Improving Efficiency: P-Graph Modeling for Business Workflows with Dynamic and Fuzzy Extensions

Andrea Tick[1] and György Molnár[2]

[1]Obuda University, Keleti Karoly Faculty of Business and Management, Tavaszmező u. 17, 1084 Budapest, Hungary, tick.andrea@uni-obuda.hu

[2]Obuda University, Kando Kalman Faculty of Electrical Engineering, Tavaszmező u. 17, 1084 Budapest, Hungary, molnar.gyorgy@uni-obuda.hu

Abstract: The solution for an error-free and cost-effective operation of complex administrative and business processes, or rather, the effectiveness of the solution, is a very important issue in both the profit-oriented and the non-profit sector. This is because the optimal effectiveness of business and administrative automation processes is a high-priority business purpose. Workflow is the most widely used modeling technique in the field of business and administrative information systems. There is no process-based method among the workflow-based modeling methods applied for elucidation, modeling and optimization of work processes that systematically leads to the optimal workflow network structures. In this paper, after a semi-systematic review on P-graph scientific literature, a new means for P-graph based modeling of administrative and business processes is introduced and various usage fields are explored. Based on the review, focusing on keywords research potential is revealed, for this specific research field. The dynamic and fuzzy extension of the P-graph modeling is justified for business processes and a fast method, with a mathematical foundation is presented for optimal workflow model generation. The model is able to handle not only the process structure, but also, the input, intermediate and output documents, taking into account the required and available resources (qualitative and quantitative) as well as the constraint parameters, the bottlenecks and the fuzzy features of real business processes.

Keywords: P-graph; business process modeling; fuzzy; workflow

1 Introduction

Early business automation focused on converting existing processes to computers applications. Business Process Management (BPM) came later, bringing a more sophisticated approach that involved planning, analyzing and optimizing workflows. Workflow modeling became a key tool for BPM due to its efficiency.

Eventually, BPM was not limited to just production processes, but also tackled administrative tasks. [1].

Various definitions were given to determine workflows. The definition of the Workflow Management Coalition (WfMC) is authentic: "The automation of a business process, in whole or part, during which documents, information or tasks are passed from one participant to another for action, according to a set of procedural rules" [2, p. 8].

Thanks to rapid developments over the last few decades, workflow has become a widely used technique in business and office information systems. Several small, medium and large organizations use workflow management systems (WfMS) to effectively handle production and management or administrative processes. WfMSs support activities such as specification, analysis, design, modeling, optimization, simulation and control of business processes.

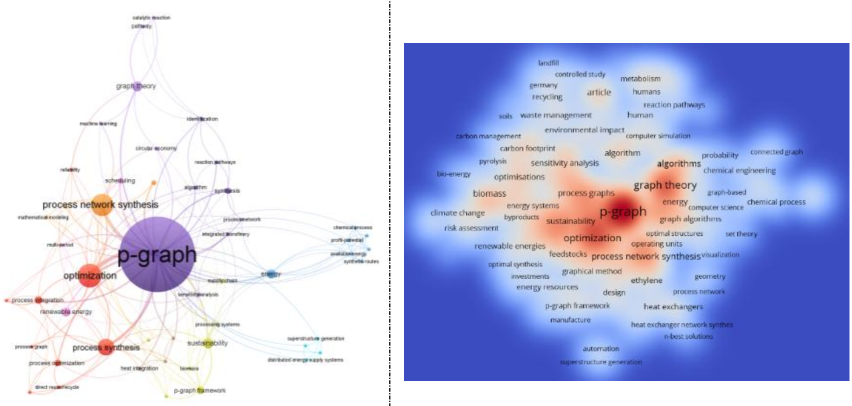
Production and administrative processes have become very complex in every industry. To ensure the stable and reliable operation of such large integrated systems, the development of a precise mathematical model is essential. It is not only important for the industry, but also for administrative organizations, government agencies, etc. to handle their business processes legally, error-free and efficiently.

One of the most significant changes in this area over the past decades has been the integration of IT into business processes in almost all areas and its dominance. This is true not only for the profit-oriented economic sector, for manufacturing companies of all sizes, but also for the non-profit sector. In addition to the service industry, IT management of processes is an indispensable necessity in public administration, education, health, etc. Hungary's updated Convergence Program 2006-2010 explicitly states that in order to provide full IT support for public services, business process models should be developed and taken into account in the legislative process. The development of these services requires not only IT developments, but also the analysis, modelling and transformation of the entire business process, of all office processes. The efficiency of a workflow system is highly dependent on its structure or network, the synthesis of the optimal network is crucial on practice and the integration of the results of related fields is a must.

This paper initially offers a semi-systematic review on P-graph modeling, based on author keywords and discusses the system of P-graph modeling in relation with it’s usage in different fields, then, process modeling with P-graph is discussed with some dynamic and fuzzy extension alternatives. Then the general keyword search helps the authors to reveal the field where P-graph modeling has already been applied, how it links to the field of fuzzy research, which allows for the identification of further research gaps. Finally, the conclusions are drawn.

