The Use of Composite Materials in the Production of Tower Cranes

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Abstract: This scientific article examines the use of innovative materials, such as composite materials, on lifting vehicles, in particular on a high-rise tower crane. Facilitation of structural components is a new field of research developed specifically for the production of special vehicles. The research work will begin primarily with identifying the dimensions of a tower crane using the example of the Chinese model QTZ 80A, which is used at many construction areas, then conducting the experimental part using software and the finite element method (FEM). The parameters of the research will be as follows: maximum deformation, maximum stress and bending of the boom of the tower crane. The materials that will be used in the research are as follows: classical steel and the following composite materials: carbon fiber and fiberglass. For the expediency of the research, the characteristics of composite materials in stress should be similar to classical materials. The experimental part of the research consists of comparing the parameters of the tower crane specified above, using different materials. The results of this study showed that a crane made of innovative composite materials and especially carbon fiber, has the lowest weight, i.e. about 20% of the weight of a classic steel crane, but has the same load capacity characteristics. The reliability of tower crane components made of composite materials is similar to the reliability of components made of steel.

Keywords: tower crane; lifting machines; composite materials; carbon fiber; fiberglass; innovations in machinery; SolidWorks; Finite element method

1 Introduction

Currently, there are two ways to improve the characteristics of special vehicles [4]. The first method is to increase engine performance, also by using electric traction, and the second, is by reducing the weight of the components. An area that is current and rapidly developing, is the integration of components, including any structural ones, specifically, through the use of composite materials [1]. Some very important parameters for this study and comparison with classical materials are the Young's Modulus, density and strength, they allow one aid decisions at the material selection stage. Engineers from all over the world, achieve various kinds of overall material characteristics, by combining them and then producing novel composite materials [2]. The research activities of researchers have gone further, since weight reduction plays a huge role in the construction sector, due to fuel economy, fast transportation of components and less time for assembling (disassembling) of a tower crane [8].

Classical methods of improving tower cranes, such as strengthening critical places are more reliable, compared to the method of using composite materials, because there, it is necessary to take into account various unpredictable phenomena. A matrix in composite materials refers to the material that binds together the reinforcement materials, such as fibers or particles, to form a composite structure [2]. One such phenomena, is the widespread often a problem of composite materials, the internal deformation of the matrix, especially locally. In fact, in the optimization process, in order to reduce the weight, the thickness of the component decreases, and therefore, the loss of stability becomes more important [3]. For example, one of the most important criteria in the design process of a tower crane boom, is the bending load [12]. Composite materials, with an optimal selection of composition, have a higher coefficient of resistance to bending [3].

The main purpose of this research is to compare classical materials with composite materials, for their variability and resistance to critical loads, bends in the design of the boom and tower of the crane [11]. Comparison of changes in these parameters for different materials used in the study. After conducting the experimental part of this study, which consists in building a model in the SolidWorks program and measuring the resultant strength characteristics, under known loads. The differences in displacement and stress of classical materials, compared to composite materials, will be clearly visible [13].

2 Hypothesis

Technical characteristics of the QTZ 80A tower crane illustrated in Table 1. The QTZ80A tower crane is a crane with a reinforced structure designed in accordance with GB/T5031 and GB5144, with a nominal load moment of 80000 kH*m and a maximum lifting capacity of 8 tons. This tower crane has a lifting capacity of 1.1 tons at a maximum boom reach of 60.2 m (Fig. 1) [10]. This is the maximum critical load on the boom structure of this tower crane. The tower crane is optimal in the price-quality segment and has all the necessary functionality in modern realities. Design and experimental calculations of this model, will allow for the creation of a methodological guide for optimizing other models of tower cranes, that follow the example of this research work. Fig. 1, shows a model made in SolidWorks.

