Topological Analyses of the Hungarian Non-Karstic Caves

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Abstract: This paper provides a quantitative study of the Hungarian non-karstic cave geometries and topologies, to define the relationship between speleogenetic processes and cave patterns. To study the planar and morphological characteristic of non-karstic caves, define similarities and differences between them, order groups we used cave-map based topological and geometrical parameters. The recognized characteristics are in a tight context with the development of cave morphology, and the morphometrical characteristics can be generalized. The topological and geometrical parameters show for a cave or a cavesection the most important cave developing force (among other forces), which can be the base point to place a cave in a speleogenetic-system. This study demonstrates that modelling topological parameters such as connectivity, density of connections and passage density can effectively classify non-karst caves into different morphological categories, revealing that most Hungarian non-karst caves exhibit simple geometry and only 11% of them show complex, networked structure.

Keywords: non-karstic caves; topology; morphology; cave developing forces

1 Introduction

The spatial organization of caves has been the focus of many karst research studies for more than a century [2, 8, 19, 20, 21, 26], and particularly in the 21st Century, geostatistic and GIS was used for the simulation and modelling to describe the geomorphological evolution and shape of cave-networks according to flow conditions, geological discontinuities and geomorphological landscape evolution [4, 5, 24, 25]. These models were based on field survey data, geometrical relationships, geological, hydrological and morphological data [7, 9, 15, 23, 27]. The investigation showed that 2D/3D modelling of caves provides an opportunity to define regular patterns for entire cave-systems [1, 6, 10, 14, 16, 22], or for a part of a cave-system [2, 11, 12, 21]. These patterns are significantly different from each other and are in a tight relationship with the speleogenetic features of a cave.

Morphometric approaches to caves were first applied by Howard in 1971 on 2D cave systems. Later several authors [1, 6, 9, 10, 16, 22] defined several indices to describe the complexity of 3D karst features. Some of the parameters used by them are applicable to any caves generally, so for the non-karstic caves, too.

Using the earlier authors' experiences and methods I try to characterize the Hungarian non-karstic caves, using two dimensional geometrical and topological data [14] [17], because the mentioned caves do not have 3D cave maps, only ground plans, sections and cross-sections. No previous studies have dealt with the topological analysis of non-karstic caves, so the parameters I calculated are only a starting point for further investigations, even in the framework of international cooperation.

2 Data and Method

The original survey data are composed of a baseline and a sequence of survey stations with local width and height (to draw cross sections). Between two consecutive topographic stations, the recorded data are: magnetic azimuth, length, and the dip of the line joining these two stations. From a topological viewpoint the cave stands about three different nodes and links, which are connecting any two nodes [14] [17]. The nodes can be section-points, internal and external nodes. The point where two passages join defines a section-point. The point where three or more passages join defines an internal node. A change of direction of a single passage, even though abrupt, does not constitute a node. An external node is defined at the dead end of a passage and at cave entrances (Fig. 1). A bedrock pillar enclosed by a loop of passages is called an island. The first step is to decomposite the original survey data to cave branches and draw a complete graph, considering nodes with two neighbor nodes. The second step is to converse a complete graph to a reduced graph, which contains only external and internal nodes, islands and links (but not section-points). Geometry (length and orientation) has no obvious meaning in topology, so the complete and reduced graphs are similar (homeotopic) [17]).

The symbol n defines the total number of internal nodes on a cave map, e as the total number of external nodes, t as the total number of links, and i as the total number of islands. Several quantitative relationships may be stated relating to these quantities if the following assumptions are made [14]:

• Cave passages are assumed to lie in a plane or to be topologically projectable onto a plane, without overlap of passages, on different levels. Most of the non-karstic caves in Hungary are on one level, so that they satisfy this assumption very well. Some caves have different levels, but no, or few loops can be made in the cave using different levels.

• The joining links in one internal node is rarely more than three. Most of the non-karstic caves in Hungary satisfy this assumption very well.