2 Review based on Article Keywords and Authorship

Despite the boom of business (process) modelling, and the integration of IT into processing literature on P-graphs is relatively scarce. Based on the specific keyword "p-graph" two databases (Scopus and Web of Science) were scanned through titles, abstracts, topics and keywords. The Web of Science gave 327 search results while Scopus listed 441 scientific papers, which can be considered few provided the size of the two datasets. The frequency of the author keywords and all keywords in these papers are displayed by VOSviewer well presents that next to the terminology used for P-graph, its usage in specific fields are listed among the keywords (Figure 1).



(a) (b)

Figure 1

Author keywords (a) and all keywords (b)

Wordcloud based on scientific papers including P-graph drawn by VOSviewer (edited by authors)

In both cases most of the papers focus on the concept of p-graph, graph theory, p-graph framework while the second most frequently related key phrase in the papers are process network synthesis and optimization, or process synthesis. Based on the size of the circles in Figure 1a very few papers deal with the application of the p-graph in business process modelling.

When all the keywords are selected the key concepts remained p-graph, graph, theory together with optimization and process network synthesis. In this case more concepts related to business process modelling occurred such as process network, operating units, optimal structures and process graphs. Similarly, to the previous keyword search the application of P-graph for business process modeling is rarely searched.

Using the same keywords, however, focusing on the authors, three main research groups could be identified, a Hungarian group of researchers led by Ferenc Friedler, a Western European and an Asian research group (Figures 2 and 3).

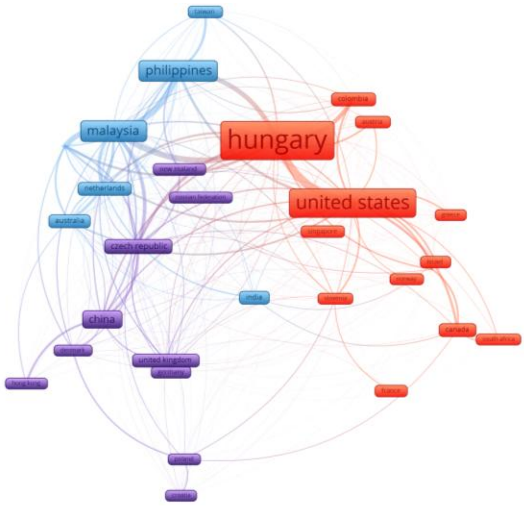


Figure 2
The network and interconnection of P-graph researchers by countries in Scopus and Web of Science, visualized by VOSviewer (edited by authors)

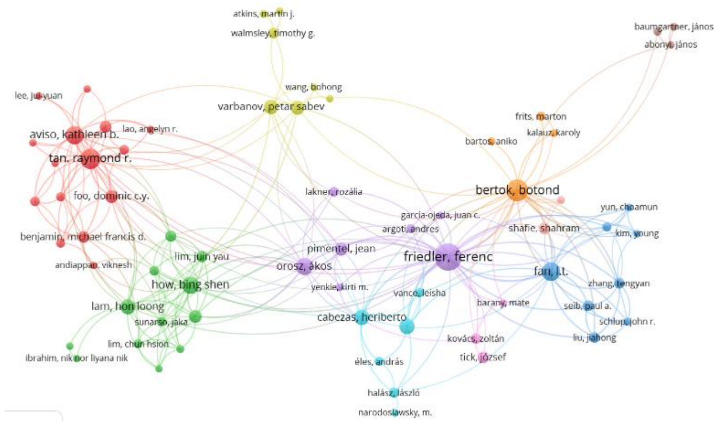


Figure 3
The network and interconnection of P-graph researchers based on scientific works in Scopus and web of Science, visualized by VOSviewer

The importance of the Hungarian research could be detected in this field, with a relatively small number of scientific papers published by researchers from all over world. It also suggests that there is future research potential in exploring the application of P-graphs in business process modelling.

The following sections gives a general overview of P-graphs and then it presents extension possibilities and some business application possibilities. Finally, P-graph scientific research is linked to fuzzy related research and the linking field is revealed.

3 Workflow Modeling Methods for P-graph Design

The specialist literature describes numerous workflow modelling methods that can be used in practice, from simple visualization methods to methods with a complex theoretical background [2]. Only two very common methods are briefly presented here.

Unified Modeling Language (UML) is a highly standardized and widespread modeling language used in software engineering as it is rich in tool systems (diagram technology). From UML’s control perspective it gives a description of process activities and their "routing" in different cases [3], which explains why UML sequence and activity diagrams are useful to model the mentioned aspects of workflows. Twenty one different patterns were identified in the research work of van der Aalst et al. that describe the behavior of business processes [3-5], while several authors gave a mapping of patterns of business process models to UML activity diagrams, as for example in [6] [7]. The use of UML reflects a more practical view that is closely related to the daily work of software developers. Although UML activity diagrams are very universally applicable, they lack certain properties and structures that are indispensable for workflow management, especially for very complex systems. UML is the No.1 in software modeling, but it is not often used as a modeling tool, especially for large business processes.

