Integrated Open/Closed Queuing Serving System, Applied to Automotive Maintenance Workshop Capacity Optimization

Dragana Velimirović[1], Milan Marković[1] and Milan Velimirović[2]

[1] Academy of Applied Studies Polytechnic,
Katarine Ambrozić 3, 11050 Belgrade, Serbia,
dvelimirovic@politehnika.edu.rs; mmarkovic@politehnika.edu.rs

[2] Crossroad Adria, Bulevar Milutina Milankovića 23, 11070 Belgrade, Serbia,
velimirovic.milan@crossroadadria.rs

Abstract: The car service system, like any other modern system, aims to achieve business efficiency and optimal utilization of available capacities. This paper analyses the concept of a car service, with scheduled maintenance revenue and predictable intervention times. It primarily focuses on the implementation of preventive maintenance and corrective interventions, especially those within the warranty period. The car service system is considered an open mass serving system with a finite number of positions in waiting line queue as well as integrated open/closed queuing system. This study further contributes to the enhancement of operational efficiency through the application of mutual and non-mutual assistance principles in service procedures. In this context, the paper presents a mathematical model of such an approach and the ACADMSS software, developed on which it was based. The results obtained from the software application are illustrated in the diagrams presented herein. This research was inspired by the challenge of optimizing the capacities of logistics centers, in the goods supply industry, where numerous parameters must be considered. In relation to car servicing, the objective of this study is to introduce a novel approach, based on operational research, that aims to achieve capacity optimization, through a combination of open and closed serving systems. The software package was tested and optimized, using input data obtained from Porsche Belgrade Ada, the authorized service center for Audi, SEAT and Cupra, as well as Nikom Auto Belgrade, the authorized service center for Fiat.

Keywords: Integrated open/closed serving System; Automotive Maintenance Workshop Capacity; Service Optimization

Nomenclature

c	- number of lifts in the service
c_t	- number of technicians in the service
m	- number of places in the waiting line in parking lot
$t_{ser.}$ **[min.]**	- average time duration of the intervention on the vehicle

λ **[vehicles/hour]**	- frequency of vehicles entering the service
λ_1 **[vehicles/hour]**	- frequency of requests submitted by lifts to be served
P_{cancel} **[%]**	- probability of system being in a state of full occupancy
P_w **[%]**	- probability of the queue existence
$\bar{n}_w$	- average number of vehicles waiting for lifts in parking lot
$\bar{t}_w$ **[min.]**	- average time duration of vehicle waiting in the queue
$\bar{t}_{op}$ **[min.]**	- avg time of a vehicle spending in the open serving system
t **[min.]**	- time vehicle spends in the whole serving system
$t_{serv.}$ **[h]**	- servicing time duration
$\bar{c}_z$	- average number of occupied lifts
n_w	- average number vehicles waiting on the lifts to be served
T_w **[min.]**	- time of the vehicle waiting on the lift to be served
$T_{tot.w}$ **[min.]**	- total vehicle waiting time in the whole serving system
c_z	- average number of technicians that are busy
$P_{non\ busy}$ **[%]**	- probability that arbitrary chosen lift won't submit the necessity to be served

1 Introduction

The aim of this work is to present the application of an integral open/closed mass serving system on the optimization of a car workshop capacity. It is designed as a service for preventive maintenance and corrective interventions, primarily those within the warranty period. In such cases, clients primarily come within the scheduled maintenance time, when the time of the intervention is predictable. A mathematical model is presented for establishing all significant parameters of both, open and closed subsystems, accordingly. The result of the study is optimization of the installed capacities of the car service, in order to arise the business profitability of the company. Such strategy secures the maintenance cost reduction in car service by decreasing the time of vehicle maintenance operation, reducing an engagement of labor, and by a quality anticipation of future maintenance in its particular duration [1].

Predictability of the type and scope of maintenance operation that needs to be carried out, as well as realistic and fair work prediction of the client's vehicles encounter for the purpose of intervention, gives a space for better planning. This strategy is also appropriate from the car services users' point of view, considering that it ensures a significantly higher reliability of transport [2] [3]. This is achieved by applying the principles of preventive intervention and replacement of elements, assemblies and sub-assemblies during their life-cycle.

Queuing theory and integrated open/closed queuing system is applied to car workshop capacity optimization in the aspect of the total number of engaged technicians (electricians and mechanics) as well as the corresponding number of working positions [4]. The primary objective of the car workshop concept is to perform contracted maintenance during the warranty period. This approach is also applicable within logistics chains. To optimize such a system, it is essential to establish correlations between key parameters, including the inflow of trucks, the average intervention time, and the required human resources [1].

This article is based on premise of meeting the needs of vehicle manufacturers, distribution centers and authorized car workshop centers in order to enhance their efficiency. Rajuvar and Dignita's [5] Arena Simulation Software provided a robust foundation for further queuing systems analysis applied to automotive maintenance workshops and service centers. Srivastava [6] and Wallace [7] proposed appropriate criteria for evaluating the profitability of car workshops, specifically addressing whether investment is more beneficial in a car repair workshop or an automotive maintenance service. Their analyses consider factors such as flat-rate manual normative time and the assumption of a constant frequency of client arrivals at the service center.

The above-mentioned approach is suitable for commercial vehicles as well as for rail and air transport. According to that, favorable strategy of the car workshop organization should be predictive intervals of car incoming flow and durations of preventive maintenance and replacement of elements, based on total productive maintenance [8].

Hong et al. [9] and Grozev et al. [10] established the foundational premises for defining the appropriate frequency of incoming vehicles to the car service and the associated flat-rate manual normative time, particularly in the context of non-stationary conditions. Newman [11] conducted an analysis of queuing analytical models, specifically focusing on those with a finite number of waiting line positions. Based on the recommendations provided in the aforementioned studies and the authors' experience, this paper employs a Gaussian distribution for input data to ensure the reliability and relevance of the proposed approach.

As stated above, every organization strives to raise the efficiency of its operations. From the data analysis of the car service, it was determined that their owners monitor data related to the employment of technicians during the workday in order to upgrade efficiency. The most significant indicators are:

- Available hours representing the time spent at the working position that does not include the time provided for a break for a hot meal and rest breaks during work
- Working (effective) hours which represent the time employees spent engaged in work orders
- Invoiced hours, time spent on contracted work orders (invoices, at the expense of the warranty period, internal orders)

The goal of any automotive maintenance workshop is to utilize technicians during the available hours, and to charge the time they spent on the work. One of the ways to achieve this is to plan or schedule work operations in the car workshop. Technical efficiency, Labor Utilization and Overall productivity is often analyzed in that context. Technical efficiency refers precisely to the ratio between the number of hours of work that have been charged and those that the technician has spent on maintenance operations. Labor utilization is the ratio between the hours spent by the technician on maintenance operations and the hours spent on the job. Overall productivity is the ratio between the hours charged and those spent by the technician in productive work. As an example, 90% is average value of labor utilization in European countries, Velimirović D. et al. [12].