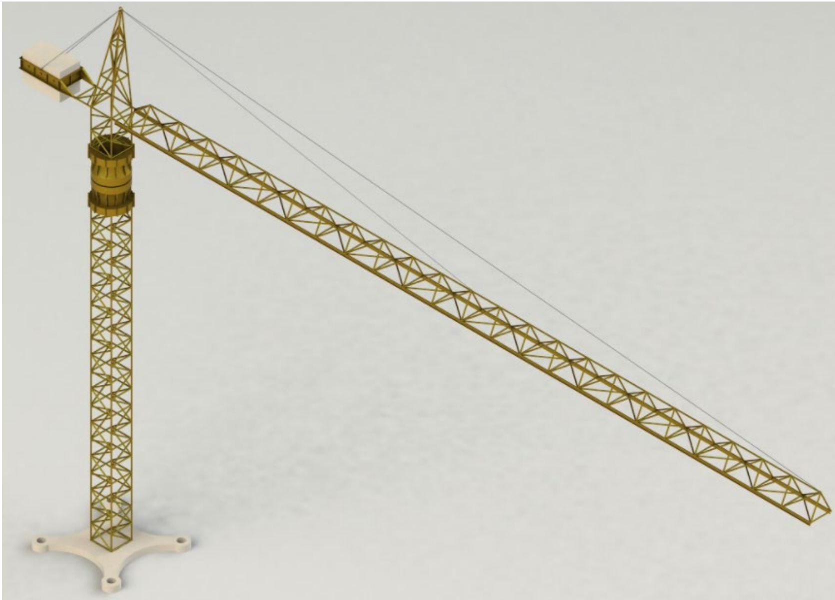

Figure 1
SolidWorks Tower crane model

Table 1
Technical characteristics of the QTZ80A tower crane

INDICATOR \ EXECUTION	**Changli** 5512
Maximum load moment	80000 kH*m
Maximum load capacity	8 t
Load capacity at maximum reach	1.1 t
Maximum reach	60.2 m
Lifting height of the free-standing crane	46 m
Maximum lifting height	150 m
Maximum lifting speed	20 m/min
The maximum lifting speed of the load	40 m/min
Speed of change of departure with cargo	10/30/60 m/min
Rotation speed, rpm	0-0.6 rpm
Weight of free-standing crane (without counterweight and ballast)	46.4 t
Weight of counterweight	15.5 t

2.1 Dimensions of the Boom of the Tower Crane and its Strength Calculation

The boom of the tower crane is the main power structure, which is a complex truss [10], with a counterweight at the other end. Each section of the boom is l = 3.5 meters, the height of the boom a = 1.84 meters and the width b = 2 meters (Fig. 2) [10]:

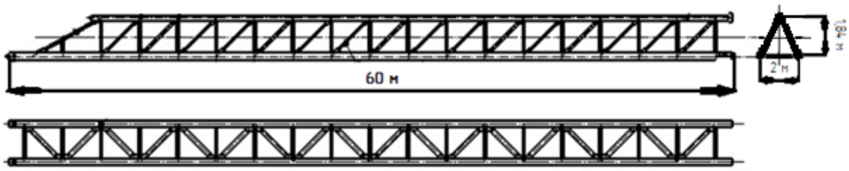

Figure 2
Boom of the tower crane

Using the deflection formula of the boom of a tower crane, it is possible to calculate the minimum value of the moment of inertia that the lever must have in order to meet the limit of permissible deviation [14]. Using the obtained value, using the tables of standard boom profiles, it is possible to determine the type and size that will be used in our study [11]. The moment of inertia formula looks like this:

$$I = \frac{F \cdot L_b^3}{3\varepsilon d_b} \quad (1)$$

where:

F – force or load acting on the boom (H); L_b – boom length (m); ε – Young's modulus; d_b – permissible deviation of the boom.

Based on the values obtained when calculating the moment of inertia, the following values can be obtained from the standard tables: cross-sectional area S_b and resistance R_b.

