expert ratings. Both methods provide a structured approach to decision-making in a fuzzy environment but differ in their characteristics and approaches. In the second part of the study, we employ the *FWASPAS* method for ranking alternatives. The *FWASPAS* method combines criterion weights and alternative ratings to obtain a final ranking of alternatives. This method involves determining the criterion weights (obtained through the *FAHP* or *FFUCOM* method) and evaluating alternatives based on those criteria. The *FWASPAS* method use criterion weight vectors and alternative rating matrices to calculate overall scores for each alternative. The alternatives are then ranked based on these overall scores. The *FWASPAS* method is suitable for situations where alternative values are unclear and exhibit different fuzzy characteristics. By using these methods, the study provides a structured approach to decision making in a fuzzy environment.

3.2 Fuzzy Analytic Hierarchy Process (FAHP)

FAHP is an extension of the classical *AHP* method that incorporates fuzzy logic to handle subjective and uncertain information. *AHP* is a decision making method that involves structuring a complex problem into a hierarchical model and comparing the relative importance of its elements [29], [32]. In *FAHP*, linguistic expressions or fuzzy numbers are used to express decisionmakers assessments regarding pairwise comparisons of criteria and alternatives in the hierarchy. Fuzzy numbers enable the representation of imprecise or vague information, allowing decision-makers to express their preferences more flexibly. *FAHP* combines the principles of *AHP* with the theory of fuzzy sets, aiming to deal with the inherent ambiguity and uncertainty in decision making processes [15], [20].

3.3 Fuzzy Full Consistency Method (FFUCOM)

The *FFUCOM* method belongs to the group of newer methods and was first proposed by Pamučar et al. [17], [31]. In this study, the *FFUCOM* method is used in conjunction with the *FAHP* method to determine the criterion weights. The working principle of the *FFUCOM* method is based on comparing the criteria during result verification and measuring the deviation from maximum consistency. According to Pamučar [32], the basic characteristics of this method are: The method allows for pairwise comparison of evaluation criteria not only using whole numbers but also decimal values, enabling a finer granularity of ratings, it employs a basic algorithm for determining criterion weights and a smaller number of comparisons is sufficient for selecting criterion weights, further facilitating the use of this method. *FFUCOM* enables decision-makers to deal with ambiguity and uncertainty in the decision making process.

3.4 Fuzzy Weighted Aggregated Sum Product Assessment (FWASPAS)

The general characteristic of the *WASPAS* method is that it enables the evaluation and ranking of alternatives with a high-level of reliability. The *FWASPAS* method was first proposed by Zavadskas et al. [15]. It is important to note that the application of the *FWASPAS* method is flexible and can be adapted to different decision making situations. The *FWASPAS* method combines *WSM* (Weighted Sum Model) and WPM (Weighted Product Model) by using a common criterion of optimality. This criterion can be determined based on a linear combination of *WSM* and *WPM* criteria. *WSM* involves the calculation of the weighted sum of criterion results (attribute values) for each alternative, while *WPM* involves the calculation of the weighted product of criterion results for each alternative. The combination of these two methods in the *FWASPAS* method aims to capture the advantages of both methods [33], [34], [35].

3.5 NSGA II algorithm for Solving the FJSP

To solve the *PIRP* problem based on the presented mathematical model, a metaheuristic approach was applied, using the *NSGA-II* algorithm specifically for addressing the *FJSP* problem [36], [37]. During the formation of the *n* possible solutions based on the population size, the solutions are categorized depending on their position within the solution space. The formed boundaries of possible solutions within the Pareto front determine the ranks of the solutions based on their population position. The formation of rank 1, rank 2, and rank n is achieved by sorting the population of parents and offspring based on the increasing level of the objective function or nondomination. The job priorities are determined based on the positions of the possible solutions and the formation of ranks within the Pareto front [38]. The main characteristic of the presented algorithm is the generation of offspring using an enhanced version of well-known crossover and mutation operators. Afterward, it aims to generate or select the next generation based on nondominated sorting and distance comparison. The overall distance value of possible solutions is calculated as the sum of the distances of each individual based on the objective function [18], [39], [40], [41], [42].