The advantage of scheduling maintenance operations in a car service is of interest for the workshop owner as well as for the clients benefit. Ideally, customers should not need to wait for an appointment. Figure 1 shows the morning and afternoon congestion recorded in Toyota car service, which creates the necessity for quick but effective acceptance and delivery of vehicles. Such congestions multiply the number of employees at the reception and delivery of vehicles [13]. In such way, the time spent with the client decreases, creating inadequate response to client's requests. Waiting for admission can be the reason for dissatisfaction among clients, Velimirović D. et al.

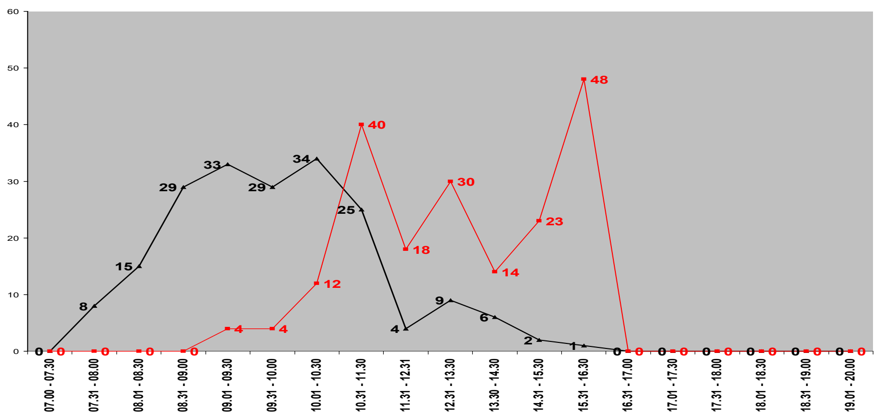

Figure 1
Unevenness of reception load during the working day [13]

From the aspect of the maintenance workshop organization, one of the biggest advantages of the scheduling process ensures the preparatory actions (preparation of documentation, preparation of parts and technicians) which affect more efficiently performances of maintenance operations. It is possible to know in advance what maintenance operations await technicians during the scheduled work; they are prepared for those operations, as well as spare parts, which reduce the waiting time for vehicle delivery.

2 Mathematical Model

This paper introduces an integrated open/closed queuing system, representing a hybrid approach that combines open and closed mass serving systems within the context of automotive maintenance. The car maintenance service is modelled as a multi-channel mass serving system with constrained queuing capacity, which serves as the foundation for the analytical framework [14] [15].

The improved approach presented in this study incorporates a closed system into the model and the software package, integrating it with the open system to enhance overall efficiency. The output from the open system represents the input to closed one, and the zone of interweaving of these two systems is the zone of workplaces where interventions on vehicles are carried out. The frequency of setting requests for servicing to technicians is established by equalizing the probability of finding both serving systems, open and closed one, in the state of none vehicle in the car workshop – none unit submits request for serving. Based on this premise, all the parameters of the closed serving system integrated with the open one are to be found, Figure 2. Here, Q represents flow of the vehicles entering the car service: $Q=Q(\lambda)$, $\lambda=1/t_d$, where λ represents average frequency of incoming vehicles flow to car service in represented period of flow sampling, while t_d represents interval between two successive vehicles entering the service in that case [16]. Number of lifts c is constant.

Mark m, Figure 2, represents total number of places on the parking lot, in the waiting line, and could be expressed by $m=g_1(\lambda, t_{serv.}, c)$, where $t_{serv.}$ represents average time duration of the intervention on the vehicle. On the other side, number of places in the queue m could also be expressed as the function of imposed probability that the vehicle will wait for acceptance in the service P_w, $m=g_2(P_w)$, Figure 2.

Mark λ_1, Figure 2, represents average frequency of submitting the request for serving by the vehicles on the lifts in closed serving system $\lambda_1=g_3(\lambda, m, t_{serv.})$. Number of technicians is expressed by $c_t=g_4(t_{serv.}, T_w)$, where T_w is the total time assessed, for vehicle waiting on the lift for serving. Marks g_1 to g_4 refer to dependences of the values in brackets.

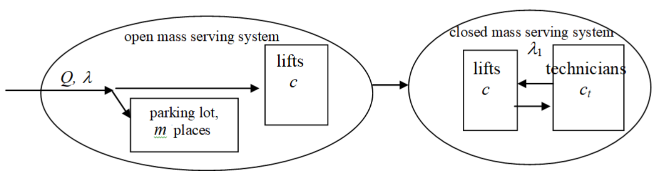

Figure 2

Coupling of open and closed queuing serving system (own editing)

2.1 Open Mass Serving System Applied to Car Service with Finite Number of Places in the Waiting Line

It is of great importance to establish the average time that elapses between two consecutive vehicles incoming [10], as well as, identical duration of the intervention on the vehicle approximately [17-19]. Output that has to be found out in analysis, including premise of non - simultaneous entry of vehicles in the system [20], is the optimization of:

- Frequency of vehicles entering the open car serving system
- Vehicle servicing time at the lift
- Number of lifts
- Number of technicians
- Number of parking places for vehicles to wait

Graph of the states in which the car workshop could be found in is represented by Figure 3. Open mass serving system is represented by vehicles entering car service consecutively with selected frequency λ, first line, Figure 3.

Such flow represents burdening of service capacity. Vehicles are getting access to the lifts or to the parking lot in the case when all the lifts are occupied. The next line on Figure 3 represents all states, in which the car service could be found depending on the number of lifts or places in the occupied waiting line. The third line on Figure 3 represents offloading of the service with ($c \cdot \mu$) intensity, when all lifts are occupied, to minimal intensity μ, when only one lift is occupied.