Petri net, the theory and methodology of which was introduced by Aalst and Hee [5] [8] to the field of workflow modeling and which has become a popular workflow modeling tool for quite some time [9], is one of the most important mathematical and graphical representations for networks, workflows and distributed systems [10]. He described the mapping of workflow management concepts to Petri nets, defined the processes, flows, controls, constructs and activities. He also focused on analyzing the workflow with Petri nets [11].

Petri nets are very suitable for studying the dynamic behavior of a workflow [12-14], but when planning the structure, there is no tool that supports the systematic generation of the model. PetriNets are also used for production systems modelling [15] or manufacturing processes [16].

4 P-graphs to Serve Process Modeling

The P-graph based modeling, as well as the generation of process networks with PNS (Process Network Synthesis) by combinatorial methods, was introduced in the publications of Friedler et al. [17-23], one of the key researchers in the network depicted in Figure 3. P-graphs are used for modeling various network structures for a long time. The P-graphs are explained in the following subsections.

4.1 Overview of P-graphs

A P-graph (process graph) is a directed bigraph, the nodes of which can be considered as operating units (***O***–operating units) and as materials (***M***–materials). The edges represent material flows between materials and operating units. The mathematical notations and the detailed definition and description of a P-graph is given by Tick and Tick et al. in [24] [25]. As the P-graph is a bigraph, its nodes form two disjoint sets, so no edges lead to nodes in the same set, the operation units and the materials are strictly separated from each other and thus a direct connection between materials or operation units is not possible. In mathematical term a node can only lead from an ***M*** material type node to an ***O*** operation unit type node if $\boldsymbol{M} \in \mathbf{input}_O$**,** namely ***O*** processes ***M*** materials. Similarly, an edge can only lead to an ***M*** material type node from an ***O*** operation unit type node if $\boldsymbol{M} \in \mathbf{output}_{\mathbf{O}}$, namely, ***O*** produces ***M*** materials. That is, the P-graph can be considered as a set of pairs of operation units and materials, such as an (***M,O***) P-graph [26, p. 78].

The nodes that are material type can be assigned to different subsets such as:

1) Subset of *raw materials*, which represents the input materials of the whole process,
2) Subset of *product-materials*, which includes the output-materials of the whole process,
3) Subset of *intermediate-materials*, which are created and used between the individual operations,
4) Subset of *by-product-materials*, which includes the "unwanted" by-products of the process.

The P-graph notations of operation units and various materials for the visual representation as well as a sample P-graph is presented in Figure 4. Four operation units $\mathbf{A_1, A_2, A_3, A_4}$ and eight materials $\mathbf{D_1, \ldots D_8}$ are presented in the graph**.** $\mathbf{D_1, D_2, D_3}$ and $\mathbf{D_4}$ are the materials available for the production of $\mathbf{D_8}$ while $\mathbf{D_5}$ and $\mathbf{D_7}$ are intermediate materials of the process. The by-product is not included in the sample P-graph in Figure 4. The mathematical definition of P-graphs is discussed in detail in [26].

As the present research focuses on improving business efficiency and on the usage of P-graphs in business workflows, their role in process network synthesis (PNS), the Maximal Structure Generator (MSG), the Solution Structure Generator (SSG), the extension of P-graphs for Workflow Modelling and business process optimization, and their fuzzy extension are discussed in the following sections.

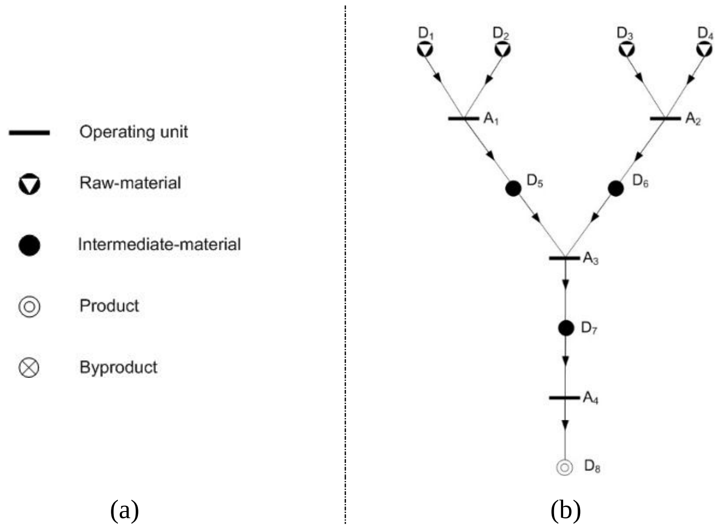


Figure 4

P-Graph notation (a) and visual representation (b) of a possible sample model with P-Graph