4 Real Case Study of Optimizing Manufacturing

The case study focuses on a furniture manufacturing company that specializes in producing custom-made furniture and veneer chairs. In this case study, the *FJSP* model is applied to address the production planning and scheduling challenges. It is important to highlight that in today's manufacturing world, there are often jobs that have a higher priority compared to others. Job prioritization plays a crucial role in ensuring that the production of a specific item is completed on time. When considering job priorities, the specific approach involves assigning weight coefficients to each job individually and ranking the jobs accordingly based on their priority. The company consists of machines that can perform various types of jobs within a predetermined time frame. The company manufactures different types of chairs, tables, and various types of furniture. Each job during production consists of a different number of operations. In the observed company, the job priority is determined and assigned manually based on the workers' experience. The idea is to apply a scientific approach to determine job priorities and optimize the sequential job scheduling on a set of machines. By applying fuzzy *MCDM* methodology to determine job priorities and achieve proper sequential job scheduling on a set of machines using the *NSGA II* algorithm, significant benefits are expected for the company in terms of job efficiency within specified deadlines. The guarantee of on-time product delivery to other companies when placing orders, increased productivity, and higher company profits are just some of the advantages of the presented model. In Figure 4, the basic concept of the scientific idea behind the observed case study can be seen through a specific example within the company.

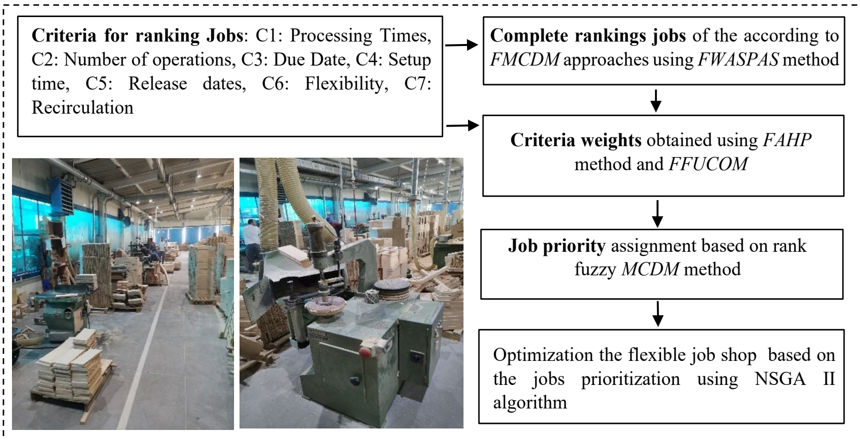

Figure 4

Concept and Schematic Representation of the Implemented Methodology

The company consists of several different facilities, one of which is a production plant for furniture manufacturing. The company uses its raw material for producing veneer, which serves as the foundation for further furniture development and production. The main objective is to implement the proposed methods into the production concept. In collaboration with the team responsible for the production plant in the furniture manufacturing company, all factors that directly and indirectly impact the production facility were examined in the first phase. In the subsequent step, a dedicated team of managers and engineers was formed to carry out the implementation of the proposed methodology. The main task of the mentioned meeting was to establish a database and define key criteria that are both characteristics of each job and directly impact the production process. Through a detailed production analysis, several key criteria were identified: C_1: Processing Times, C_2: Number of operations, C_3: Due Date, C_4: Setup time, C_5: Release dates, C_6: Flexibility, C_7: Recirculation. Based on the collected data and the implemented procedure for database formation and criteria selection, a normalized matrix representing the characteristics of each job individually was created. The normalized matrix is presented using fuzzy numbers and is displayed in Table 2.