Figure 1 Principle of the considered data structure from original cave survey data

Two important quantitative relationships between the topologic parameters come from graph theory [3] [13]:

$$i=t-(n+e)+1 \tag{1}$$

$$t = l - i + 2n \tag{2}$$

Because there are two equations and four variables (t, e, n and i), determination of any two for a particular cave pattern indirectly specifies the other two. The number of external nodes and the number of islands are easy to measure on cave maps and are subject to little error. Using the (1), (2) equations the number of internal nodes and links can be calculated. In addition, with the above-mentioned four parameters I used the following index-numbers for the characterization of the non-karstic caves [14]:

- β=n+e, the index-number of connectivity of passages. For a cave network with a very high degree of interconnections, many external nodes and few or no islands, the value of β is large.
- Ac/Ap, where Ac is the area of cave passages and Ap is the area within cave periphery (Fig. 2). This parameter shows the extension and complexity of the cave area. Generally for a cave network with a very high degree of interconnections, many external nodes this value is near to one. If a geometry of a cave is less complex, this value is under 0.7. It ios only applicable with β together, because the value of it will be close to one not only by cave-networks, but by simple, one-passage caves as well.

- L/Ap, where L is the total length of cave passages. This parameter shows passage density. Caves with the same passage density have the same total length of passage in equivalent planimetric areas. It is only applicable with β together, because the value of it will be the same by cavenetworks, and by one-passage formed caves, too.
- t/Ap, where t is the total number of passage links in a cave. This
 parameter shows link density. Caves with the same link density have an
 equivalent degree of branching and interconnecting of passages. Beside
 β, this parameter is the most important for the characterization of cave
 topology and geometry.



Figure 2 The derivation of the Ac (blue) and Ap (red) parameters

For my investigation I used data from the Hungarian National Cave Registry (www.termeszetvedelem.hu) and from the list of the non-karstic caves in Hungary, which is maintained by the Hungarian Speleological Society Volcanspeleological Collective (nonkarstic.geo.info.hu). The latter registry includes 1299 non-karstic caves known up to now, among them some caves which were recorded in the past, have been already destroyed up to now. Furthermore, listed are caves of the border regions of Austria and Slovakia, which belong to similar geomorphological and geological formations. Also included in the list are artificial or artificially widened cavities, which are considered as caves by the local population. They represent abandoned mine workings, rock shelters, former hermitages, underground commercial premises which are several hundred years old, rock-chapels and other cavities used for religious purposes throughout history. However, the list does not include cellars, former cave-dwellings in built up areas, basements, former military bunkers, industrial objects, or air-raid shelters. Most of the non-karstic caves in Hungary (928 caves) developed in igneous rocks: andesite, basalt, rhyolite and in their tuffaceous formations as well as in granite. Some 287 caves are known in sedimentary rocks, mainly in sandstones and conglomerates and 84 caves are formed in metamorphic rocks, in calcareous phyllites and in green schists. Taking into account only the caves with natural origin, 871 caves will remain. Most of them are only 2-3 meters long and can be described with two external nodes and one link; so, they are not appropriate for further topological analyses. From these caves I choose the geometrically most considerable 180 (21% of the total number) for further investigations; the total length of them is 3203 meters. From the 180 caves, 52 (29%) are registered only by the Volcanspeleological Collective, while 128 ones (71%) are recorded in both registers. Figure 3 shows the geographical distribution of these caves.



Figure 3 The geographical distribution of caves used for the investigations (Source of the map [18])

3 Analysis of the Non-Karstic Caves

The database contains the following data about caves: name, identification number of the cave in a region, geographical location, administrative district, length, vertical expansion, i, e, n, t, β , Ac, Ap, L/Ap, t/Ap and Ac/Ap parameters. The results of the histogram analysis of the i, e, n, t parameters show Figure 4.