4.2 Process Network Synthesis (PNS) with P-graph

In process networks the core function is to produce ***P*** products from ***R*** raw materials, for which purpose all plausible ***O*** operation units and intermediate materials must be determined in the first phase. When ***P, R*** and ***O*** are determined, the number of materials in the network ***M*** is also defined. In order to generate the optimal solution structure through process-network synthesis axioms need to be defined. The properties defined by the axioms help improve the efficiency of combinatorial search during the process as there is an exponential relation between the number of operation units and the number of combinatorically possible networks. As for instance, in the case of a process-network synthesis with 30 operation units [22], the number of potentially possible structures is 2^{30}-1, which is exactly 1,073,741,823. Provided an average PC is used, with computational capacity of 10^{-2} seconds, then the calculation of the 1 billion combinations would take around four months. These axioms are listed below [17]:

- **(S1)** Each final product is represented in the graph.
- **(S2)** An **M**-type node has no input if and only if it represents a commodity.
- **(S3)** Each **O**–type node represents an operation unit which is defined in PNS.
- **(S4)** There must be at least one path from each **O**-type node (operation unit) that leads to an **M**–type node representing a product.
- **(S5)** If an **M**–type node belongs to the graph, then there must be at least one path that leads to an **O**–type node, or a path that leads from an **O**–type node to the given **M**–type node.

The properties defined in the axioms (S1...S5) are necessary but not sufficient criterion for selecting the optimal structure. The defined axioms enable a reduction in the number of structures by omitting and eliminating redundant and invalid structures, thus leaving the combinatorically possible structures fulfilling the axioms. With the help of this practical restriction, the search field has been drastically reduced. Taken the above example, the number of structures is reduced from 30 billion to 110. Therefore, the actual processing time is reduced from 4 months to 18 seconds.

The exclusion of the structures which are definitely not among the optimal solution structures cannot be carried out efficiently with conventional methods of process synthesis like the "super-structure methods" such as MILP or MINLP, due to their exponential nature of algorithms. The "Maximal Structure Generation" (MSG) polynomial algorithm, developed by Fiedler et al. [18] using the 5 axioms, generates the maximal structure that contains the subsets of all combinatorically possible structures.

4.3 MSG (Maximal Structure Generation)

The question arises what the maximum structure generator is that helps to map the maximum structure of the synthesis problem (**P, R, O**). It contains all combinatorial structures that enable the generation of the defined products from certain raw materials, but no further combination is possible. Therefore, the maximum structure definitely contains the optimal structure. Four main stages can be defined in the algorithm:

1) ***Input phase*:** The synthesis problem **(P, R, O)** is defined such that **M**={all plausible materials}, **P={**the final products}, **R={**all raw materials} and **O={**all operation units}. **M** contains not only the intermediate materials assigned to the operation units and defined in the set of **O**, but also the raw materials specified in **R** and the end products specified in **P**.
2) ***The elaboration of the input structure of the network phase*:** Performed by linking all similar (same type) nodes of materials.
3) ***The elimination phase*:** Taking into account the 5 axioms, the materials and operation units that cannot be connected to the maximum structure are eliminated. The elimination is carried stepwise, from the lowest level of the input structure, with the raw materials. The nodes of materials and operation units are faced with the 5 axioms step by step. Certainly, the elimination of a node often leads to the elimination of other related nodes.
4) ***The reconnection phase*:** The nodes are reconnected from level to level, starting with the highest level where the final products are located.

The maximum structure generated in this way contains all combinatorically possible structures with their elements fulfilling the 5 axioms.

4.4 SSG (Solution Structure Generation)

As the maximum structure is generated by MSG, the optimal structure needs to be found. Since the maximum structure generated by the MSG algorithm contains all such combinatorically possible network structures that can produce the end product from the given raw materials, it also contains the optimal network structure. In most business cases, optimization means finding the most cost–effective solution.

The Solution Structure Generation (SSG), used for the production of all the solution structures, is a mathematical tool based on the application of Decision–Mapping (DM) developed and discussed by Friedler et al. [22] and detailed in [24] [26] [27], With the help of the MSG and the decision mapping for P-graph, the steps of the SSG algorithm can be defined, which procedure generates all solution structures, i.e. all the combinatorically possible solution structures. In the input phase of the SSG algorithm, the quantities of products (**P** set), raw materials (**R** set) and the other materials (**M** set) required for the generation are specified.

After the input phase, the computer algorithm systematically and combinatorically selects the active sets in a recursive manner and performs decision mapping on them. The algorithm works until all active sets have been selected and executed.

5 Extension of P-graph for Workflow Modeling (WFS)

Efficient management of complex business processes is essential for both for-profit and non-profit organizations. Workflow modeling, a common technique in business information systems, provides a cost-effective and error-free solution to streamline these processes. However, there's currently a lack of a comprehensive methodology that focuses on process improvement.