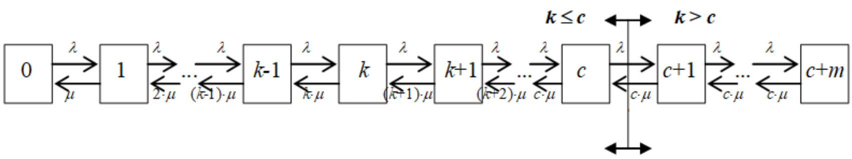

Figure 3
Graph of the states of the system with m places in the queue (own editing)

Such model of offloading represents case of mutual assistance absence. The marks on Figure 3 represent:

k	Number of units, vehicles, in the car service; it could be less or equal to number of car lifts, c: $k \leq c$, or greater than c: $k > c$, $k=k+r$, $r=1 \ldots m$
r	Number of vehicles waiting on the parking lot
c	Total number of lifts, channels in car service
$\boldsymbol{\lambda}$ **[1/h] = 1/t_d**	Frequency of incoming vehicles

t_d [h]	The average time elapsed between two consecutive cars entering the system
μ [1/h]=1/$t_{serv.}$	Frequency of completion of vehicle servicing per unit channel, frequency of car lift offloading
$t_{serv.}$ [h]	Average time duration of the intervention on the vehicle
$c \cdot \mu$ [1/h]	The maximum frequency regarding to offloading the car lifts from the ($c + m$) state to zero one when no vehicles are in car service

Probability that k vehicles take place in the car service could be found in cases:

- where $k \leq c$, $r = 0$
- where $k > c$, $r = k - c$, $r_{max} = m$

Parameters of importance for car service functioning are shown on Figure 3. Probability that k vehicles are in the service could be expressed in the form:

$$P_{k \leq c} = \frac{1}{k!}\rho^k \cdot P_0,\ r = 0, m = 0 \text{ and } P_{c+r} = \alpha^r \cdot \frac{\rho^c}{c!} \cdot P_0 = \alpha^r \cdot P_c, r > 0, m = 1....\infty \quad (1)$$

Here $\rho = \lambda \cdot t_{serv.}$ represents utilization intensity of a car lift. Indexes of probability P denote state of the car service, actually, number of vehicles in the car service. Probability that not a single vehicle is in car service P_0, is determined by the recurrent equation: $\sum_{i=0}^{c+m} P_i = 1, m = 0,1...\infty$, and equations (1). According to that and to the graph, Figure 3, this probability follows in the form:

$$P_0 = \frac{1}{\sum_{k=0}^{c}\frac{\rho^k}{k!} + \frac{\rho^c}{c!}\sum_{r=1}^{m}\alpha^r} = \frac{1}{\sum_{k=0}^{c}\frac{\rho^k}{k!} + \frac{\rho^c}{c!}\alpha\frac{1-\alpha^m}{1-\beta}}, \sum_{r=1}^{m}\alpha^r \rightarrow \alpha \cdot \frac{1-\alpha^m}{1-\alpha}, \alpha = \frac{\rho}{c} \quad (2)$$

Probability that number of the cars expressing the necessity for servicing is equal to the number of lifts, according to (2), is in the form:

$$P_c = \frac{\rho^c}{c!} \cdot P_0 \quad (3)$$

From diagram, Figure 4, it is clear that the number of vehicles waiting for servicing, n_w, increases by reducing the number of lifts, c. Dependence is presented as function of utilization intensity per lift ρ [21]. It is obvious that dependence changes from linear character, for the case of single lift car service, to exponential curve for car services with 10 lifts.

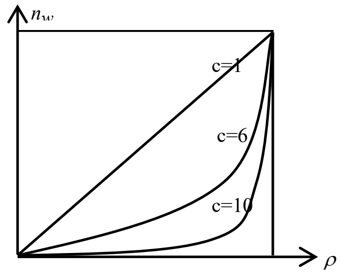

Figure 4
Dependence n_w of ρ, for various values of c, [22] and [23]

The probability of car service being in state when all car lifts are occupied as well as all places in parking lot, P_{cancel}, corresponds to the service being in the state of full occupancy. That probability must be in defined limits and is criteria of efficiency of the car service. If that probability is too low, car service capacity will be under utilized and vice versa. However, probability of serving possibility: P_{serv}=1-P_{cancel} corresponds to the state when, at least, one place in parking lot is unoccupied, according to (1) and (3):

$$P_{cancel} = P_{c+m} = \frac{\rho^{c+m}}{c^m \cdot c!} P_0 = \alpha^m \cdot P_c,\; P_{serv} = \sum_{k=0}^{c+m-1} P_k = 1 - P_{c+m} = 1 - \alpha^m \cdot P_c \qquad (4)$$

Probability of the car service being in the state when all lifts are occupied: $P_{tot.ch.oc}$, for m=const, is as follows:

$$P_{tot.ch.oc} = P_c + P_{c+1} + P_{c+1} + ... + P_{c+m} = \sum_{r=0}^{r=m} P_{c+r} = P_c \cdot \sum_{r=0}^{r=m} \alpha^r = P_c \frac{1-\alpha^{m+1}}{1-\alpha}, P_{tot.ch.oc/m\to\infty} \qquad (5)$$

$$P_{tot.ch.oc} = \frac{P_c}{1-\alpha}$$

The average number of occupied lifts could be found out by polynomial dependence, where the first term refers to conditions of incomplete car lifts occupancy and the other two conditions corresponding to full occupancy:

$$\bar{c}_z = \sum_{k=1}^{c} k \cdot P_k + c \sum_{r=1}^{m} P_{c+r} = \rho \cdot P_{serv} = \rho\left(1 - \alpha^m \cdot P_c\right) \qquad (6)$$

Probability of queue existence is expressed by:

$$P_w = \sum_{r=1}^{m} P_{c+r} = P_c \cdot \sum_{r=1}^{r=m} \alpha^r = P_c \cdot \alpha \frac{1-\alpha^m}{1-\alpha} \qquad (7)$$

That represents the probability that: $c + 1$, $c + 2$, $c + 3$... $c + m$, vehicles wait for serving. The average number of vehicles waiting in car service, time duration of their waiting and total number of the vehicles in workshop n_s are, according to (4) and (6), in the form:

$$\overline{n}_w = \sum_{r=1}^{m} r P_{c+r} = P_c \alpha \cdot \frac{1-\alpha^m \left(m \cdot (1-\alpha)+1\right)}{(1-\alpha)^2} = n_s - \overline{c}_z, \qquad \bar{t}_w = \frac{\overline{n}_w}{\lambda}, \qquad n_s = \sum_{k=1}^{c+m} k \cdot P_k \tag{8}$$

The average time duration of car lifts occupancy, as well as average time that vehicle has to spend in open serving system is in the form:

$$\bar{t}_{zk} = \frac{\overline{c}_z}{\lambda}, \quad \bar{t}_{op} = \frac{1}{\lambda}\left(\bar{n}_w + \overline{c}_z\right) \tag{9}$$