Table 2
Job characteristic ratings-decision matrix

C_n	C1 [times]	C2 [-]	C3 [-]	…	C5 [-]	C6 [-]	C7 [-]
J_n	min	min	min	…	min	min	max
J_1	(14, 16.5, 18.6)	8	(2, 8, 10)	…	(2, 4, 7)	(0.25, 0.5, 0.75)	(0, 0, 0.25)
J_2	(16, 18.5, 20)	8	(7, 15, 19)	…	(7, 8, 9)	(0.5, 0.75, 1.0)	(0, 0.25, 0.5)
J_3	(7.5, 8.5, 9.6)	4	(9, 15, 23)	…	(9, 12, 13)	(0.25, 0.5, 0.75)	(0, 0.25, 0.5)
J_4	(6.5, 7.5, 8.7)	5	(3, 12, 16)	…	(13, 14, 15)	(0.5, 0.75, 1.0)	(0.25,0.5, 0.75)
J_5	(16, 17.5, 21.5)	9	(15, 16, 18)	…	(15, 16, 18)	(0.5, 0.75, 1.0)	(0.25, 0.5,0.75)
J_6	(7.5, 8.5, 9)	6	(15, 17, 25)	…	(1, 2, 4)	(0.75, 1.0, 1.0)	(0.5, 0.75, 1.0)
J_7	(3.5, 6.5, 7.8)	3	(17, 21, 29)	…	(1, 3, 5)	(0.75, 1.0, 1.0)	(0.5, 0.75, 1.0)
J_8	(6.5, 7.5, 8.6)	3	(18, 22, 28)	…	(3, 5, 10)	(0.25, 0.5, 0.75)	(0, 0.25, 0.5)
J_9	(7.5, 8.5, 9.6)	3	(19, 23, 29)	…	(10, 13, 15)	(0, 0.25, 0.5)	(0, 0, 0.25)
J_{10}	(15.5, 17.5, 19)	8	(22, 25, 32)	…	(15, 16, 17)	(0.25, 0.5, 0.75)	(0.25, 0.5,0.75)
J_{11}	(12.5, 13.5, 15)	7	(23, 25, 33)	…	(17, 19, 20)	(0, 0.25, 0.5)	(0.25, 0.5,0.75)
J_{12}	(18.5, 19.5, 22)	9	(26, 27, 37)	…	(20, 21, 27)	(0.75, 1.0, 1.0)	(0.25, 0.5,0.75)

To estimate and calculate the fuzzy weights in Table 3, the data from the normalized matrix in Table 2 were used as input parameters. Table 2 presents the relationship between criteria C_n and jobs J_n as characteristics of each job. By assessing the opinions of three experts and using *FAHP* and *FFUCOM* methods, the weights of fuzzy numbers W_l, W_m and W_u were obtained. The resulting values are presented in Table 3.

Table 3
Criteria weights of considered criteria

			C_1	C_2	C_3	C_4	C_5	C_6	C_7
FAHP	Expert 1	W_l	0.045	0.006	0.554	0.000	0.395	0.000	0.000
		W_m	0.045	0.006	0.554	0.000	0.395	0.000	0.000
		W_u	0.045	0.006	0.554	0.000	0.395	0.000	0.000
	Expert 2	W_l	0.181	0.046	0.436	0.000	0.337	0.000	0.000
		W_m	0.181	0.046	0.436	0.000	0.337	0.000	0.000
		W_u	0.181	0.046	0.436	0.000	0.337	0.000	0.000
	Expert 3	W_l	0.208	0.131	0.390	0.000	0.271	0.000	0.000
		W_m	0.208	0.131	0.390	0.000	0.271	0.000	0.000

		W_u	0.208	0.131	0.390	0.000	0.271	0.000	0.000
FFUCOM	Expert 1	W_l	0.087	0.047	0.175	0.058	0.113	0.070	0.064
		W_m	0.150	0.121	0.322	0.122	0.143	0.121	0.098
		W_u	0.150	0.121	0.322	0.122	0.143	0.126	0.098
	Expert 2	W_l	0.057	0.035	0.133	0.054	0.151	0.070	0.060
		W_m	0.129	0.102	0.273	0.115	0.261	0.101	0.101
		W_u	0.129	0.104	0.273	0.115	0.261	0.130	0.101
	Expert 3	W_l	0.057	0.035	0.133	0.054	0.151	0.070	0.060
		W_m	0.129	0.102	0.273	0.115	0.261	0.101	0.101
		W_u	0.129	0.104	0.273	0.115	0.261	0.130	0.101
Arithmetic mean	Sum	W_l	0.106	0.050	0.303	0.028	0.236	0.035	0.031
		W_m	0.140	0.085	0.375	0.059	0.278	0.054	0.050
		W_u	0.140	0.085	0.375	0.059	0.278	0.064	0.050

Based on the obtained results and the calculation of fuzzy criteria weights for each method separately, as well as the assessment of the expert team, the arithmetic mean of the results from the two fuzzy methods was formed by summing them. The results can be seen in Table 3. In the next step, the *FWASPAS* method was used for job ranking, using the data from Table 3. The results of the ranked jobs are presented in Table 4.