Further having density values ρ, beam length L_b and cross-sectional area S_b can find the weight of the arrow P_b:

$$P_b = S_b \cdot \rho \cdot L_b \quad (2)$$

Then calculate the weight per 1 meter of the boom structure:

$$p_b = S_b \cdot \rho \quad (3)$$

General deformation of the boom d_b it is the sum of three components: the deformation of the boom due to the load d_l, deformation due to the weight of the boom itself and the deformation that appears when shifting d_s [9]:

$$d_b = d_l + d_p + d_s = \frac{F \cdot L_b^3}{3\varepsilon I_b} + \frac{F \cdot L_b}{G S_b} + \frac{F_{p_b} \cdot L_b^4}{8\varepsilon I_b} \quad (4)$$

Based on the regulatory documents on the design and dimensions of the crane boom, the following value of the ratio of length to the total permissible deviation is given, which should be greater than or equal to 800 [8]. In order for the boom to comply with the restriction imposed on the bend, it is necessary to make sure that:

$$\frac{L_b}{d_b} \geq 800$$

Maximum total stress σ_b at the end of the boom is determined by the amount of stress from the load σ_l and from the weight σ_p the boom of the tower crane itself can be calculated using the following formula [8]:

$$\sigma_b = \sigma_l + \sigma_p = \frac{F \cdot L_b}{R_b} + \frac{F_{p_b} \cdot L_b}{2 \cdot R_b} \quad (5)$$

Next, it is necessary to calculate the critical load on the boom of the tower crane, which has a triangular shape in cross section [8]. To calculate the forces that bend the boom of a tower crane, it is necessary to find the cross-sectional area of the boom S_b:

$$S_b = \frac{a_1 \cdot b_1 - a_2 \cdot b_2}{2} \quad (6)$$

Next, calculated the geometric characteristics of the cross section of the arrow, the moment of inertia of the arrow J_b:

$$J_{bx} = \frac{b_1 \cdot a_1^3 - b_2 \cdot a_2^3}{24} \quad (7)$$

And the moment of resistance to bending of a hollow triangle W_b:

$$W_{bx} = \frac{b_1 \cdot a_1^3 - b_2 \cdot a_2^3}{2(6 \cdot a_1)} \quad (8)$$

From the strength condition , the permissible torque of the boom with a triangular section is as follows M_b:

$$[M_{bx}]_{[\tau]} = [\tau] \cdot W_{bx} \quad (9)$$

where:

$[\tau]$ – permissible tangential stress (Pa)

From the rigidity condition, the permissible torque for the boom section of the tower crane M_b the following:

$$[M_{bx}]_{[\theta]} = [\theta] \cdot G \cdot J_{bx} \cdot \varphi \quad (10)$$

where:

$[\theta]$ – permissible twisting angles

φ - load reliability coefficient

To find the permissible torque, it is necessary to know the modulus of elasticity under torsion, the formula is as follows G:

$$G = \frac{\varepsilon}{2(1+v)} \tag{11}$$

where:

v – Poisson 's ratio

It is necessary to find the torsion parameter β:

$$\beta = \sqrt{\frac{\varepsilon \cdot \pi^2 \cdot W_{bx}}{G \cdot J_{bx} \cdot L_b^2}} \tag{12}$$

The next characteristic is load parameter on the boom of the tower crane $E_{\square}$:

$$E_{\square} = \frac{2\mu \cdot \beta}{\pi \cdot a} \tag{13}$$

where:

$\mu = \frac{a}{2}$ – this is the place where the load begins on the cross section of the boom.

Further, based on the parameters found above, you can find the critical load F_b for a triangular-shaped cantilever beam with a concentrated load of 1.1 tons:

$$F_b = (11\left(1 + \frac{1.2E}{\sqrt{1+1.2^2E^2}}\right) + 4(\beta - 2)\left(1 + \frac{1.2(E-0.1)}{\sqrt{1+1.2^2(E-0.1)^2}}\right))\frac{\sqrt{EW_xGJ_x}}{L_b^3} \tag{14}$$

2.2 Dimensions of the Crane Tower and its Strength Calculation

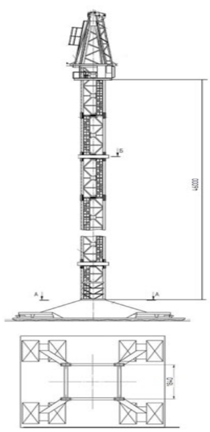

Figure 3
Tower of the crane

To determine the exact dimensions of the crane tower, used a rectangular shape [12] width a_t and length b_t which is 1.84 meters. The height of the tower is 46 meters h_t (Fig. 3). One end of the tower is rigidly attached to the ground by the foundation and a beam is rigidly attached to the other end, for more visual load parameters [12].