2.2 Closed Mass Serving System

A closed mass serving system applied to car maintenance service, Figure 2, is comprised of car lifts which submit requests to be served and technicians who serve them. Such system is defined by graph of states, Figure 5 [22-24]. It illustrates procedures of burdening and offloading car workshop capacity by transferring it from lower to higher burden states, and vice versa. Each state is defined by the probability that 1, 2 ... *c,* lifts submit serving request, each with a frequency λ_1. Time for the vehicle to be served by the technicians, is marked by $t_{ops.}$. The symbols on Figure 5 are:

- c_t total number of technicians servicing vehicles on the car lifts
- c total number of lifts in the closed serving system which express necessity to be served when the vehicle is on it, each with frequency: λ_1
- r number of lifts with car waiting for serving, r = 1..... (c_t - c)
- k number of vehicles, e.g. car lifts, which submit necessity to be served

The first line on graph, Figure 5, represents transferring system from lower to higher state, by decreasing intensity, from $c_t \cdot \lambda_1$ to λ_1. Next line, X_0, X_1, ... X_c, denotes all states in which system could be found, while the third line represents system offloading by varying the intensity from maximum $c_t \cdot \mu$ to μ when mutual assistance of the technicians does not exist.

Constant values of $c_t \cdot \mu$, in the last line, Figure 5, rely to intensity of offloading frequency from all *c* states to zero one, when mutual serving assistance of the technicians exists.

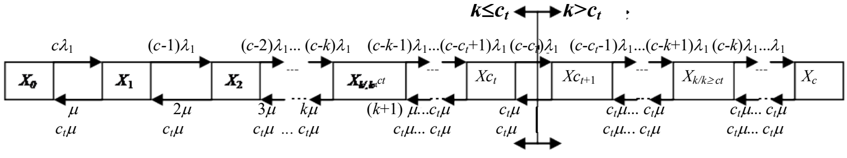

Figure 5

Graf of closed mass serving system (own editing)

Probability that k car lifts submit request to be served by technicians is in the form:

$$P_k = \frac{c!}{k! \cdot (c-k)!} \cdot \alpha_1^k P_{0_1}, \alpha_1 = \frac{\rho_1}{c_t}, k \le c_t, P_k = P_{c_t+r} = \frac{c!}{c_t^{k-c_t} \cdot c_t! \cdot (c-k)!} \cdot \alpha_1^k \cdot P_{0_1}, r \in (c_t+1, c) \tag{10}$$

In equations (10), $\rho_1 = \lambda_1 \cdot t_{serv.}$, represents intensity of each car lift utilization while $\alpha_1 = \rho_1 / c_t$ represents intensity of the whole closed serving system utilization. Estimated probability comprises all combinations of units, lifts, which submit necessity to be served in the same time:

$$\frac{c!}{k! \cdot (c-k)!} = \frac{c \cdot (c-1) \cdot \ldots \cdot (c-k+1)}{k \cdot (k-1) \cdot \ldots \cdot 1 \cdot (c-k) \cdot \ldots \cdot 1} = C_k^c = \binom{c}{k}. \tag{11}$$

Probability P_{01} that none car lift submits the necessity to be served is in the form:

$$P_{0_1} = \frac{1}{\sum_{k=0}^{c_t} C_k^c \rho_1^k + \sum_{r=1}^{c-c_t} \frac{c!}{c_t^r \cdot c_t! \cdot (c-c_t-r)!} \cdot \rho_1^{c_t+r}}, k = c_t + r, r \in (1, c-c_t) \tag{12}$$

Characteristic parameters of car workshop being explored in the optimization calculation, when the car workshop is considered as serving system closed type, are as follows:

- average number of technicians being busy c_z, is in the form:

$$c_z = \sum_{k=1}^{c} k \cdot P_k + c_t \cdot \sum_{r=1}^{c-c_t} P_{c_t+r} \tag{13}$$

- average number of vehicles on the lifts waiting to be served is as follows:

$$n_w = \sum_{k=c_t+1}^{c} (k-c_t) \cdot P_k = \sum_{r=1}^{c-c_t} r \cdot P_{c_t+r} \tag{14}$$

- total number of vehicles in closed system, according to (10)(12)(13), are in the form:

$$n = c_z + n_w = \sum_{k=1}^{c_t} k \cdot P_k + c_t \cdot \sum_{r=1}^{c-c_t} P_{c_t+r} + \sum_{k=c_t+1}^{c} (k-c_t) \cdot P_k = \sum_{k=1}^{c_t} k \cdot P_k + \sum_{k=c_t+1}^{c} k \cdot P_k$$

$$n = \sum_{k=1}^{c} k \cdot P_k, \tag{15}$$

- average number of technicians being unoccupied is as follows:

$$C_{free} = \sum_{k=0}^{c_t} (c_t - k) \cdot P_k = c_t - c_z, \tag{16}$$

- time for vehicle waiting on lift for service T_w, from (10)(13)(14), is in the form:

$$T_w = \frac{n_w}{c \cdot \lambda_1} = \frac{1}{c \cdot \lambda_1} \sum_{k=c_t+1}^{c} \left(k - c_t\right) \cdot P_k, \tag{17}$$

- probability that arbitrary chosen lift will not submit the necessity for serving $P_{nonbusy}$, due to (13) and (14), is as follows:

$$P_{nonbusy} = 1 - \frac{c_z + n_w}{c} = 1 - \frac{n}{c}, \tag{18}$$

- the total vehicle waiting time, due to (8) and (17), is in the form:

$$T_{tot.w} = \bar{t}_w + T_w \tag{19}$$

3 Car Service for Maintenance of Vehicles as an Integrated Open/Closed Queuing System

In this paper, an automotive maintenance car service is modelled as an integrated open/closed service system. It functions as an open service system in the sense that vehicles enter the system and may wait in the parking lot before receiving service. However, from the perspective of vehicles positioned on lifts, requiring servicing by technicians, the system operates as a closed one. Vehicles enter the system at a frequency of λ [1/h], Figure 6 [25-27], reflecting the arrival rate of incoming service requests.