Based on Figure 4 we can declare, that in the Hungarian non-karstic caves' islands, so bedrock pillars are not typical. In 173 caves (96%) there was no island, in 5 (3%) caves there was one island, and only in two caves can we define two and four islands. An example for one island cave is the Pók-lyuk (Spider Hole, cadastral number 5240-17, Ajnácskői Mountain), for two island caves: the Budaligeti-barlang (Cave of Budaliget, 4773-9, Budai Mountain), for four island caves: the Pulai-bazaltbarlang (Basalt Cave of Pula, 4430-11, Bakony Mountain).



Histograms of the i, e, n, t parameters

Of the caves studied, 48% (87) were characterized by having only two external nodes, i.e., the caves could be topologically described by a single edge and two nodes. 26% (46) of the caves were characterized by the presence of 3 external nodes, which also implies an internal node, i.e., the cave was topologically mostly described by Y-shapes. Caves with more complex passages are also characterized by a higher number of external nodes, which accounted for 26% (46) of the caves studied. The Csörgő-lyuk (Clanking Hole, 5230-1, Mátra) with 50 external nodes, 48 internal nodes and 97 links; and the Szilváskői-barlang (Cave of Szilváskő 5240-5, Ajnácskői Mountain) with 9 external nodes, 9 internal nodes and 18 links are examples of more complex non-karstic caves.

In line with the previous indicator, 48% (87) of the caves studied have no internal nodes. A single internal node was found in 26% (46) of the caves, 12% (20) had two nodes, and caves with more than two internal nodes accounted for 14% (26) of the sample. An example for a non-karstic cave with many internal nodes is the Bölcső-hegyi-barlang (Cave of the Mount Bölcső, 4900-35, Visegrádi Mountain),

which is the deepest (-33 m) non-karstic cave in Hungary (in 2023) and has 14 internal nodes.

The number of links is also an index-number of cave complexity. Only one link describes 87 caves (48%), three links - 47 caves (26%), and more than three links - 46 caves (26%). The caves with many links, external and internal nodes, with a complex geometry are, for example, the Betyár-barlang (Outlaw Cave, 5223-4, Cserhát Mountain) with 7 external nodes, 5 internal nodes and 11 links; and the Rózsa Sándor barlangja (Cave of Rózsa Sándor, 5500-87, Tokaj-Eperjesi Mountain Chain) with 6 external nodes, 4 internal nodes and 9 links.

Figure 5 shows β -values, the index-number of connectivity of passages (networking).



The index-number of connectivity of passages

The meaning of Figure 5 is the same as above mentioned by Figure 4; only 47 caves (26%) can we declare as network-shaped cave with complex geometry. For further topological investigations there is no need to use i, e, n and t parameters; it is enough to analyze the β number. An example for the network-shaped non-karstic cave with a large β -value (32) is the Jéggombás-barlang (Ice-mushroom-cave, 4900-44, Visegrádi Mountain).

The L/Ap parameter shows passage density (Figure 6). This value is between 0.10 and 0.60 by 93 caves (52%), that means, the length of the caves is relatively small in proportion to the areas within the cave's periphery.



Figure 6 Histogram of the passage density (L/Ap)

This index-number characterizes the wool-sack caves and other weathering caves, fissure caves, gravity induced caves, talus-caves etc. A good example for the relatively small cave with large periphery is the Pokol-lik (Hell Hole, 4430-13, Bakony Mountain), which is a gravity induced cave, and the L/Ap number of it is 0.35. The passage density number is over 0.60 for 87 caves (48%). It does not mean that all these caves are network-shaped caves. Network-shaped caves and one-passage caves have also a large L/Ap value, so to have a right conclusion, the passage density index-number can be used only with the β number together. Good examples are the Jakab-hegyi Forrás-kőfülke (Spring Rockshelter on the Mount Jakab, 4420-175, Mecsek Mountain, 4.6 meters long), and the Sólyom-kő bérci Hasadék-barlang (Fissure Cave of Sólyom-kő Ridge, 5500-98, Tokaj-Eperjesi Mountain Chain, 11 m long). By these caves the L/Ap numbers are 1.35 and 1.39, but the β numbers range respectively 2 and 8. This shows that the geometry of a gravity induced cave is more complex than by the Rockshelter cave. Figure 7 shows the related L/Ap and β values, except the Csörgő-lyuk (above mentioned, β =98), which is the longest non-karstic cave in Hungary (428 m). For most caves the value of the β number is under 6, and the related value of the L/Ap number is lower than 0.8. This means, that the most Hungarian non-karstic caves are not network-shaped.