The total compressive force F and the moment M, due to the lifting weight of the tower itself, the weight of the load and the weight of the boom, can be calculated as [11]:

$$F_{tot} = F_t + F_p + F_b \tag{15}$$

$$M = L_b\,(F_b + \frac{F_P}{2}) \tag{16}$$

The moment of inertia for the crane tower will be calculated:

$$I_t = \frac{M \cdot L_t^3}{2\varepsilon d_t} \quad (17)$$

where:

M – moment of torsion that the beam exerts on the end of the tower (N*m); L_t – tower length (m); ε – Young's modulus; d_t – permissible deviation of the tower.

Having obtained the values of the moment of inertia I_t, resistance modulus R_t and cross-sectional area S_t crane tower, total weight P_t and weight per unit length p_t it can be calculated using the following formulas [9]:

$$P_t = S_t \cdot \rho \cdot R_t \quad (18)$$

$$p_t = S_t \cdot \rho \cdot 1m \quad (19)$$

Tower deformation caused by compressive force d_t, along the vertical axis is equal to:

$$d_t = \frac{F_{tot} \cdot L_t}{\varepsilon \cdot S_t} \quad (20)$$

While the displacement of the crane tower is transverse to the vertical axis:

$$d_M = \frac{M \cdot L^2{}_t}{2\varepsilon \cdot I_t} \quad (21)$$

Maximum total stress σ_t at the end of the tower is determined by the amount of stress from the load σ_F and from the weight σ_p the boom of the tower crane itself and the moment of inertia σ_m it can be calculated using the following formula [8]:

$$\sigma_t = \sigma_m + \sigma_p + \sigma_F = \frac{M}{I_t} + \frac{F_t}{S_t} + \frac{F_p}{S_t} \quad (22)$$

Since the crane tower is made of profiles made either from classical steel materials or from modern composite materials, they are also strongly influenced by bending during torsion, the crane tower, with a sudden sideways deviation, begins to twist out of the load plane. To avoid critical positions of the tower, it is necessary to calculate the maximum bending resistance in the same way as with the boom [12].

$$S_t = a_t \cdot b_t - a_{t2} \cdot b_{t2} \quad (23)$$

The next calculated the geometric characteristics of the cross section of the tower, the moment of inertia of the boom J_t:

$$J_{ty} = \frac{a_1 \cdot b_1^3 - a_2 \cdot b_2^3}{12} \quad (24)$$

And the moment of resistance to bending of a hollow rectangle W_t:

$$W_{ty} = \frac{a_1 \cdot b_1^3 - a_2 \cdot b_2^3}{6 \cdot b} \quad (25)$$

From the strength condition, the permissible torque of the tower with a rectangular cross section is as follows M_t:

$$M_{ty_{[\tau]}} = [\tau] \cdot W_{ty} \quad (26)$$

where:

$[\tau]$ – permissible tangential stress (Pa)

From the rigidity condition, the permissible torque for the tower section M_t is next:

$$M_{ty_{[\theta]}} = [\theta] \cdot G \cdot J_{ty} \cdot \varphi \quad (27)$$

where:

$[\theta]$ – permissible twisting angles; φ - load reliability coefficient;

To find the permissible torque, it is necessary to know the modulus of elasticity under torsion, the formula is as follows G:

$$G = \frac{\varepsilon}{2(1+v)} \quad (28)$$

where:

v – Poisson's ratio

It is necessary to find the torsion parameter β_t:

$$\beta = \sqrt{\frac{\varepsilon \cdot \pi^2 \cdot W_{ty}}{G \cdot J_{ty} \cdot L_t^2}} \quad (29)$$

The next find the load parameter on the base of the tower crane E_t:

$$E_t = \frac{2\mu \cdot \beta}{\pi \cdot a} \quad (30)$$

where:

$\mu = \frac{t}{2}$ – this is the place where the load begins on the cross section of the tower.