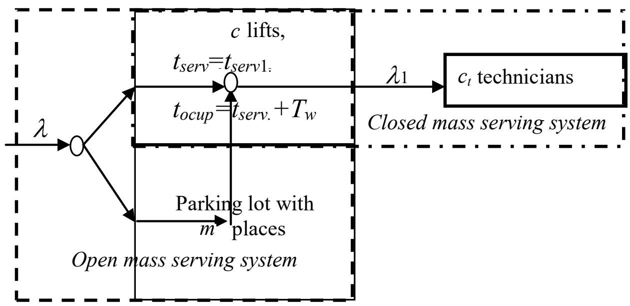

Figure 6

Integrated open/closed queuing system

Vehicles are in state of being served in the area of c working positions – lifts, or in state of waiting on total m parking places of parking lot, Figure 6. Total c lifts represent channels in open mass serving system, [28]. From the perspective of the car workshop, vehicles enter the system at a frequency of λ [1/h] to receive service at c lifts. Conversely, the working positions, represented by the lifts, generate requests for vehicles to be serviced by c_t technicians at a frequency of λ_1. Intervention time duration on the vehicles lasts, on average, $t_{serv1}=t_{serv.}$, and that time duration is less or equal to lift occupation time duration: $t_{ocup}=t_{serv.}+T_w$, where T_w represents the waiting time of the vehicle on the lift, Figure 6.The probability of state with none vehicle in the car workshop which is considered as an open serving system, (2):

$$P_0 = \frac{1}{\sum_{k=0}^{c}\frac{\rho^k}{k!}+\frac{\rho^c}{c!}\sum_{v=1}^{m}\alpha^r} = \frac{1}{\sum_{k=0}^{c}\frac{\rho^k}{k!}+\frac{\rho^c}{c!}\alpha\frac{1-\alpha^m}{1-\alpha}}, \quad \alpha=\frac{\rho}{c}, \rho=\lambda\cdot t_{serv}, \tag{20}$$

is equalized with probability that none lift expresses need to be served by technicians in the system of serving, considered as closed type, (12) and Figure 6:

$$P_{0_1} = \frac{1}{\sum_{k=0}^{k=c_t}\frac{c!\,\rho_I^k}{k!(c-k)!}+\sum_{r=1}^{c-c_t}\frac{c!\,\rho_I^{c_t+r}}{c_t^r c_t!(c-c_t-r)!}}, \rho_I=\lambda_I t_{serv}, k=c_t+r, \text{if } k<c_t \Rightarrow r=0. \tag{21}$$

The result of equating expressions (20), P_0 and (21), P_{01} is the frequency of requests expressed by each of the total of c lifts, according to the total of c_t technicians: λ_1, equation (22):

$$\frac{1}{\sum_{k=0}^{c}\frac{\rho^k}{k!}+\frac{\rho^c}{c!}\alpha\frac{1-\alpha^m}{1-\alpha}} = \frac{1}{\sum_{k=0}^{k=c_t}\frac{c!\,\rho_I^k}{k!(c-k)!}+\sum_{r=1}^{c-c_t}\frac{c!\,\rho_I^{c_t+r}}{c_t^r c_t!(c-c_t-r)!}} \Rightarrow \lambda_1. \tag{22}$$

With the obtained value of λ_1 all parameters of the car workshop considered as closed serving system are possible to explore in order to find out adequate capacities of it as a whole.

4 Analyzes of the Results Obtained by Application of Integrated Open/Closed Queuing Serving System

Based on the presented methodology, the ACADMSS software is developed at the Academy of Applied Studies Polytechnic, Serbia, derived in Quick Basic Compiler, and applied to data provided from the declared car workshops with capacity of fixed number of working positions, lifts, and number of places on the parking lot.

Concept of the software is based on independent open and closed serving system analysis. Input data for open serving system are frequency of units entering the system, serving time, number of places in waiting line and possibility of existence of mutual assistance of the channels in some percentage. In contrast, input data for closed serving system are number of units that submit requests for serving, number of channels that serve them and serving time. Output from module for open serving system is, partly, input for closed. Parameters that represent output from integrated open/closed serving system module are integrally achieved, some of them are listed in the following content. Software consists of modules and subroutines in order to form analyzes for: open, closed, integrated open/closed serving system and of modules for optimization of engaged resources in distribution of information, in transportation of passengers and goods, for passenger and cargo terminals, warehouses as well as for vehicle maintenance services.

4.1 Estimation of the Optimal Number of Technicians

In the first case, the implementation of the ACADMSS software package was carried out for the Porsche Belgrade Ada car service to optimize the number of engaged technicians, considering 12 car lifts and 20 parking spaces.

The selected vehicle arrival frequency is 3.75 vehicles per hour, while the average intervention duration per vehicle is 138 minutes. The optimal number of technicians ranges between 6.2 and 6.5 on average. This represents intersection of the curves expressing dependence on the average number of unoccupied technicians C_{free} (16) and the vehicle waiting time on the lift T_w (17) to the number of technicians, as illustrated in Figures 7 and 9. Simultaneously, for this number of engaged technicians, the average vehicle waiting time before being serviced on a lift is approximately 6 minutes, with approximately one vehicle typically waiting on a lift. The number of idle technicians remains below one, which is considered satisfactory.

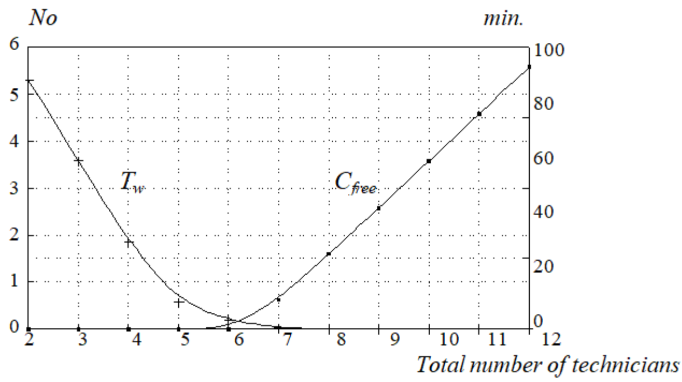

Figure 7

Number of unemployed technicians and Waiting time of the vehicle on lift to the Total number of engaged technicians in the case of mutual assistance

The calculations were conducted for two scenarios: one in which mutual assistance among technicians is implemented at a rate of 30% (Figures 7 and 8), and another in which mutual assistance is absent (Figures 9 and 10).