The interrelation between values L/Ap and β

The Ac/Ap ratio shows the extension and complexity of the cave area. This can be called, as the proportion of the gross and net cave area (Figure 8).



Figure 8

The histogram of the extension and complexity (Ac/Ap ratio) of the cave area

This value is under 0.70 for 108 caves (60%), and only for 72 caves (40%) are larger. For 30 caves (17%) this number is over 0.90. Net-shaped caves, rockshelters or one-passage caves have also a large Ac/Ap value, so to have the right conclusion it can be considered only with the β number together. An example can be the weathering, developed Kiss-barlang (Little Cave, 5500-3, Tokaj-Eperjesi Mountain Chain), which is 12 m long; and the above mentioned Csörgő-lyuk. The Ac/Ap values are respectively 0.75 and 0.73, but the β numbers range 8 and 98. Figure 9 shows the related Ac/Ap and β numbers except the Csörgő-lyuk (β =98).



Figure 9 The interrelation between Ac/Ap and β values

The value of the Ac/Ap ratio is in most cases between 0.4 and 0.8, and the related β values are lower than 6. This means, that among the Hungarian non-karstic caves only a few is considerable in the viewpoint of their expansion and complexity.

Beside the β index-number, the link density (t/Ap) is the most important cave topology parameter (Figure 10). This parameter makes it possible to analyze the complexity of caves, because caves with the same link density have an equivalent degree of branching and interconnecting of passages. The analysis of such parameters, as: length, depth, vertical size of a cave, the β and t/Ap values; can for example inform us about the difficulty level of a planned cave-tour.



Figure 10 The histogram of link density (t/Ap)

Link density is very small in the case of 160 caves (89%), and this means that they are not difficult geometry qualifiable, network-shaped caves with complex topology. Only 20 caves (11%) are characterized by the large value of the link density (over 0.60). Figure 11 shows the interrelation between t/Ap and β numbers (except the Csörgő-lyuk, β =98). For most caves the link density value is lower than 0.5, and the related β values are less than 5. These caves have a simple geometry and topology and visiting them does not lean special requirements for the cavers. Only 20 caves, the 11% of the Hungarian non-karstic caves have difficult geometry and topology, they are network-shaped with complex branching and different levels of passages, and this finding has not been previously confirmed or documented in the literature.



Figure 11 The relationship between link density and cave-complexity

Figure 12 shows together the four above mentioned topological parameters "t/Ap", "L/Ap", "Ac/Ap" and " β " (without the large β values, over 18). The index-number of connectivity (β) is for the most caves ranges between 2 and 4. The link density (t/Ap), so the complexity value is in the most cases less than 0.4. The value of the passage density (L/Ap) and area expansion (Ac/Ap) is between 0.5 and 0.8 for most caves. These parameters are showing the simple geometry and topology of the Hungarian non-karstic caves, aside from 20 caves (11% of the total number of investigated caves), which are typically network-shaped caves with different levels and complex topology.





The relationships between connectivity (β), area expansion (Ac/Ap), link and passage density (t/Ap, L/Ap) for the Hungarian non-karstic caves

After the investigation of the topological and geometrical parameters, the geological, morphological and cartographical conditions of a cave, it can be possible to conclude for the main cave developing force. Finishing the analyses, I summarize the topological parameters of example caves of the most frequent genetical types of Hungarian non-karstic caves (Table 1).