The P-graph for workflow modeling has been introduced analogously to process-network modeling [28] [29] as well. This approach goes beyond just outlining the basic structure of a process, like traditional models. It takes into account factors like resource usage and time to pinpoint bottlenecks, critical moments, and areas with excess resources during real-world operations. By considering these dynamic aspects, the model can be fine-tuned to optimize performance under normal conditions.

In the case of a workflow, documents **D** and activities **A** are used as basic elements instead of materials and operation units. Documents are categorized depending on their roles in the workflow. Therefore, there are general documents **D**, input documents **ID** (input elements of the workflow), product documents **PD** (outputs of the workflow), and intermediate documents **MD** (documents created between the different processing stages and/or in the process and used later) [30].

In order to execute an activity, the necessary resources must be introduced. In P-graph-based workflow modeling, resources are represented as a particular input **S** of the given activity **A**. Figure 5 presents the workflow elements and a possible workflow for business processes.

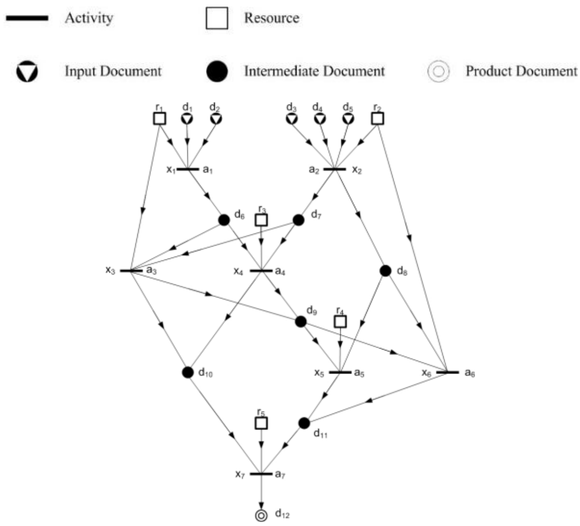


Figure 5
Workflow elements and the visual representation of a possible workflow

Administrative work, where documents and files are processed, differs significantly from material processing [1]. Documents are opposed to materials (powder, gas, liquid, etc.) discrete and quantified elements of the system. These characteristics influence the processes in the workflow. Although the handling and processing of documents is quite different from the previous application of PNS, the structure of the network can be generated in the same way. Certain special restrictions must be introduced if P-graphs are to be used for administrative purposes and not for network synthesis. However, there are special restrictions in the process.

Constraints on the documents: An *input document* has two properties, the available quantity (**f_i**), (number of documents) and the time of availability of the input documents (**t_i**). The time duration that is considered for one shift is 8 hours long, so (**t_i**) must be less than 8. *Intermediate documents* are created in the workflow of activities and used by other activities. This means that for each document it must be true that at most only as many documents can be used as have been created. This inequality is specified in the mathematical model for each intermediate document as a constraint. For the *product documents* the prescribed quantities of product documents, that must be generated, need to be determined in advance.

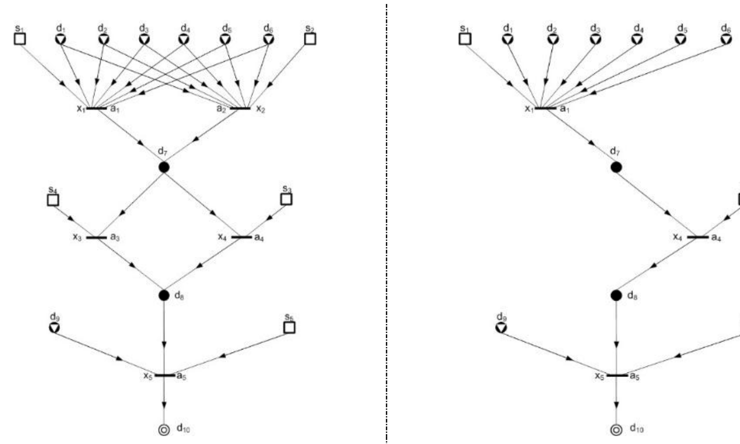
Restrictions on time: *Limited Resource Availability*: Unlike the typical 8-hour workday assumption, resources have finite availability. This could be due to factors like employee breaks or equipment downtime. *Defining Availability Periods*:

The model incorporates these limitations by specifying availability periods for each resource. *Time-stamped Documents*: Documents within the workflow are assigned timestamps to ensure they are produced and completed within these availability constraints. *Production vs. Availability*: The earliest availability time of an *intermediate document* must be after (or equal to) the time it takes to produce it. Similarly, the final *product document* cannot be available before its creation activity is finished. The model considers realistic break times and creates a schedule that ensures all documents are completed within these limitations.