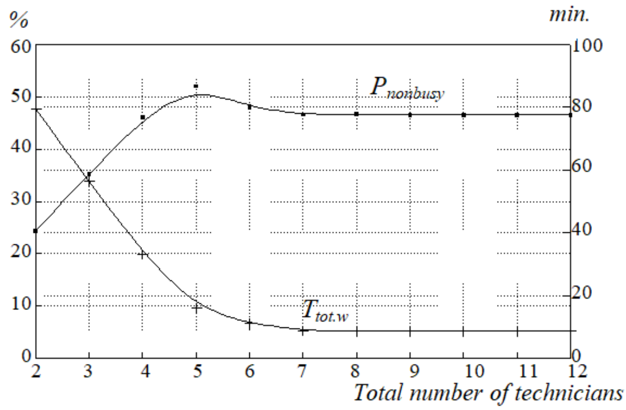

Figure 8

Probability of absence of necessity for serving and Total time waiting to Total number of technicians in the case of mutual assistance

Dependence on the probability that the arbitrary chosen lift is not occupied $P_{nonbusy}$ (18), to the number of engaged technicians, Figure 8, expresses the constant value corresponding to more than seven technicians engaged. In the same figure is notable descending curve representing dependence on the total vehicle waiting time $T_{tot.w}$ (19), to constant value less than 10 minutes, non-dependent on total number of technicians. In that zone total waiting time corresponds only to waiting time in the parking lot. Unlike the diagram in Figure 8, in the diagram shown in Figure 10 the probability that the arbitrary chosen lift is not occupied $P_{nonbusy}$ tends to a constant value of 46% for the number of engaged technicians greater than 10.

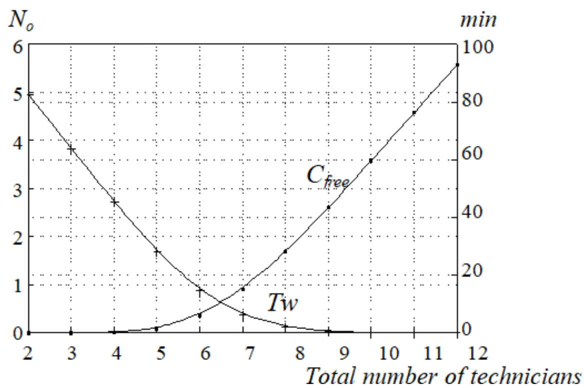

Figure 9

Number of unemployed technicians and Waiting time of the vehicle on lift to the Total number of engaged technicians in the case of mutual assistance absence

The intersection of the curves in Figure 9 corresponds to average 6.5 engaged technicians and to lower efficiency comparing to the case when mutual assistance exists, namely to 0.8 non busy technicians comparing to 0.2 in the case of existence of mutual assistance.

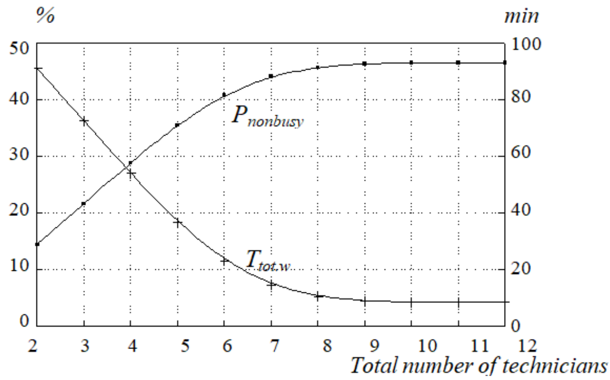

Figure 10
Probability of absence of necessity for serving and Total time waiting to Total number of technicians in the case of mutual assistance absence

4.2 Car Workshop Operation Parameters when Varying the Frequency of Vehicle Entering the Car Service

For second analysis, integrated open/closed mass serving system is used in order to establish the optimal frequency of vehicles entering to car service, adequate to installed capacity of 12 car lifts and 7 technicians while intervention duration time is 138 minutes. Mutual assistance of technicians is not taken into account. It is established that, by the aspect of closed serving system, optimal frequency of incoming vehicle is in the rate of 3.5 to 3.8 vehicles per hour. The authors established that waiting probability P_w and average number of vehicles waiting in the parking lot $\bar{n}_w$, raise rapidly with a of frequency >4 vehicles/hr, as in Table 1.

Table 1
Incoming vehicle variations frequency λ
c= 12 c_t= 7 m= 20 $t_{serv.}$= 2.3, λ_1= 0.5625, mutual assistance in rate: 0 [%]

λ	P_{cancel}	P_w	$\bar{n}_w$	$\bar{t}_w$	t	$\bar{t}_{op}$	$\bar{c}_z$	n_w	T_w	c_z	$P_{nonbusy}$
1	0.00	0.00	0.00	0.00	138.01	138	2.30	0.00	0.01	2.17	81.90
2	0.00	0.14	0.00	0.05	138.60	138	4.60	0.02	0.54	3.81	67.99
3	0.00	5.00	0.07	1.54	142.22	139	6.89	0.18	2.68	5.15	55.51
4	0.03	34.63	0.95	14.28	158.42	152	9.19	0.61	6.13	6.10	44.03
5	2.27	100	6.38	76.67	224.20	212	11.23	1.42	9.52	6.69	32.32

From the Table 1 it is visible that only 8 lifts of total 12 are engaged and total time for vehicle spent in service is about 150 minutes for proposed frequency of vehicles entering the car service. In this case average number of engaged technicians is 6 which is satisfying and enough close to the optimal result got by minimum of the envelope curve, Figure 10. According to that, authors consider that selected frequency of 3.75 vehicles per hour is acceptable but only for closed system. Varying of frequency λ is realized from 1 to 5 [vehicles/h] as presented on Figure 11 and Figure 12.

Diagrams on Figure 11 express probability of the case when all lifts and places in parking lot are occupied P_{cancel} in the rate of over 2% for high frequencies and regresses to zero value for frequencies less than 4 units per hour. Value of P_{cancel} is acceptable for proposed values of frequency of vehicle arrivals and servicing time. Based on the findings presented above, the installed capacity ensures that the number of vehicles waiting for lift in the parking lot, which has a capacity of 20 spaces, remains below 1 on average.