NAME	ID#	i	e	n	t	β	L/Ap	t/Ap	Ac/Ap	Genetical type
Vidróczki- barlang (Vidróczki Cave)	5230-10	0	2	0	1	2	0.35	0.09	0.73	gas bubble cavity
Nógrádszakáli 1. sz. fatörzsbarlang (1st. Tree-mold Cave of Nógrádszakál)	5240-23	0	2	0	1	2	0.65	0.15	0.52	tree-mold cave
Arany-barlang (Gold Cave)	VC reg.	0	2	0	1	2	0.16	0.00	0.38	crystal chamber
Szarkádi- barlang (Cave of Szarkád)	4463-14	0	2	0	1	2	0.08	0.04	0.79	alkaline solution cave

 Table 1

 Topological parameters of Hungarian non-karstic caves representative for various genetical types

Kámori- rókalyuk (Foxhole Cave of Kámor)	5210-40	0	2	0	1	2	1.28	0.11	0.79	fumarola cave
Gödrösi- explóziós- barlang (Gödrösi Explosion Cave)	4463-5	0	2	0	1	2	0.71	0.11	0.87	gas explosion cave
Lépcső-menti- orgonaköz (Lépcső-menti Organgap)	4450-9	0	3	1	3	4	0.20	0.30	0.42	weathering cave
Zsivány- barlang (Zsivány Cave)	4510-2	0	4	2	5	6	0.60	0.21	0.39	wool-sack cave
Ülés-pataki- sziklakapu (Ülés Creek Natural Bridge)	VC reg.	0	2	0	1	2	0.47	0.15	0.71	natural bridge
Tátikai-kőlik (Stonehole of Tátika)	4430-23	0	2	0	1	2	1.43	0.0.4	0.61	gravity induced cave (one-passage cave)
Csörgő-lyuk (Clanking Hole)	5230-1	0	50	48	97	98	1.10	0.25	0.73	gravity induced cave
Vízesés- barlang (Waterfall Cave)	VC reg.	0	2	0	1	2	0.76	0.22	0.74	erosion cave
Szilváskői- barlang (Cave of the Szilváskő)	5240-5	1	9	9	18	18	1.09	0.29	0.88	consequence cave

Conclusions

This paper delivers a quantitative study of the Hungarian non-karstic cave geometries and topologies, to define the relationship between speleogenetic processes and cave patterns. For the study of the planar and morphological characteristic of non-karstic caves, to define similarities and differences between them and to order groups, we used cave-maps based on topological and geometrical parameters. The recognized characteristics are in a tight context with the development of cave morphology, and the morphometrical characteristics can be generalized. This study shows the importance of modelling the topological parameters of nonkarstic caves for defining a geometrical and morphological characteristics-based system. The topological and geometrical parameters can be helpful in the identification of the important factors of cave (or cave sections) development, which can be the basepoint to place them in a speleogenetic system.

From the Hungarian non-karstic caves, I choose the "geometrically", most significant, 180 (21% of the total number) for further investigation; the total length of them is 3203 meters. I used eight topological parameters to describe them: total number of external and internal nodes, islands and links; connectivity, area expansion, link and passage density. The topological investigations showed that knowing the total number of external and internal nodes, islands and links have the same topological meaning as connectivity, so for further investigations, it is enough to analyze the latter.

Link and passage density index-numbers and the area expansion parameter can be used for the classification, only with connectivity together, because the different non-karstic cave genetical types have notwithstanding the distinction's similarities, too, in the topological and geometrical parameters. Considering the topological parameters, I can declare, that most Hungarian non-karstic caves, have a simple geometry and topology, therefore, visiting them does not impose any special requirements for the cavers.

Only 20 caves, 11% of the Hungarian non-karstic caves have difficult geometries and topologies. They are network-shaped, with complex branching and different levels of passages.

The investigation of the topological, geometrical parameters, geological, morphological and cartographical conditions of a cave can be possible to accomplish, for the main cave developing force.

In finishing the study, I summarize the topological parameters of the most frequent genetical types of Hungarian non-karstic caves.

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