Leveraging graph theory, this approach builds upon existing methods for P-graph modeling and PNS generation. It automates the creation of optimal workflow structures. An objective function is used to evaluate different structures and identify the one that achieves the best outcome based on defined optimization criteria. This function essentially quantifies the level of improvement when comparing various workflow structures. In the case of administrative processes, the following objective functions can be used:

1) ***Shortest time*:** It is essential that the expected number of documents are produced in the shortest possible time.
2) ***Cost efficiency*:** Costs are allocated to resources; the aim is to generate an optimal workflow structure that performs the given task with minimal resources (costs).
3) ***Optimizing Resource constraints*:** The availability of resources is usually limited in time; their capacity is limited. This approach aims to create the best possible workflow structure by considering these limitations.

Using well-defined objective functions like the ones mentioned earlier, special algorithms (MSG, SSG, and B&B (Branch–and–Bound) can systematically generate optimal workflow structures [22] [23]. These functions guide the process towards the most efficient outcome, whether it is minimizing processing time (achieved through parallel activities) or using the fewest resources. For illustration purposes, consider workflow for processing speeding tickets. One is optimized for the fastest processing time (Figure 6a). This structure likely involves parallel activities (a_1-a_2 and a_3-a_4) to eliminate bottlenecks. For instance, tasks like reviewing evidence (a_1) and recording the violation (a_2) could be done simultaneously. Figure 6b is optimized for minimal resource usage. This structure might prioritize a sequential approach, minimizing the need for extra staff [25].



(a) (b)

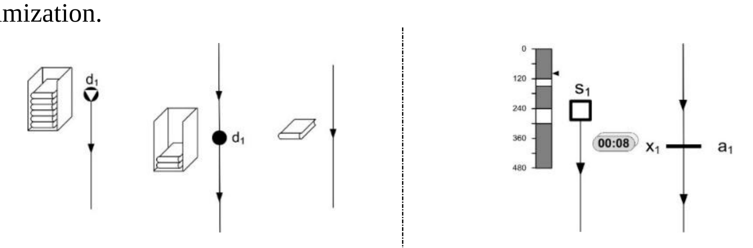
Figure 6

Workflow optimized for (a) the shortest possible time and (b) minimal resources

6 Model Extension to Handle Dynamic Behavior

While workflow models excel at static analysis of components and their relationships, they lack insight into real-world behavior and does not provide information about dynamic behavior. To address this, we introduce a simulation procedure with enhanced notation for visualizing document flow and timing [31] [32] (see Figure 7).

Administrative workflows deal with distinct documents, often with linked copies. Since resources have limited availability and time management is crucial, these processes heavily focus on document handling, resource allocation and time optimization.



(a) (b)

Figure 7

Notation extension for input documents, intermediate documents and document flows (a) and notation extension for resources and activities (c)

6.1 Document Management

Documents are identified individually in an administrative process. During the processing process, documents are used that are assigned to each other and can therefore only be processed in the same step. For example, in Figure 5, activity a_1 can only be carried out if all 6 documents $d_1...d_6$ are available at the same time. If a document is not available (e.g., vehicle registration document not available) then a_1 cannot start and the documents are placed at the end of the queue (FIFO) because they cannot be processed. This means that for each "input" (input documents) a FIFO must be defined where the documents are sorted according to arrival time.

In the case when all documents ($d_1...d_6$) are available for a_1, then the task of a_1 is carried out, a new document is created and placed in the FIFO of d_7. The processed documents leave the FIFOs belonging to $d_1...d_6$. The documents are not necessarily synchronized in FIFOs, i.e., before a_1 starts, the first document from d_1 should be taken and the documents belonging to the document from the FIFO of d_1 should be systematically searched for in FIFOs $d_2...d_6$. If not all related documents are to be found, the document from d_1 is placed at the end of the FIFO and the process is repeated with the next document. The dynamic behavior of the FIFO is symbolized by the book stack animation (see Figure 8).

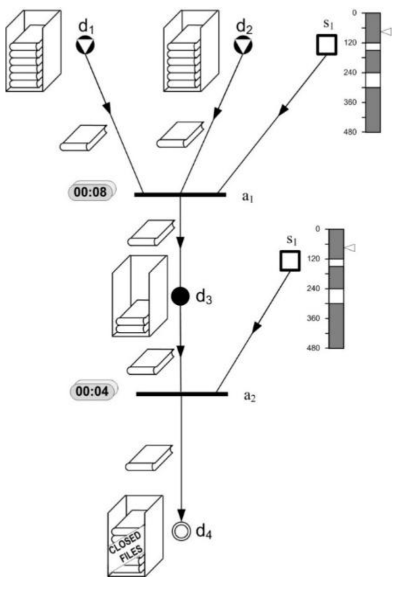


Figure 8

Administrative business process P-Graph based workflow[32]

6.2 Time and Resource Management

Time management in simulations, especially real-time ones, can be tricky. For administrative processes, "simulated time" is derived from real-time, ideally with a variable ratio. A common approach is 1 hour = 1 minute, compressing an 8-hour workday into 8 minutes for easier document flow observation. Activities have execution times defined down to the minute. In simulation mode (active state), a countdown timer beside each activity icon shows the remaining processing time (see Figures 7 and 8). There are already a number of IoT-based or different robotics technology solutions for time and resource management [33] [34].