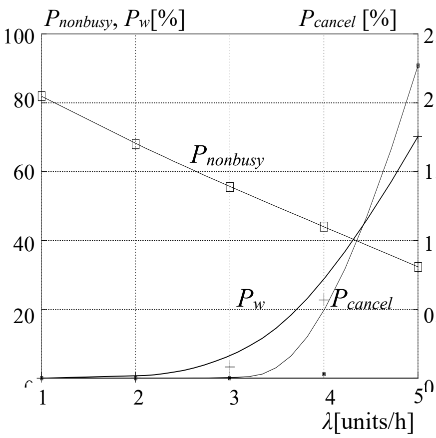

Figure 11
Probability of the case when all lifts and all places on parking lot are occupied P_{cancel}, of the case of queue existence P_w, and of the absence of the need for the serving on the lift and $P_{nonbusy}$, to frequency of vehicles requiring servicing

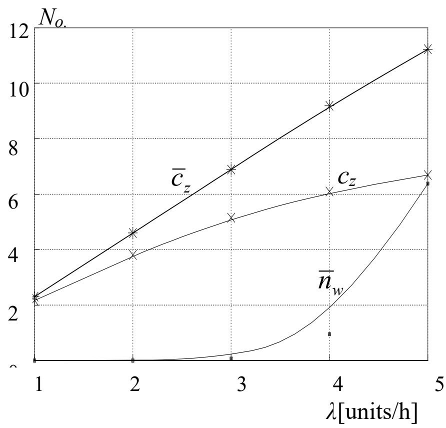

Figure 12
Number of vehicles waiting for lifts $\bar{n}_w$, No. of lifts being occupied $\bar{c}_z$ and No. of technicians being busy c_z, to frequency of vehicles requiring servicing

4.3 Car Workshop Operation Parameters when Varying the Servicing Duration Time

In the third case, the goal of implementation of the software package is to establish the adequate servicing time according to installed capacities relevant to 12 lifts and 7 technicians while selected frequency is 3.75 vehicles per hour. It is established that for proposed servicing time of 138 minutes total waiting time $T_{tot.w}$ and probability of the system being in the state of full occupancy P_{cancel} are too low, and rapidly increase when the servicing time surpasses 3 hours, Table 2. Dependences on the crucial parameters to servicing time in the case of frequency equal to 3.75 units per hour are presented by diagrams, Figure 13 and Figure 14. From diagram, Figure 13, it is obvious that, for proposed servicing time probability of the case when all lifts and places in parking lot are occupied P_{cancel} is too low, while the probability for queue existence P_w obtains value of only 20%.

Table 2

Servicing time variation frequency

c= 12 c_t= 7 m= 20 λ= 3.75, λ_1= 0.5625, mutual assistance in rate: 0 [%]

$t_{serv.}$	P_{cancel}	P_w	$\overline{n}_w$	$\overline{t}_w$	t	$\overline{t}_{op}$	$\overline{c}_z$	n_w	T_w	c_z	$P_{nonbusu}$
1	0.00	0.01	0.00	0.00	60.09	60	3.75	0.00	0.09	3.23	72.98
2	0.00	6.54	0.15	2.55	125.60	122	7.49	0.26	3.05	5.44	52.43
3	1.63	81.36	5.44	87.12	279.37	264	11.06	1.37	12.24	6.66	32.93
4	20.08	99.25	16.11	257.79	497.79	480	11.98	1.78	17.83	6.81	28.37
5	36.00	99.98	18.22	291.56	591.56	568	11.99	2.31	23.15	6.91	23.05

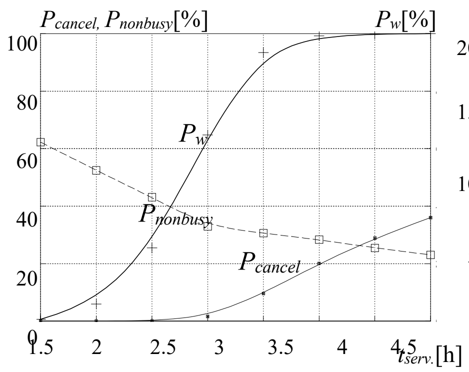

Figure 13

Probability of the case when all lifts and all places in parking lot are occupied P_{cancel}, of queue existence P_w, and of the absence of the need for servicing on the lift $P_{nonbusy}$, to servicing time duration

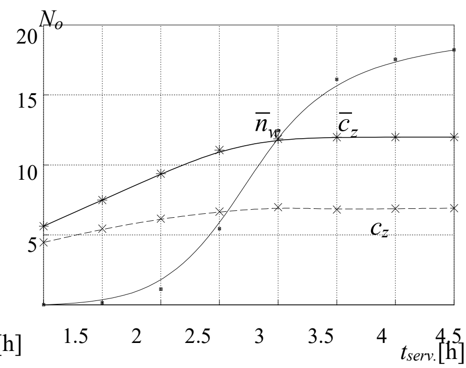

Figure 14

Number of vehicles waiting for lifts $\overline{n}_w$, No. of lifts being occupied $\overline{c}_z$ and No. of technicians which are busy c_z, to servicing time duration

Probability that arbitrary car lift will not submit request for serving $P_{nonbusy}$, regresses from 75% to 25%, obtaining approximately 45% for proposed servicing time equal to 138 minutes.

Curve that represents number of vehicles waiting for lifts $\overline{n}_w$, Figure 14, has cubic character and increases from zero to total number of places in the parking lot, 20.

Curve corresponding to number of lifts being occupied $\overline{c}_z$ increases strive to reach full capacity, 12 elevators, when the service maintenance time increases to 5 hours. The same trend is visible to number of technicians who are busy, c_z that increases to 7 units respectably. It is obvious that the value for c_z is approximately constant for servicing time greater than 3 hours, and reaches full capacity.

All the presented calculations demonstrate that the designed maintenance system is reliable and facilitates an efficient response to client requests, which is a key factor in ensuring customer satisfaction.

The validity of the calculations presented in the Paper has also been confirmed using data from Nikom Auto Centre Belgrade. Based on this verification, it has been proven that the calculations incorporating the proposed methodology are accurate and applicable to such optimization processes.

Conclusions

The concept of a superposed, integrated, open/closed serving system is developed and presented in this paper. Additionally, the method for realizing such coupling is described. Based on the developed model, the results can be summarized as follows:

The primary contribution of this paper is the introduction of a novel approach to automotive workshop capacity optimization through an integrated open/closed serving system. This approach employs the superposition of two serving systems by equating the probability of no service requests being generated in both the open and closed systems.

The ACADMSS software is formed with calculations based on the mathematical model presented in the paper and with the results illustrated in the accompanying diagrams.

The validity of the proposed methodology is confirmed by its alignment with recommended values for Labor Utilization parameters found in the cited literature, as well as through its application to optimizing the capacity of Porsche Belgrade Ada car service, which is the focus of this study.

The optimal number of technicians is determined for scenarios both with and without mutual assistance among technicians. From this perspective, the study explores the relationship between key parameters of automotive maintenance workshops, specifically as a function of vehicle arrival frequency and service duration.

The approach presented in this paper is not limited to vehicle maintenance facilities, but is also applicable to capacity planning in a wide range of industries, including, warehouse operations and various domains within mechanical, traffic engineering and management.