Table 4

Complete rankings of the alternatives according to different criteria weights (Fuzzy WASPAS)

J_n	1	2	3	4	5	6	7	8	9	10	11	12
K_i	0.287	0.192	0.195	0.274	0.163	0.347	0.369	0.218	0.131	0.141	0.119	0.131
Rang	**3**	**7**	**6**	**4**	**8**	**2**	**1**	**5**	**10**	**9**	**12**	**11**

The results of job ranking can be seen in Table 4, which serves as input data for Phase 4 and the preparatory phase of the optimization process and job prioritization based on the conducted multi-criteria decision analysis and job ranking. The next phase involves the assignment of priorities to jobs according to the ranked jobs from Table 5. Job priorities are assigned as weight coefficients to each job and are part of the input parameters for the optimization process, as described in Section 2. Table 5 presents the input parameters of the previously obtained results, as well as the results of the measured processing times for each operation individually. The total execution times of operations on the machines were obtained by measuring them over multiple intervals and recording the average value from the measurement intervals for each operation individually.

The symbol "-" in Table 5 indicates that the operation cannot be processed on that machine. For example, job J_2 with operation O_{21} cannot be executed on machine M_3. Based on the input parameters from Table 5 and *FJSP* optimization, optimization results were obtained in the sequential form. The results are first presented graphically in Figure 5 and then tabular form in Tabele 6, providing a clearer insight into the performance of the proposed method.

Table 5
Priority Jobs and input parameters of the FJSP problem

Priority Jobs	Jobs	Operations	Processing times							
			M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
3	J_1	O_{11}	4	2	3	4	7	-	8	-
		O_{12}	4	-	1	-	4	8	-	7
		...	…	…	…	…	…	…	…	…
		O_{17}	-	7	-	2	1	4	-	3
		O_{18}	8	-	11	12	-	6	8	7
7	J_2	O_{21}	-	5	-	6	-	2	5	8
		O_{22}	6	-	4	8	9	-	3	4
		O_{23}	-	5	4	6	7	6	-	7
		...	…	…	…	…	…	…	…	…
		O_{28}	8	-	7	10	9	11	-	7
6	J_3	O_{31}	5	-	4	6	-	4	5	6
		O_{32}	-	2	8	-	4	5	3	5
		O_{33}	3	-	5	4	5	6	5	-
		O_{34}	8	-	11	-	-	7	-	6
4	J_4	O_{41}	8	7	6	9	-	-	7	6
		O_{42}	7	-	5	-	4	8	9	-
		...	…	…	…	…	…	…	…	…
		O_{45}	9	-	12	-	6	8	-	6
8	J_5	O_{51}	6	8	-	5	7	-	8	6
		O_{52}	-	13	11	15	-	12	9	15
		...	…	…	…	…	…	…	…	…
		O_{58}	18	16	-	16	18	16	-	13
		O_{59}	9	-	6	7	-	8	11	8
2	J_6	O_{61}	8	7	-	9	-	8	7	-
		O_{62}	-	5	-	4	5	-	4	8
		O_{63}	6	-	4	5	7	6	-	6
		...	…	…	…	…	…	…	…	…
		O_{66}	-	8	-	8	-	7	6	5
1	J_7	O_{71}	8	-	6	7	5	-	4	5
		O_{72}	-	3	4	-	4	6	-	4
		O_{73}	3	5	-	4	6	-	4	-
5	J_8	O_{81}	-	4	-	6	8	-	6	8
		O_{82}	5	-	6	5	7	8	-	2
		O_{83}	6	-	-	8	6	4	8	-
10	J_9	O_{91}	-	9	5	-	4	-	6	8
		O_{92}	3	-	-	6	-	2	-	4
		O_{93}	-	9	7	-	5	-	8	-
9	J_{10}	O_{101}	10	-	14	-	13	-	-	14
		O_{102}	-	12	13	12	-	15	15	17
		...	…	…	…	…	…	…	…	…
		O_{107}	8	6	-	8	4	5	6	6

		O_{108}	-	6	3	-	9	7	-	6
		O_{111}	8	11	12	-	14	-	15	-
		O_{112}	-	18	12	13	-	17	12	14
12	J_{11}	...	…	…	…	…	…	…	…	…
		O_{116}	5	-	8	9	-	5	-	6
		O_{117}	-	6	4	-	7	6	5	-
		O_{121}	3	-	4	5	3	-	6	-
		O_{122}	5	8	7	-	7	6	-	7
11	J_{12}	...	…	…	…	…	…	…	…	…
		O_{128}	-	5	6	-	5	-	-	8
		O_{129}	6	-	8	6	-	7	5	8