Further, based on the parameters found above, you can find the critical load F_t for a rectangular cantilever beam with a concentrated load of boom weight, ballast and cargo [9]:

$$F_t = (11\left(1 + \frac{1.2E}{\sqrt{1+1.2^2E^2}}\right) + 4(\beta - 2)\left(1 + \frac{1.2(E-0.1)}{\sqrt{1+1.2^2(E-0.1)^2}}\right)) \frac{\sqrt{EW_{ty}GJ_{ty}}}{L_t^3} \quad (31)$$

The indicators found above allow you to compare the results obtained during the experimental part. All acceptable values obtained theoretically show the difference. The SolidWorks software is the most optimal for solving this problem [13].

It is necessary to choose the right materials for high-quality calculation and add them to the algorithms of the program.

3 Material Selection

This study examines three types of materials: classical steel, fiberglass and carbon fiber composite materials. As steel, selected the classic model St45 [16], the following carbon fiber and fiberglass are used as composite materials (Table 2) [16].

The properties of carbon and glass fibers depend on the surrounding medium, raw materials and the process of creating these materials [6]. Composite materials in the modern world have become popular as the most effective lightweight material for their use in the design industry of heavy machinery [7]. Now engineers are working on reducing fuel consumption by facilitating the design.

Table 2

Material and its characteristics

Type of material	Young's modulus (MPa)	Poisson's ratio v	Shear modulus G (MPa)	Density p (kg/m3)	Tensile strength σR (MPa)	Fluidity σyi (MPa)	Compression strength Xc (MPa)
Steel 45	210,000	0.28	79,000	7800	550	241	/
Carbon Fiber	250,000	0.34	76,865	1600	1700	/	670
Fiberglass	87,000	0.32	32,954	2610	344	/	1240

Composite materials are widely used in the aerospace industry, which is an indicator for their use in other industries [4][5]. Composite materials have a high modulus of elasticity and a specific tensile strength. Due to its light weight and strength characteristics, composite materials are conquering all branches of machine building [6]. Composite materials have many advantages such as high strength at low weight, corrosion resistance and good insulation [2]. However, they also have disadvantages that can be significant in the production of cranes:

Repair difficulty: In case of damage, composite materials may be more difficult to repair. Traditional repair methods such as welding are often not applicable to composites, and special methods and materials are required [1].

Technological complexity: The production and processing of composite materials requires specialized equipment and knowledge. This may require additional staff training and investment in new equipment [1].

Although composites are resistant to corrosion, they may have problems with durability under fatigue loads. Cracks and delaminations that occur over time can lead to a decrease in structural strength. These factors make the choice of composite materials in crane manufacturing difficult and require careful cost-benefit analysis.

4 Experimental Section

The tower crane model was created using modern SolidWorks software [13]. It has a user-friendly interface for creating models and their strength calculation. After creating a model of a tower crane (Fig. 4) [14], experiments were carried out on the displacement and tension of the tower crane under load at the free end of the boom 11000 N (1100 kg), using the finite element method (FEM) [15], which divides the tower structure into a grid and calculates each cell as a separate section and subsequently combines data [17]. At the same time, the crane tower is rigidly attached to the ground with the help of a foundation. For simplicity of calculations, the boom was rigidly attached to the crane tower[2]. The rotation platform and the trolley of the tower crane were not designed in the drawing, as they are additional elements that load the program and complicate calculations [15].

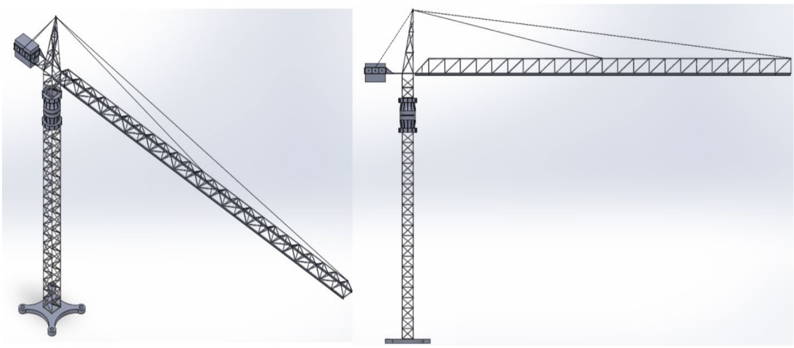

Figure 4
Tower crane in the SolidWorks program

The first material is classic steel of the St45 brand [16], it is shows below that the crane deviated from the initial coordinates with a load of 1.1 tons and a ballast weight of 15 tons by 53 centimeters, which is a good indicator (Fig. 5):