Acknowledgments

The authors gratefully acknowledge to Porsche Belgrade Ada (https://www.porsche-ada.rs/) and Nikom Auto Centre (https://www.nikomauto.rs/) for data got from their practice used to verify proposed model. Responsibility for the contents of the paper rests with the authors.

References

[1] Song W., Lei Z., Le Q., Li F., Wu J.: Maintenance Personnel Optimization Model of Vehicle Equipment Based on Support Task Mathematical Problems in Engineering, p. 13, 2021, https://doi.org/10.1155/2021/5547784

[2] Pophaley, M. & Vyas R.: Plant maintenance management practices in automobile industries: a retrospective and literature review. Journal of Industrial Engineering and Management, 3 (3), 2010, pp. 512-541, doi:http://dx.doi.org/10.3926/jiem..v3n3.pp. 512-541

[3] Arena F., Collotta M., Luca L., Ruggieri M., & Gaetano F.: Predictive Maintenance in the Automotive Sector: A Literature Review. Mathematical and Computational Applications, 27(1), 2022, pp. 1-21; https://doi.org/10.3390/mca27010002

[4] Srinivas R., Chakravarthy R. S., Dudin N. A.: A queuing model for crowd sourcing, Journal of the Operational Research Society, 68 (3), 2017, pp. 221-236, https://doi.org/10.1057/s41274-016-0099-x

[5] Rajuwar M. K. & Diganta K.: Simulation of Queuing System for Car Service Centre using Arena Simulation Software, International Journal of Production Engineering, 4(2), 2018, pp. 1-12, DOI:10.37628/IJPE

[6] Srivastava S.: Queuing Theory in Workshop, International Journal of Science, Technology & Management 4 (1), 2015, pp. 88-95, ISSN (online): 2394-1537

[7] Wallace R. B., D. N. & Prabhakar M.: Case studies in reliability and maintenance, Wiley-inter-science, 2002, ISBN: 978-0-471-41373-8

[8] Ahuja I. & Khamba J.: Total productive maintenance: Literature review and directions. International Journal of Quality & Reliability Management, 25 (1), 2008, pp. 709-756, https://doi.org/10.1108/02656710810890890

[9] Hong J, Kim B. & Oh S.: The Relationship Benefits of Auto Maintenance and Repair Car Service: A Case Study of Korea. Published online Behavioral sciences (Basel), 10(7):115, 2020, doi: 10.3390/bs10070115

[10] Grozev D., Milchev M. & Georgiev I.: Study the work of specialized car service as queue theory. Mathematical Modeling, 4 (1), 2020, pp. 31-34, WEB ISSN 2603-2929

[11] Newman E.: Relevance of the Queuing Theory to Car Serviced - Based - Organizations SSRN Product & Services, 2016, http://dx.doi.org/10.2139/ssrn.2757278

[12] Velimirović, D., Velimirović, M. & Stanković, R.: Role and Importance of Key Performance Indicator Measurement (KPI). Serbian Journal of Management, 6(1), 2011, pp. 63-72, doi:10.5937/sjm1101063V

[13] Velimirović M., Velimirović D. & Popović P.: Market and performance implications of fast fit car service concepts in automotive maintenance systems. Journal of Applied Engineering Science, 20 (1), 2022, pp. 285-292, DOI:10.5937/ jaes0-33637

[14] Petrovic V., Mrdak V. & Luković B.: Mass serving theory application to the analysis of maintenance system functioning. Military technical Courier, 61 (2), 2013, pp. 159-181, DOI: 10.5937/vojtehg61-2001

[15] Antonova, P. V.: The development of a multi-channel mass service system with limited queuing using the parallel library of the.net platform. Computational nanotechnology, 10 (3), 2023, pp. 44-50, https://doi.org/10.33693/2313-223X-2023-10-3-44-50

[16] Gautam N.: Analysis of queues: Methods and applications, CRC Press, 2012

[17] Zukerman M.: Introduction to queuing theory and stochastic telegraphic models. EE Department, City University of Hong Kong, 2016

[18] Taha H. (2017) Operations research: An introduction, Pearson Education, 10th Edition, 2017, ISBN 9780134444017

[19] Sztrik J.: Basic Queuing Theory. Globe Edit, Omni Scriptum GmbH, KG, Saarbrucken, Germany, University of Debrecen, Faculty of Informatics, 2016

[20] Adeyinka A. M., & Kareem B.: The Application of Queuing Theory in Solving Automobile Assembly Line Problem. International Journal of Engineering Research & Technology, 7 (6), 2018, pp. 344-351, DOI: 10.17577/IJERTV7IS060206

[21] Martyshkin A.: Study of distributed task manager mathematical models for multiprocessor systems based on open networks of mass servicing. Journal of Interdisciplinary Research, 8 (1), 2008, pp. 309-314

[22] Marković B. & Marković M.: Fundaments of Mass Serving Systems in Engineering, Academy of Applied Technical Studies Belgrade, Serbia, 2022

[23] Vukadinović S. (1975) Elements of Mass Serving Theory. Naucna Knjiga, Belgrade (in Serbian)

[24] Malmborg J. C.: Interleaving dynamics in autonomous vehicle storage and retrieval systems, International Journal of Production Research, 41 (5), 2003, pp. 1057-1069, https://doi.org/10.1080/0020754021000033887

[25] Dudin S., Kim C., Dudina O. &,1 Baek J.: Queuing System with Heterogeneous Customers as a Model of a Call Center with a Call-Back for Lost Customers. Mathematical problems in engineering, (Artical ID 983723), 2013, pp.1-14, https://doi.org/10.1155/2013/983723

[26] Fukunari M., Malmborg J. C.: A network queuing approach for evaluation of performance measures in autonomous vehicle storage and retrieval systems. European Journal of Operational Research, 193 (1), 2009, pp.152-167, https://doi.org/10.1016/j.ejor.2007.10.049

[27] Kuo P. H, Krishnamurthy A. & Malmborg J. C.: Design models for unit load storage and retrieval systems using autonomous vehicle technology and resource conserving storage and dwell point policies, Applied Mathematical Modelling, 31 (10), 2007, pp. 2332-2346, https://doi.org/10.1016/j.apm.2006.09.011

[28] Riechi J., Mácian V., Tormos B. & Avlia C.: Optimal fleet replacement: A case study on a Spanish urban transport fleet. Journal of the Operational Research Society, volume 68 (6), 2017, pp. 886-894, DOI:10.1057/s41274-017-0236-1