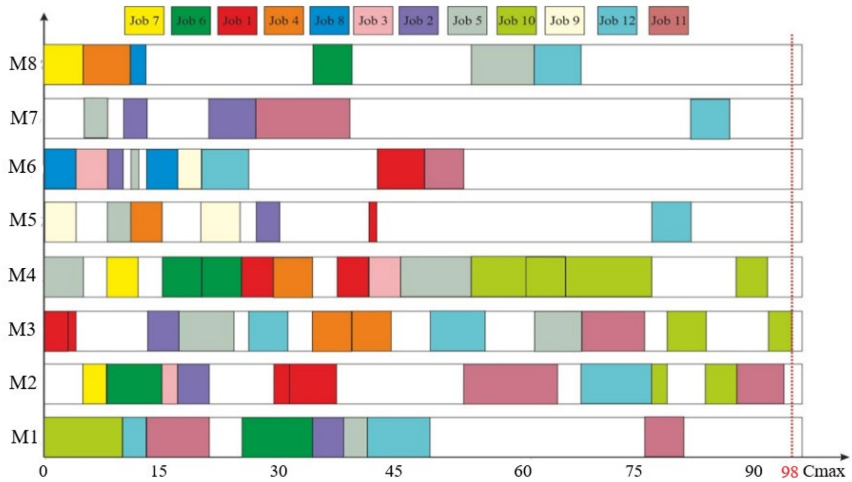

Figure 5

Graphic representation of the results and the optimal output sequence of jobs for the machines

Specifically, the presented case study revealed that it outperformed the company's existing method by an average of 23 percent.

Table 6

Performance Improvement Over the Existing Method in the Observed Company

	Makespan - C_{max}	An improvement over the existing method (%)
The traditional method based on the experience of the workers	121 minutes	-
After applying the *FMCDM* and *FJSP* model	98 minutes	23%

Conclusions

The problem of resource planning and scheduling in production is a complex process due to the dynamic environment and the nature of the production setting. Such an optimization process can be very challenging, especially when all external factors that directly and indirectly impact decision making are involved. The paper presented an integrated hybrid decision making approach based on well-known multicriteria fuzzy *MCDM* methods. This research demonstrated the applicability of *FMCDM* methods (*FAHP, FFUCOM, FWASPAS*). In this study, we employed the *FAHP* method and the *FFUCOM* method to obtain criterion weights based on expert ratings. Both methods provide a structured approach to decision making in a fuzzy environment but differ in their characteristics and approaches. In the second part of the paper, the FWASPAS method was used for ranking alternatives. The case study in the paper focused on a furniture manufacturing company that produces customized furniture and chairs made of veneer. The *FJSP* model was applied in this case study for production planning and scheduling. It is important to highlight that in today's manufacturing world, there are often jobs that have higher priority compared to other jobs. The priority of jobs is of great importance to ensure that the product to be manufactured is completed on time. When it comes to job priority, the specific idea is to assign weight coefficients to each job individually and rank the jobs according to their priority. This paper provides a comprehensive methodology for solving the flexible job shop scheduling problem by combining the fuzzy *MCDM* approach and the *NSGA II* algorithm. The experimental results illustrate the effectiveness of the proposed methodology and highlight the specific benefits that a company can achieve by implementing this approach in optimizing the production process. This approach enables efficient job completion according to priorities defined based on orders, improving the organizational capabilities of the company and ensuring accurate product delivery. Specifically, the presented case study revealed that it outperformed the company's existing method by an average of *23* percent, by giving priority to high-throughput products for early delivery. The integration of these methods results in an optimal job schedule in the production environment, leading to more efficient utilization of resources and increased productivity. By using objective criteria and the fuzzy *MCDM* approach, the subjectivity in assigning job priorities is eliminated, ensuring consistency and reliability in the decision making process. Moreover, this approach enables improved on-time product delivery, and optimized due dates for finished products allow the company to plan deliveries and enhance customer satisfaction. Future research directions could include considering the dynamic environment when making decisions about job priorities and resource scheduling in real-time using artificial intelligence tools.

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