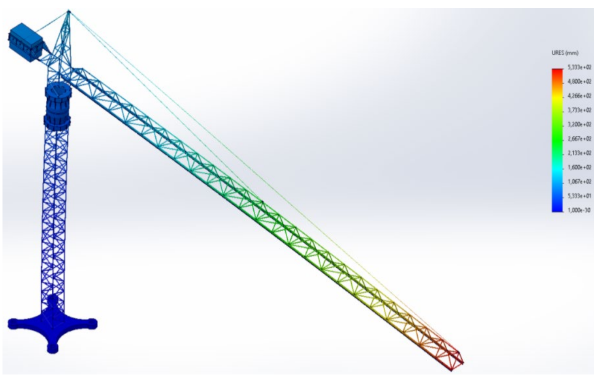

Figure 5
Displacement of a steel tower crane

The criteria that are taken to calculate the truss stress (Fig. 5) of a tower crane are based on the Mises-Hencky theory, also known as the theory of the energy of shape change. In the calculus of principal stresses $\sigma_1, \sigma_{2,}$ and σ_3, the Mises stress is expressed as [17]:

$$\sigma_{mises} = \sqrt{\left(\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}\right)} \quad (32)$$

This theory states that any plastic material is destroyed in critical places where the stress reaches the yield point. In order to accurately determine the yield strength, a standard temperature of +15 degrees Celsius was set.

In this drawing, the exceeded values are at the ends of the rods that hold the boom, other materials with a different yield strength are used for steel ropes. The average values for structural elements are about 45 MPa, which is the norm at 241 MPa of permissible stress.

The next material to be considered is a carbon fiber, the offset from the initial coordinates of the boom of a tower crane made of this composite material is 67.79 centimeters, which is acceptable (Fig. 6) [15]:

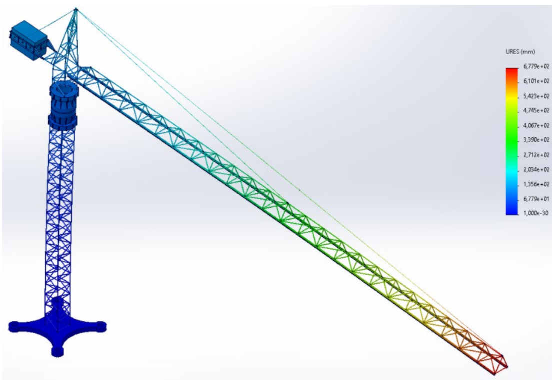

Figure 6
Displacement of a carbon fiber tower crane

To calculate the tension of a tower crane made of composite material, tensile strength indicators are required, since they do not have fluidity.

The last material used as an experiment for the construction of a tower crane is fiberglass, which has poor displacement indicators from the starting point, about 3.7 meters (Fig. 7):

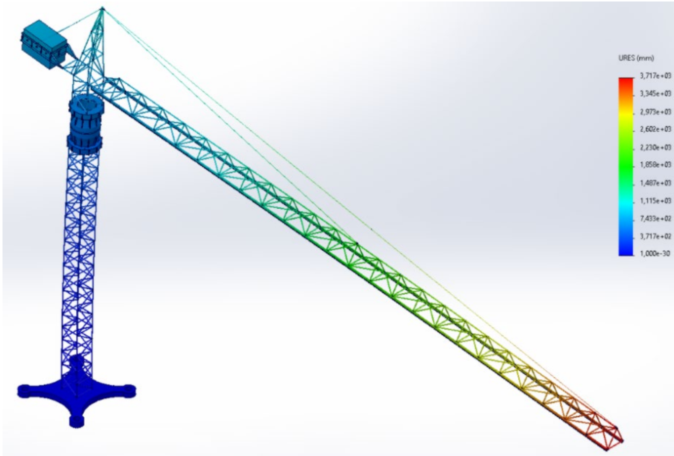

Figure 7
Displacement of a fiberglass tower crane

To calculate the tension of a fiberglass tower crane, tensile strength indicators are also needed, since they do not have fluidity. The average stress of the structural elements was 60 MPa, which is a tolerance with a tensile strength of 344 MPa. But this material showed poor indicators of displacement from the initial coordinates, which is critical, since it creates additional rolling, which increases the load on the structure.