Limited resource availability is crucial for realistic workflow simulations and needs to be incorporated in the behavior of the system. As human resource gives the source of the system, staff breaks, unlike a typical 8-hour workday, impact performance. Therefore, we define resource schedules with availability periods (dark) vs. breaks (light) before simulation (see Figure 7 and 8 for examples).

Activities only proceed when both required documents and resources are available. Missing elements cause a "standby state" (red color). If an activity requires multiple resources, all must be available simultaneously (like an AND operation). Resource notation details are in Figures 7 and 8 (Triangle indicates actual time). Such workflow processes are usual in business process modeling. Therefore P-graphs can be used to optimize administrative processes [1], to make supply chain management more efficient in Logistics [35], however, as the keywords presented in the review of P-graph application, it can be utilized in circular industry, in recycling, in manufacturing industry [36] in distributed energy supply system optimization and in the energy sector (see Figure 1).

7 The Extension of the P-graph with Fuzzy Elements

So far P-graph modeling has been explored, its application to workflows, and its role in business process optimization. This chapter explores how P-graph based workflow models can be extended using fuzzy logic. This extension aims to create more realistic process models by incorporating the ambiguity and inherent vagueness of real-world situations. We will discuss the need for this extension, different approaches, selection criteria, and the analysis results of the chosen solution.

P-graphs were originally used for precisely defined processes. However, administrative workflows involve judgment calls within legal boundaries [37-39]. While some aspects of setting fines are clear-cut (speed, existing points, prior offenses), others are subjective (remorse, cooperation). Fuzzy extensions handle these subjective factors that cannot be rigidly defined in algorithms. Fuzzy extensions address this by incorporating ambiguity as in for instance in decision making [40] This creates a new type of P-graph model with fuzzy operational units and document types.

Few studies explore applying fuzzy logic to workflow models, making them more flexible for handling ambiguity in real-world processes [37] [41]. Zirpins et al. [42] stands out for using fuzzy decision criteria instead of rigid rules. Existing literature offers limited approaches in this area. The present fuzzy extension assigns fuzzy sets to model elements to better reflect real-world uncertainties: (1) *Documents:* Allow "approximate" availability times instead of exact ones (Figure 9a), (2) *Resources:* Availability windows are represented by fuzzy sets to reflect uncertainty (Figure 9b) and (3) *Activities:* Execution times remain precise, as these are typically well-defined steps.

Activities process documents using assigned resources. All inputs (documents and resources) must be available to start, modeled by t-norms in fuzzy sets (often using Zadeh minimum) [28] [43]. These t-norms define an Operation Time Window (OTW) based on availability. The OTW must be wider than processing time for the activity to proceed.

Continuing with the resource-extended example from Section 4 (Figure 5), we determine fuzzy input sets (availability) for each activity ($a_1...a_7$). These include input documents ($d_1...d_7$), resources ($r_1...r_5$), and intermediate/product documents ($d_6...d_{12}$). The t-norm (OTW) considering all inputs is defined. Redundant steps (documents processed by multiple activities) use the maximum t-conorm (union) to combine fuzzy sets. The fuzzy set for the final document (d_{12}) indicates the approximate time it can be produced considering document and resource availability within the workflow. Figure 9c illustrates the operation of activity a_7.

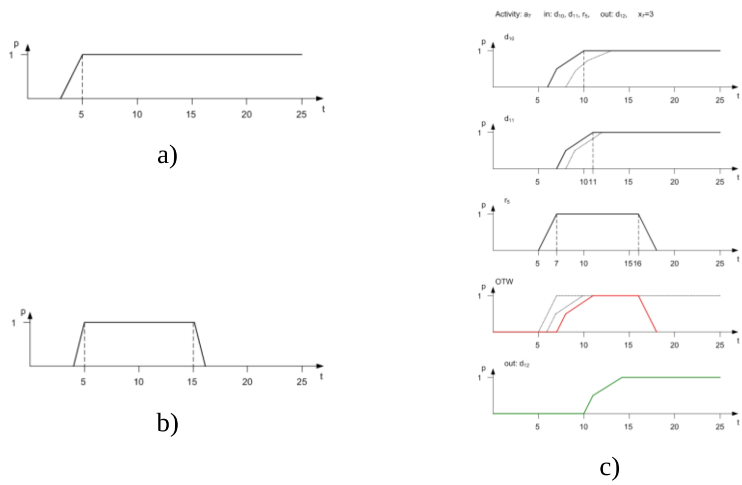


Figure 9

The fuzzy features ((a) and (b)) and the operation of **a_7** activity (c)

For workflow network synthesis, the fuzzy set of the final document can be used as the objective function to evaluate networks generated by the Accelerated Branch and Bound (ABB) algorithm [44] [45]. The key requirement is a non-zero fuzzy set for the final document, indicating successful product creation under given job availabilities (fuzzy sets). When evaluating networks, we can consider at weighing (a) the exact time of product availability, (b) the total network activity cost (minutes weighted by cost) (c) the number of intermediate documents (weighted by cost) and (d) the resource availability cost (sum of fuzzy set integrals). By considering these factors alongside the successful creation of the final product (non-zero fuzzy set), we can identify the optimal workflow network for a given scenario [28].