After the results of this experiment, carbon fiber became the most optimal material for replacing classical steel, it has similar deformation indicators and resists tension well.

5 Results

From the results obtained, it can be noted that it is possible to significantly reduce the weight of the crane by using composite materials. For example, the weight of a steel tower crane [2] is 1000 kg, then with the same quantity of materials, based on the density of composites, the weight of a carbon fiber crane is 205 kg, and the weight of a fiberglass crane is 334 kg. As a percentage, compared to the heaviest steel crane, the weight of the carbon fiber crane is reduced by 79.5%, the weight of the fiberglass crane is reduced by 66.6%.

Another result obtained by us is that the structural limitations caused by the deflection of the beam are much more important than the strength caused by the applied load and critical loads.

Also, when choosing a material for creating a tower crane, it is necessary to take into account other environmental influences, that are not specified in this article.

After all, changes in temperature, wind speed, duration of work and fatigue of materials, which also greatly affect the strength of the structure and can have a detrimental effect.

After a thorough analysis of the materials, it is necessary to make calculations from an economic point of view. That is, to prove the feasibility of building tower cranes from composite materials. It is necessary to calculate the costs of production, for the purchase of materials. From the table below, it can be noted that a kilogram of carbon fiber is the most expensive on the materials market, but this is compensated by its volume, a kilogram of fiberglass has almost the same price as steel, but has poor technical characteristics (Table 3).

Table 3

Comparison of purchase price

Material	Price per kilogram of profile ($)	Weight (Kg)	Total price ($)
Steel	10	1000	10000
Carbon Fiber	72	205	14760
Fiberglass	15	334	5010

From this table, it can be observed that the price of carbon fiber per kilogram is 7 times more, and the price of fiberglass is 1.5 times more than steel. But if you take with the volumes necessary for the construction of a tower crane, carbon fiber will cost 1.5 times more expensive, and fiberglass is twice cheaper. From this it can be concluded, that when purchasing materials for construction, the costs will be insignificant.

Conclusions

The results of this study have shown that the introduction of modern composite materials in engineering industries, is an integral part of modern manufacturing. In particular, the tower crane, which has been made of classical steel for a very long time, was considered. The materials that have been proposed for modernization, are carbon fiber and fiberglass. These composite materials are the most popular in modern mechanical engineering. The Chinese tower crane QTZ80A was used as a model, which was loaded with 1.1 tons of cargo, at a boom height of 60 meters [10].

In the experimental section of this work, at nominally the same load of the tower crane, the material of its structure was changed, which changed the parameters of deformation and stress [3]. The experimental part showed that carbon fiber has similar parameters in terms of reliability as a steel tower crane, but weighs 5 times less than the classic one. Fiberglass, although it has a low weight, it also has a low tensile strength, which is inappropriate. The weight of the crane plays a big role, as savings during installation (dismantling) and transportation of the tower crane is very important. Also, the result obtained herein, show that the structural limitations caused by the deflection of the beam are much more important than the strength caused by the applied load and critical loads. In addition, the relationship between the deterministic safety factor and the reliability of components is highly nonlinear.

Table 3 showed that composite materials are expensive, but since these materials have a larger volume, they are used less when creating a tower crane [6].

Further research on this topic is needed, since other important components of the crane were not examined, such as engines, rotary platforms, trolleys, cables, etc. Also, the general economic assessment of the manufacture of a composite crane and its use, were not discussed.

Based on the results of the research, composite materials are the future for this industry, and with further study of this topic, proper funding, a significant technical progress can be achieved.

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