8 Interconnection of P-graph Extension with the Fuzzy Society

The integration of the fuzzy logic and the fuzzy extension of P-graphs, on the one hand, was an obvious and at hand choice during the elaboration of the P-graphs, since a worldwide known research group operates in Hungary led by Imre Rudas. Having run a review on his research work and focusing the search on the keywords 'fuzzy sets' and 'fuzzy' in Scopus (14 records) and Web of Science (65 records), an international research community could be detected with Asian and American hubs, in which Imre Rudas plays a determining role.

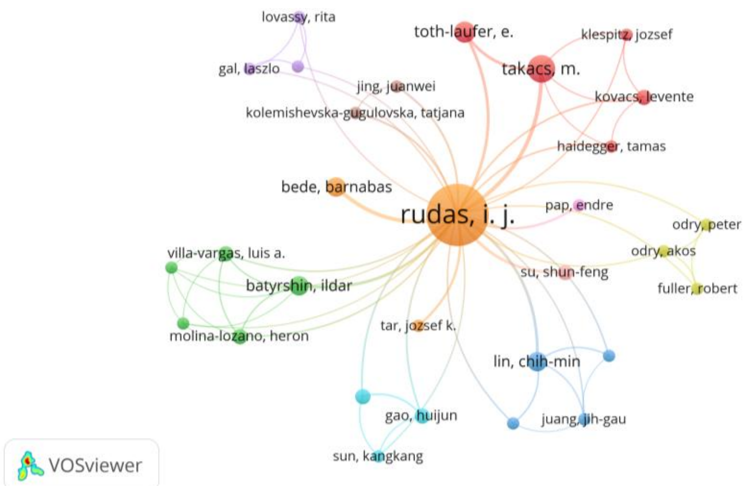


Figure 10
Fuzzy Society research network centered in Hungary (developed by authors in VOSviewer)

On the other hand, checking the interrelationship of p-graph research work and research on fuzzy systems (keywords: 'p-graph' and 'fuzzy') a two-pole network could be identified based on the Web of Science (8 records) and Scopus (9 records) databases (Figure 11). What related these two poles is exactly the process network synthesis that incorporates all the applications as business processes, business applications etc., which were discussed in the present paper. The small number of records in these databases has revealed that there is future research potential in this field.

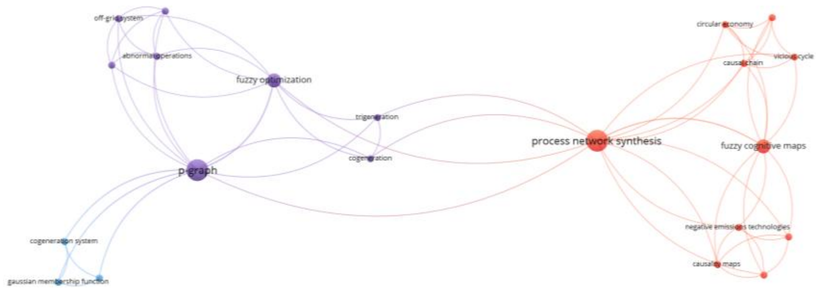


Figure 11

The interrelationship of fuzzy and P-graph research groups in Hungary based on keywords "fuzzy' and 'p-graph' (developed by authors, displayed by VOSviewer)

Conclusions

Compared to previous workflow models, the introduced extended P-Graph-based workflow modeling, has two major advantages. First, the automatic and systematic generation of optimal structures and thus the possibility to regenerate the optimal structure in real time when conditions change and second, the dynamic analysis of the workflow, which gives significantly more information about the behavior of the system and the properties of the workflow structure (bottlenecks, lack of resources, etc.), as well as the management of uncertain elements and incidents by fuzzy approach and functions.

The visual representation of the workflow, the animation of the process, as well as the traceable document flows and the representation of the resources and documents make the simulation and thus the investigation of the behavior of the workflow easily traceable and informative. This makes it possible to modify the workflow structure, the distribution of resources and thus, to better adapt the workflow to the environment and to test heuristic solutions. It can be concluded that the application of P-Graph based workflow offers a clean and efficient method to model and optimize business processes. The presented P-Graph-based system model, is not only suitable for the complete control of business processes, but also helps and supports the optimization of the decision-making process, that determines the return on business and development investments.

The research herein has its limitations, as two databases were selected for the review on P-graph and fuzzy research fields and the number of scientific papers were relatively limited. However, the results determined a research gap, since future research of P-graphs with fuzzy extension can be conducted to optimize the workflows, in several fields of business and industry. The strong presence of the two research groups – P-graph and fuzzy – could provide a solid base for further efforts in this field.

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